Shapes of Raindrop Size Distributions

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ABSTRACT

The "instant" shape of raindrop size distributions (measured during 1 min or less) usually differs from the exponential, generally in the direction of monodispersity. Experimental results are presented for both widespread and thunderstorm rain. It is shown that the measured shape depends significantly on the sample size, and that adding many "instant" distributions from different conditions leads to an exponential distribution such as proposed by Marshall and Palmer. This transition is examined, as well as the sample size needed for a well-defined shape.

1. Introduction

Many studies (e.g., Fujiwara 1965) in the past have demonstrated that "instant" raindrop size distributions (accumulated during 1 min or less) are often far from exponential. The concentration N of drops per unit volume and per diameter interval may easily deviate by an order of magnitude from the exponential distribution of best fit. On the other hand, the distribution formed by adding the number of drops in each size range from many of these "instant" distributions is in good agreement with an exponential distribution as has already been shown by Marshall and Palmer (1948).

In this work the above two rules are numerically substantiated and the transition of the shape of distributions measured during short and long periods is explored. With this in mind, the shape was parameterized according to the method described in the next section. Averaging shape factors, instead of the drops in the individual classes, leads to the average shape of "instant" samples as described in Section 3. By varying the timelength of the samples the influence of its duration on the shape is examined in Section 4. As a result we find that the sample size is an important factor when describing shapes of drop size distributions.

Because there are different ways of increasing the sample size, e.g., by a longer sampling interval or by increasing the area of the measuring device, the influence of various ways of averaging is investigated in Section 5.

2. Measuring the shape

The shape of raindrop distributions may quantitatively be described by means of related factors as shown by Joss and Gori (1976). These factors indicate the curvature of the distributions when plotted in log number versus linear diameter. Drop size regions in which the curvature is described may be chosen by selecting the integrals on which the specific shape factor is based. For example, in order to determine $S(Z\sigma)$, the curvature over the region of the drops which contributes most to the optical extinction coefficient σ and the radar reflectivity Z [as defined by Eqs. (1) and (2)], we first calculate the diameters $D(\sigma)$ and $D(Z)$ defined by (3) and (4). In the case of an exponential distribution these diameters belong to the drops of maximum contribution to σ and Z, respectively:[1]

$$\sigma(\text{mm}^2\ \text{m}^{-3}) = (\pi/4)\int_0^\infty N(D)D^2dD \qquad (1)$$

$$Z(\text{mm}^6\ \text{m}^{-3}) = \int_0^\infty N(D)D^6dD \qquad (2)$$

$$D(\sigma)(\text{mm}) = \int_0^\infty N(D)D^2dD \Big/ \int_0^\infty N(D)DdD \qquad (3)$$

$$D(Z)(\text{mm}) = \int_0^\infty N(D)D^6dD \Big/ \int_0^\infty N(D)D^5dD. \qquad (4)$$

The shape factor $S(Z\sigma)$ is defined by Eq. (5a), but since $D(Z)$ is equal to $3D(\sigma)$ in the distribution $N = N_0 \times\exp(-\Lambda D)$, independently of N_0 and Λ, (5a) may be

[1] See Appendix A.

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simplified to (5b):

$$S(Z\sigma)=\frac{\left|\dfrac{D(Z)-D(\sigma)}{D(Z)+D(\sigma)}\right|_{\text{observed}}}{\left|\dfrac{D(Z)-D(\sigma)}{D(Z)+D(\sigma)}\right|_{\text{exponential}}} \qquad (5a)$$

$$S(Z\sigma)=2\left|\frac{D(Z)-D(\sigma)}{D(Z)+D(\sigma)}\right|_{\text{observed}} \qquad (5b)$$

Following Joss and Gori (1976) we note the following:

$S(Z\sigma) \approx 0$ means that a monodisperse distribution is a suitable approximation of the drop size region which mainly contributes to the radar reflectivity factor Z and to the surface σ per unit volume.

$S(Z\sigma) < 1$ indicates that the drops responsible for the main contribution to Z and σ are closer together in diameter than in the exponential distribution.

$S(Z\sigma) = 1$ represents an exponential distribution of drops contributing to Z and σ.

$S(Z\sigma) > 1$ shows that the drops responsible for the main contribution to Z and σ are further apart in diameter than in an exponential distribution.

Because of limitations of the instrument [in these experiments the RD-69 disdrometer (Joss and Waldvogel, 1967) was used], the measured distribution is truncated both below 0.3 mm and above 5.5 mm diameters. A truly exponential distribution in nature and recorded within the above boundaries will result in a shape factor somewhat smaller than unity. This deviation from unity depends on upper and lower limits of the measuring range and on Λ of the exponential distribution. For example, for $5 > \Lambda > 1$ the shape factor $S(Z\sigma)$ of the above truncated exponential distribution would be $0.7 < S(Z\sigma) < 0.9$. Apart from $S(Z\sigma)$, which reflects the average shape of the distribution, $S(W\sigma)$, $S(R^*W)$ and $S(ZR^*)$ were calculated to give more detail about the shape. Here W stands for the liquid water content defined by (6) and the quantity R^* is closely related to the rain intensity defined by (7):

$$W[\text{mm}^3\ \text{m}^{-3}]=(\pi/6)\int_0^\infty N(D)D^3dD \qquad (6)$$

$$R^*[\text{mm}^4\ \text{m}^{-3}]=\int_0^\infty N(D)D^4dD. \qquad (7)$$

3. The shape of observed distributions

Since the shapes of "instant" distributions differ from those distributions accumulated during long intervals, examples of the shape characteristic of different sampling periods and different rain intensities will be presented in this section.

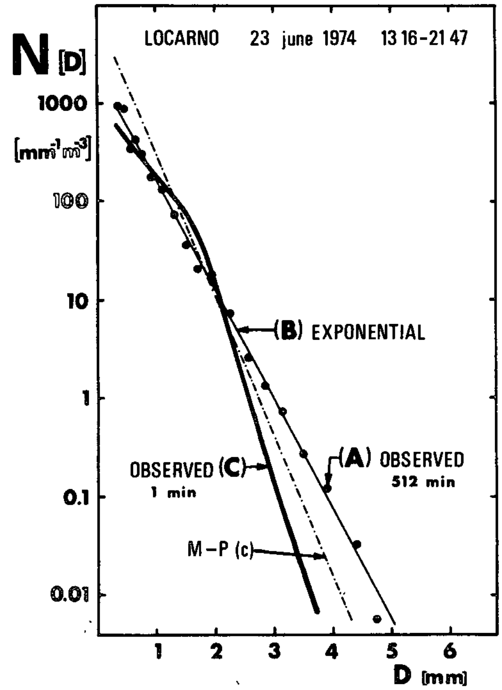

Fig. 1. Average "instant" shape (C) of 512 1-min distributions together with the single accumulated distribution (A) during the same 512 min, the best-fit-exponential distribution (B) and the Marshall–Palmer (M-P) distribution calculated for the same rain intensity (also see Table 1).

Widespread rain was recorded on 23 June 1974 during 512 min at the Osservatorio Ticinese (370 MSL), Locarno, Switzerland, with the RD-69 disdrometer. If all the drops in each drop class of this rain are added the resulting distribution is shown by the points A in Fig. 1. Parameters of this distribution are given in Table 1. These results are in excellent agreement with the exponential distribution B of the fit given by $N = 2080\exp(-2.55D)$, but differ from the Marshall–Palmer (M-P) distribution c where N_0 is set to 8000 $\text{m}^{-3}\ \text{mm}^{-1}$ and $\Lambda = 4.1R^{-0.21}$ (observed rain intensity R=2.9 mm h^{-1}. The following procedure was used to derive the average "instant" shape C in Fig. 1:

- Parameters of 512 individual 1 min samples were calculated.
- The average of each parameter was calculated.
- The retransformation of the average parameters to a distribution $N(D)$ was calculated (see Appendix B).

The results presented in Fig. 1 and Table 1 confirm, for a case of continuous widespread rain, the two rules stated in the Introduction:

1) The shape of a sample accumulated during a long period (in this case 8.5 h) is very close to exponential as shown by the agreement of parameters of A and B in Table 1.

TABLE 1. Basic and derived parameters pertinent to drop size distributions, specifically including the diameters of maximum contribution and shape factors relevant to Figs. 1 and 2. As the distributions are truncated by the instrument below 0.3 and above 5.5 mm, the shape factors of a true exponential distribution (M-P) is less than 1. The derived parameters are to be inserted in Eq. (15) of Appendix B to obtain the drop concentration $N(D)$. M-P is Marshall-Palmer.

	Curve	Basic parameters												Derived parameters				Type
		σ	W	R	Z	$D\sigma$	DW	DR^*	DZ	$S(W\sigma)$	$S(R^*W)$	$S(ZR^*)$	$S(Z\sigma)$	Λ	N_0	C	X_0	
FIG. 1 Widespread rain	A	188	153	2.9	2 088	0.90	1.22	1.59	2.31	0.75	0.92	0.93	0.88					Observed (512 min)
	B	189	153	2.9	2 102	0.92	1.22	1.58	2.33	0.71	0.90	0.97	0.87	2.55	2 080	0.00	0.0	Retransformed
	—	188	153	2.9	1 288	0.98	1.22	1.46	1.82	0.48	0.56	0.52	0.55					Observed (1 min)
	C	198	160	2.9	1 191	0.99	1.22	1.41	1.74	0.50	0.53	0.51	0.55	3.20	10 100	0.25	5.5	Retransformed
	c	328	213	3.4	1 408	0.76	0.98	1.24	1.83	0.62	0.84	0.96	0.83	3.28	8 000			M-P ($\Lambda = 4.1 \times 2.9^{-0.21}$)
FIG. 2 Thunderstorm rain	—	88	63	1.0	313	0.85	1.07	1.26	1.49	0.47	0.46	0.36	0.47					Observed (1 min)
	D	89	62	1.0	266	0.88	1.05	1.20	1.42	0.44	0.46	0.43	0.47	4.10	14 240	0.40	6.0	Retransformed
	d	159	86	1.2	296	0.66	0.81	1.01	1.47	0.53	0.76	0.93	0.77	4.10	8 000			M-P ($\Lambda = 4.1 \times 1^{-0.21}$)
	—	538	501	10.0	6 053	1.06	1.40	1.67	2.08	0.67	0.59	0.45	0.61					Observed (1 min)
	E	541	501	10.0	6 061	1.11	1.39	1.65	2.07	0.57	0.59	0.57	0.61	2.70	14 369	0.18	5.5	Retransformed
	e	745	610	11.6	8 580	0.92	1.23	1.59	2.35	0.72	0.90	0.96	0.87	2.53	8 000			M-P ($\Lambda = 4.1 \times 10^{-0.21}$)
	—	2460	3690	100	237 000	1.72	2.24	2.81	3.59	0.65	0.65	0.66	0.70					Observed (1 min)
	F	2550	3824	100	209 000	1.72	2.25	2.71	3.42	0.67	0.65	0.58	0.66	1.53	9 615	0.13	5.0	Retransformed
	f	3260	4173	103	205 700	1.37	1.92	2.49	3.45	0.83	0.90	0.81	0.86	1.56	8 000			M-P ($\Lambda = 4.1 \times 100^{-0.21}$)

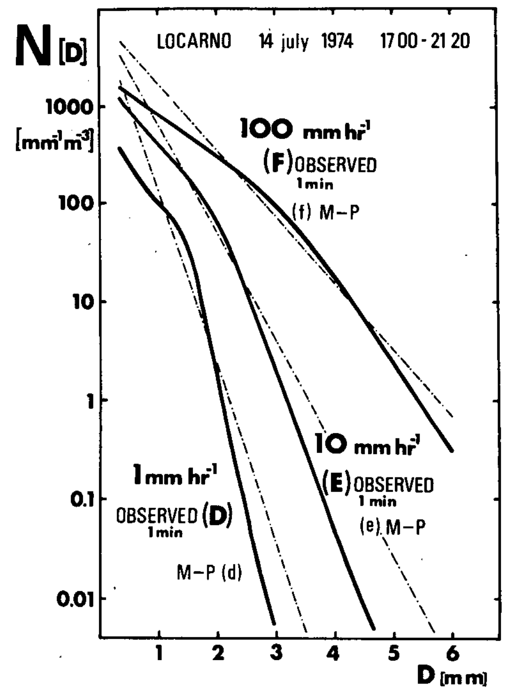

FIG. 2. Average "instant" shape of distributions having 1, 10 and 100 mm h^{-1} rain intensity together with the exponential distributions given by Marshall and Palmer (1948) for the corresponding intensity (also see Table 1).

2) The typical shape of a short period ("instant" sample), here taken as the average shape of 512 1 min samples representing exactly the same data as above, deviates markedly from exponential. This can be seen by the disagreement of shape parameters of C with c, A and B in Table 1.

These findings are not peculiar to widespread rain, as shown by Fig. 2 for a thunderstorm recorded on 14 July 1974 at the Osservatorio Ticinese (some data of this storm have previously been presented in Joss and Gori (1976). A total of 256 1-min samples covering weak rain as well as peaks of over 100 mm h^{-1} were analyzed using multiple-polynominal regression techniques. The average shapes at 1, 10 and 100 mm h^{-1} are plotted in Fig. 2, after retransformation as described in Appendix B. The related parameters may be found in Table 1 which also contains the corresponding M-P distributions for comparison. For both these data and for those from the earlier case of widespread rain, we find the following main results:

1) At all rain intensities the average shape of the "instant" distribution deviates from exponential toward monodispersity. In other words, there are markedly less small and large drops than predicted by the M-P distribution or by the best-fit exponential distribution.

2) This deviation is more pronounced at low rainfall rates than at large ones.

4. Duration of sample and shape of distribution

In order to examine how the shape of a distribution depends on the length of time during which the distribution was accumulated, we analyzed the 512 1 min samples of the widespread rain which were discussed in the previous section. Those data were analyzed for ten different intervals; that is, the average shape factor $S(Z\sigma)$ of 512 1 min samples was first calculated, then the average of 256 2 min samples was calculated, etc., until at least the single-shape factor of the total 512 min sample was determined. This way exactly the same data were analyzed each time, the only difference being the length of time during which the drops were accumulated. The results in Fig. 3 indicate that the shape is indeed influenced by the sample size; the instantaneous shape is between monodisperse and exponential, and changes to a shape which is very close to exponential when all the drops of a whole rainfall are accumulated in a single distribution. The shape factor of the true exponential best-fit distribution (truncated at the drop size limits of the instrument), yields a $S(Z\sigma)=0.87$ for B, which differs insignificantly from $S(Z\sigma)=0.88$ for the measured distribution A. Whether longer storms would lead to correspondingly higher shape factors is yet to be answered. None of the shorter storms exceeded the shape factor of the corresponding exponential distribution. All yielded, the same trend (monodisperse toward exponential), however.

This result is not a characteristic of the widespread rain alone, as demonstrated by the results of similar analyses for 256 min of thunderstorm data, also given in Fig. 3.

5. Ways of averaging

In Section 4 we discussed the influence of increasing sample size by adding drops of samples contiguous in time. This is equivalent to increasing the sampling duration and we will call it Case a. In order to examine how else the sample size influences the shape, three other ways of increasing the sample size are considered as follows:

Case b. Samples of similar rain intensity are combined (influence of rain intensity variations minimized).

Case c. Samples taken at random are combined (influence of the autocorrelation in the time domain minimized).

Case d. Samples taken at the same time and location but on different instruments are combined (influence of sample size only), all other possible variations minimized).

The same data as in Case a, but differently arranged, were used in Cases b and c. In Case d additional data (more disdrometers) were used in order to ascertain that findings described here were not due to insufficient sample sizes. Limited sample sizes will introduce fluctuations in numbers of drops of different classes and might falsely indicate a significant tendency toward monodisperse distributions.

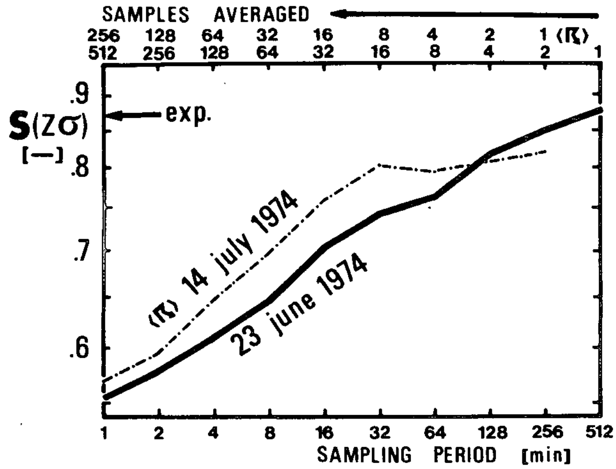

FIG. 3. Variation of the average shape $S(Z\sigma)$ as a function of the sampling period for 512 min widespread rain and 256 min of thunderstorm rain. As the distributions are truncated by the instrument below 0.3 and above 5.5 mm, the shape factors of a true exponential distribution is less than 1.

Case b: Contiguous samples of rain intensity

The 512 1 min samples of the widespread rain were arranged in increasing order of rain intensity. The first sample represented the weakest rain, the last one the heaviest. These data were again analyzed in ten different ways (as in Section 4), but here the number of samples contiguous in rain intensity, where drops were added, was increased each time by a factor of 2. The parameters of these distributions were calculated and averaged ten times, each time yielding a single value in Fig. 4. Case a of Fig. 3 is repeated for comparison.

The starting and ending values of both curves, corresponding to sampling periods of 1 min and of 512 min, must coincide because the arrangement of samples has no importance for these cases. As an example, point P of Fig. 4 indicates an average shape factor $S(Z\sigma)=0.65$ for 128 distributions, each one consisting of four 1 min samples of similar rain intensity. The first distribution contains the 4 min of weakest intensity, the 128th consists of the four heaviest. Only a small fraction of the total rain intensity change is spanned by any of the four distributions which were added together.

If the shape depended on rain intensity alone, the dotted curve would be nearly parallel to the abscissa for small sampling periods, since adding drops and dividing by 4 in each class would have little influence on the shape. As the contrary is true and the curve for samples contiguous in rain intensity rises more rapidly than for samples contiguous in time, we must

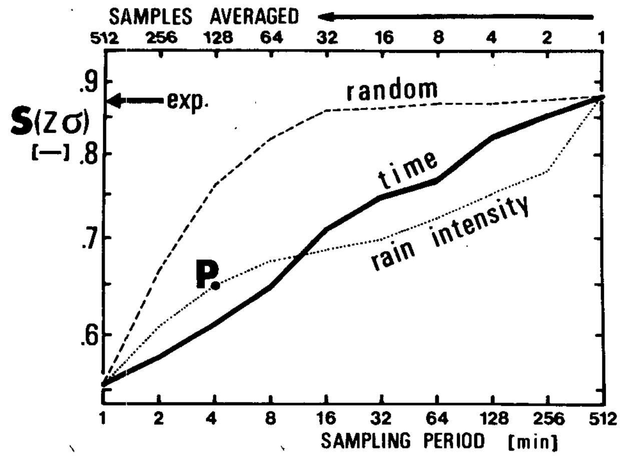

FIG. 4. Variation of the average shape $S(Z\sigma)$ as a function of the number of minutes added (sampling period) for 512 min of widespread rain of 23 June 1974: "time," samples arranged as recorded; "rain intensity," same 512 1-min samples rearranged in sequence of increasing rain intensity before analyses; "random," same 512 1-min samples rearranged at random before analyses. As the distributions are truncated by the instrument below 0.3 and above 5.5 mm, the shape factors of a true exponential distribution is less than 1.

conclude that the shape varies less between samples successive in time than at a specific rain intensity. This is true for small numbers of contiguous samples. When adding between 8 and 16 contiguous samples the two curves cross while for larger numbers of contiguous samples $S(Z\sigma)$ increases less when the samples added are contiguous in rain intensity. The character of the two curves suggest that, at any fixed rain intensity, the average shape will never reach exponentiality [$S(Z\sigma)\sim 0.7$] and that true exponential distributions are obtained when adding *many* 1 min samples of *different* rain intensity.

Case C: Samples taken at random

In both cases analyzed so far, the added samples—contiguous either in time or in intensity—were not mutually independent. To answer the question of how the shape would change without this correlation, a pseudo-random series was constructed by taking the samples in the following order: 1, 33, 65, . . ., 481; 2, 34, 66, . . ., 482; 3, 35, 67, . . ., 512. Assuming that after 32 min the storm had decorrelated, as shown by the rain intensity autocorrelation coefficient $r=0.27$, the arrangement obtained in this way was equivalent to selecting samples at random but was much easier to handle on a small computer. Once this random series was obtained, the analyses were carried out exactly as in the previous cases and the results plotted in Fig. 4. At small sampling periods this curve shows the steepest increase of the shape factor, indicating, as expected, that individual samples added were less well-correlated, since the tangent at the origin provides some measure of the decorrelation between the samples added.

Case d: Samples taken at same time and location

In all cases described, increases of sampling periods were accompanied by proportional increases of total drop numbers and were therefore statistically better defined samples. Thus the question arises, whether this effect influenced the results previously discussed and if so, how well the shapes of 1 min samples are defined. To answer this question, four disdrometers, having surfaces of 50 cm², were mounted within 1 m of each other during widespread rain. The data from each disdrometer were then analyzed as in Section 4. Results corresponding to the heavy line in Fig. 3 were obtained from all four instruments and the values obtained coincided within 1%, indicating that the instruments measured the same data. Drops pertaining to each minute and to the four instruments were then added, thus simulating data of a "200 cm² disdrometer." Those data were then analyzed according to the method of Section 4.

For a 1 min sampling period $S(Z\sigma)$ of the 200 cm²-disdrometer was but 3.5% larger than that of the four 50 cm² disdrometers analyzed individually. This difference was reduced to 1.8% for a 2 min sampling period and was not detectable for larger sampling periods. These results show that limited sample sizes in fact impose a tendency toward monodispersity, but the effect of the order of 3.5% on $S(Z\sigma)$ for a sample size of 0.3 m² s^{-1} is small and hence the shape is considered to be well defined.

COMPARISON OF RESULTS

Table 2 shows the percentage increase I of the shape factor $S(Z\sigma)$ when the sample size (product of area F and exposure time T) is increased four times using the four different ways previously discussed. In each case the sample size and with it the total number of drops analyzed is increased from 0.3 to 1.2 m² s, thus reducing the fractional standard deviations given in Table 3 of Appendix A by a factor of 2. Table 2, however, does not show the fractional standard deviation, but the bias produced by the change of sample size. The samples taken at random (Case c) demonstrate that there are strong changes during the storm between distributions, which when added, result in a pronounced tendency from the curved shape of "instant" distributions toward exponential ones. Distributions of similar

TABLE 2. Percentage change of the shape factor $S(Z\sigma)$ for an increase of sample size of a factor of 4 for four different ways of contiguity.

Case	Contiguous in	F (m²)	T (s)	I (%)
a	Time	0.005	60 to 240	10.3
b	Rain intensity	0.005	60 to 240	17.4
c	Random	0.005	60 to 240	37.8
d	Space	0.005 to 0.02	60	3.5

rain intensity (Case b) show less correlation than do successive distributions in time (Case a), thus indicating greater similarity within a precipitation cell than between adjacent cells. The agreement between distributions recorded simultaneously 1 m apart (Case d) shows that if a small bias of $\sim 3.5\%$ is acceptable, a sample size of only 0.3 m^2 s is sufficient to define the shape of a distribution.

6. Conclusions

1) "Instant" distributions, i.e., distributions accumulated during 1 min or less, deviate strongly from exponential distributions in the direction toward monodispersity. This is the case for widespread as well as for thunderstorm rain. This result bears directly on cloud models in which the dispersity of the raindrop distribution enters as an important factor, since collection and breakup rate are proportional to the difference in fall velocity between particles. For the narrower "instant" distributions these rates are reduced as compared to the exponential one.

2) Exponential distributions are found when adding *many* "instant" distributions from *different* conditions. Therefore, the approximation of real distributions by exponential ones will be adequate in all applications where an average over time or space is relevant, such as in radar where the average over a large pulse volume is taken, or in applications involving attenuation, where the average distribution over a path is relevant.

3) The tendency of the shape from the typical "instant" shape to the exponential one of the sum-distribution is more pronounced, if the correlation coefficient of individual distributions is small.

4) When comparing two sum-distributions, the first made up from a given number of "instant" distributions recorded together (same time → same raincell) and the second one made up of the same number of "instant" distributions with similar rain intensity (in general from different raincells) we find the following results:

- When up to 8 samples are added, the sum-distribution, containing the samples recorded at the *same time*, is less exponential. This points up the short-time continuity of the shape within raincells.
- When more than 16 samples are added, the sum-distribution containing the samples with similar rain intensity, is less exponential. This indicates the existence of a typical shape for a given rain intensity.

5) The shape of raindrop distributions can be adequately defined by the proposed shape factors. An alternative method which has been used in the past is to fit an exponential to the data by least-squares regression and examine the size of the residual. The use of the shape factors proposed here has the following advantages over the exponential method:

- A measure of the amount of curvature is obtained rather than simply a statistical measure of the departure from a straight line.
- With only simple calculations the data are already weighted in the particular region of interest Without weighting, the least-squares fit would give undue significance to the extreme ends of the diameter range.

In comparison to the $B(n)$ values proposed by Atlas and Ulbrich (1974), the main advantage of the shape factors proposed here lies in the simplicity of their interpretation in terms of curvature (in relation to the exponential distribution). $S(W\sigma)$, $S(RW)$ and $S(ZR)$ indicate the curvature in the regions of the distribution where the contribution to the indicated integral quantities is maximum and $S(Z\sigma)$ represents the average curvature of the whole distribution. Another advantage lies in the fact that the proposed shape factors may be calculated from maximum-contribution diameters or mean diameters which have a considerably smaller fractional standard deviation (see Appendix 1) than median diameters which play an important role in the definition of the $B(n)$ values.

6) As demonstrated by four disdrometers set up side by side, the shape of a distribution having a sample size of 0.3 m^2 s is well defined.

7) If the shapes of distributions are compared, the bias introduced by different sampling periods and sample sizes should be recognized.

8) It is expected that the average "instant" shape and its change with increasing sampling period contain information about the precipitation process such as cellular structure, turbulence, drop sorting and collision and breakup of drops.

APPENDIX A

Mean, Median and Maximum Contribution Diameters

For calculating the shape factors, Joss and Gori (1976) used the median diameters, defined for σ and Z as

$$DM(\sigma):\ \sigma/2=(\pi/4)\int_0^{DM(\sigma)} N(D)D^2dD, \tag{9}$$

$$DM(Z):Z/2=\int_0^{DM(Z)} N(D)D^6dD. \tag{10}$$

In discussing the results with P. L. Smith, he proposed that, instead of using the medians as Joss and Gori (1976) did, mean diameters computed from equations similar to (3) and (4) (Smith *et al.*, 1976) should be used. For the present work this suggestion was modified by introducing the modal or maximum contribution diameters as defined by Eqs. (3) and (4). As P. L. Smith

TABLE 3. Fractional standard deviation (%) or coefficient of variation $V=S_x/X$ of parameters, based on mean (mea), median (med) and maximum (max) contribution diameters (estimated using samples with size of 0.3 m² s).

	D_σ	D_W	D_R	D_Z	$S(W\sigma)$	$S(RW)$	$S(ZR)$	$S(Z\sigma)$	σ	W	R	Z
V mea	5.4	6.7	8.0	10.6	15.8	19.2	22.5	17.3				
V med	7.1	7.8	8.5	10.6	23.3	37.6	34.6	20.5	9.0	12.5	16.3	32.5
V max	4.1	5.4	6.7	9.3	13.5	15.8	19.4	13.5				

(1977, private communication) has pointed out, we have now used all of the common measures of central tendency:

Atlas (1953) median volume diameter D_0
Smith *et al.* (1976) mean D_m
Joss and Gori (1978) mode D_w.

When comparing the various properties of these three measures it is important to note that the shape factors as defined in Eq. (5) may be calculated with any one of the three and that the conclusions of this work are not affected by the choice. The main advantage in using the maximum contribution or the mean diameter instead of the median diameters lies in the smaller fractional standard deviation (or coefficient of variation V), as demonstrated by the results in Table 3. These results are calculated from data recorded simultaneously with four disdrometers spaced 1 m apart during 512 min of widespread rain.

Because V for any derived parameter is proportional to the inverse square root of the sample size, we may estimate the sample size needed to obtain a given V. This sample size is the smallest for the maximum contribution parameters and largest for the median parameters. On the average less than half the sample size is needed when using maximum contribution diameters as compared to median diameters. This is partly due to the fact that maximum contribution diameters are smaller than median or mean diameters and drops with smaller diameters are more numerous and their concentration is therefore statistically better defined; i.e.,

$$D\text{ mea} > D\text{ med} > D\text{ max}. \tag{11}$$

In Table 3 the order of V for all diameters and shape factors is

$$V\text{ med} > V\text{ mea} \gtrsim V\text{ max}. \tag{12}$$

The reason for the reversed order in (12) as compared to (11) lies in the fact that fluctuations in drop concentration around the median diameter significantly increase the fractional standard deviation V med, especially for the shape parameters (derived from *two* diameters).

When comparing maximum and mean parameters the following two points may be mentioned:

1) Mean diameters are identical to maximum contribution diameters for the next higher moment, e.g., $D(\sigma)$ mea $=D(W)$ max and $S(W\sigma)$ mea $=S(R^*W)$ max. This is strictly true only for exponential distributions. While moments one to six must be calculated for the maximum contribution diameters used in this paper, moments two to seven would be needed for the corresponding mean parameters.

2) Maximum contribution diameters as defined by equations similar to (3) indicate the true location of the maximum contribution for exponential distributions only, whereas mean diameters are defined independent of the distribution. At first glance this may seem a strong point in favor of mean parameters; in reality it is not, because the contributing region of real distributions is much wider than the differences between the real maximum contribution diameter and the one defined by (3), for non-exponential distributions.

Concluding, we find only a small difference between mean and maximum contribution diameters. Both are better than using median diameters and less work to calculate. We chose the latter ones, because better use is made of the information contained in the distributions; the value of the data is reduced more by the errors resulting from limited sample size for large drops (weighted heavily in the seventh moment) than from inaccuracies in measuring small drops (important in the first moment).

APPENDIX B

Retransformation of the Distribution

In order to know the average shape of the "instant" distribution, we need a mathematical expression specifying the drop concentration $N(D)$. Calculating the basic parameters for this expression should yield values (referred to as "retransformed" in the last column of Table 1), which are as close as possible to the ones obtained when averaging the basic parameters of the observed "instant" distributions (referred to as "observed 1 min" in the last column of Table 1).

Because of the exponential nature of most rain drop size distributions, $N(D)$ can be written

$$N(D)=N_0^*\exp(-\Lambda D). \tag{13}$$

The easiest way to approximate the measured instant distribution is to let N_0^* depend on D. The variations of N_0^* as a function of D is controlled by the two dimensionless parameters X_0 and c, i.e.,

$$N_0^*=N_0/[1+c(X_0-\Lambda D)^2]. \tag{14}$$

Inserting (14) into (13) yields

$$N(D)=N_0\exp(-\Lambda D)/[1+c(X_0-\Lambda D)^2]. \quad (15)$$

Note that N_0 is a constant as originally proposed by Marshall and Palmer (1948). Here, however, it no longer has the meaning of the intercept. N_0^*, on the other hand, is the intercept at $D=0$ of the tangent to the point at the diameter D of the distribution. As compared to the exponential distribution, Eq. (15) reduces the drop concentration for the very small and the very large drops, just as observed in nature. X_0 and Λ determine the diameter D_c at which the curvature of the distribution is maximum, i.e.,

$$D_c=X_0/\Lambda, \quad (16)$$

and the parameter c determines the amount of curvature.

In order to perform the retransformation, N_0 was tentatively set to 10 000 m^{-3} mm^{-1}, and X_0, c and Λ were varied. All the basic parameters of Table 1 for many sets of Λ, X_0 and c were calculated. Then the set with values closest to the measured average "shape factors" and the measured "maximum contribution diameters" was selected. Finally, the true N_0 was found by using the measured "integral parameters" making use of the fact that changing N_0 of a distribution only changes the integral parameters proportionally and leaves the shape, and with it the maximum contribution diameters, unchanged.

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