A Simple Approximation for the Variance of Meterological Time Averages

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ABSTRACT

A simple method for approximating the variance of meteorological time averages is presented. Graphs of the characteristic time between independent estimates and the ratio of the variance of time-averaged data to that of unaveraged data for a first-order autoregressive process are shown.

1. Introduction

Estimating the variance of meteorological time averages is an important problem in climate studies. For example, we need to know how much one January mean temperature can differ from another due to statistical sampling alone before we can make quantitative climate predictions. In fact, knowledge of this natural variability due to statistical sampling variation is necessary in order for us to even reliably detect true climate changes. The problem has been discussed by Leith (1973) in the context of climate modeling and climate change.

2. First-order autoregressive process

Brooks and Carruthers (1953, p. 326) discuss the effect of persistence on the variance of time averaged data. Jones (1975) presented equations for estimating the variance of meteorological time averages by fitting autoregressive models to the data that take this persistence into account. Here we present two graphs that illustrate some relevant features of the simplest of these, a first-order autoregressive or Markov model. Jones (1975) shows that the variance $\sigma_T{}^2$ of a time average of length T for a discrete first-order autoregressive process is given by

$$\sigma_T{}^2=\frac{\sigma^2}{T}[1+2(1-1/T)\gamma+2(1-2/T)\gamma^2+\ldots+(2/T)\gamma^{T-1}]. \quad (1)$$

If T is the averaging time in days and σ^2 the variance of daily data, then γ is the lag 1-day autocorrelation.

3. Characteristic time between independent estimates

The quantity in the brackets in (1) is a measure of the persistence in the data and it may be considered to be a characteristic time between independent estimates (T_0), i.e.,

$$T_0=[1+2(1-1/T)\gamma+2(1-2/T)\gamma^2+\cdots+(2/T)\gamma^{T-1}]. \quad (2)$$

This characteristic time T_0 has been computed for 5-, 10- and 30-day averages for different values of γ (WMO, 1966). For very long time averages and $\gamma=0.74$, Leith (1973) determined T_0 to be about 6.67 days. Here we evaluate T_0 for various positive values of γ and for differing averaging times using (2). The results are presented in graph form in Fig. 1. For $\gamma=0$

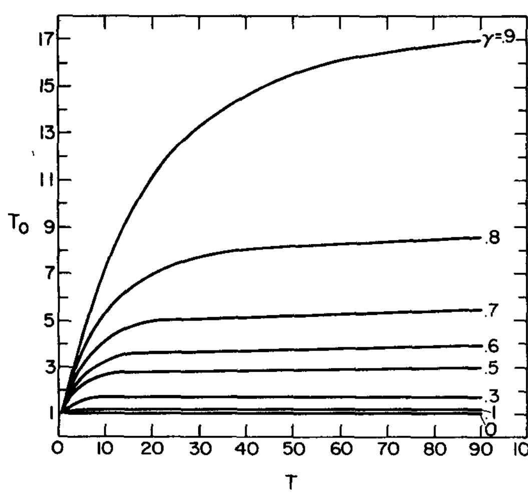

FIG. 1. The characteristic time between independent estimates (T_0) for differing averaging times (T) of a first-order autoregressive process.

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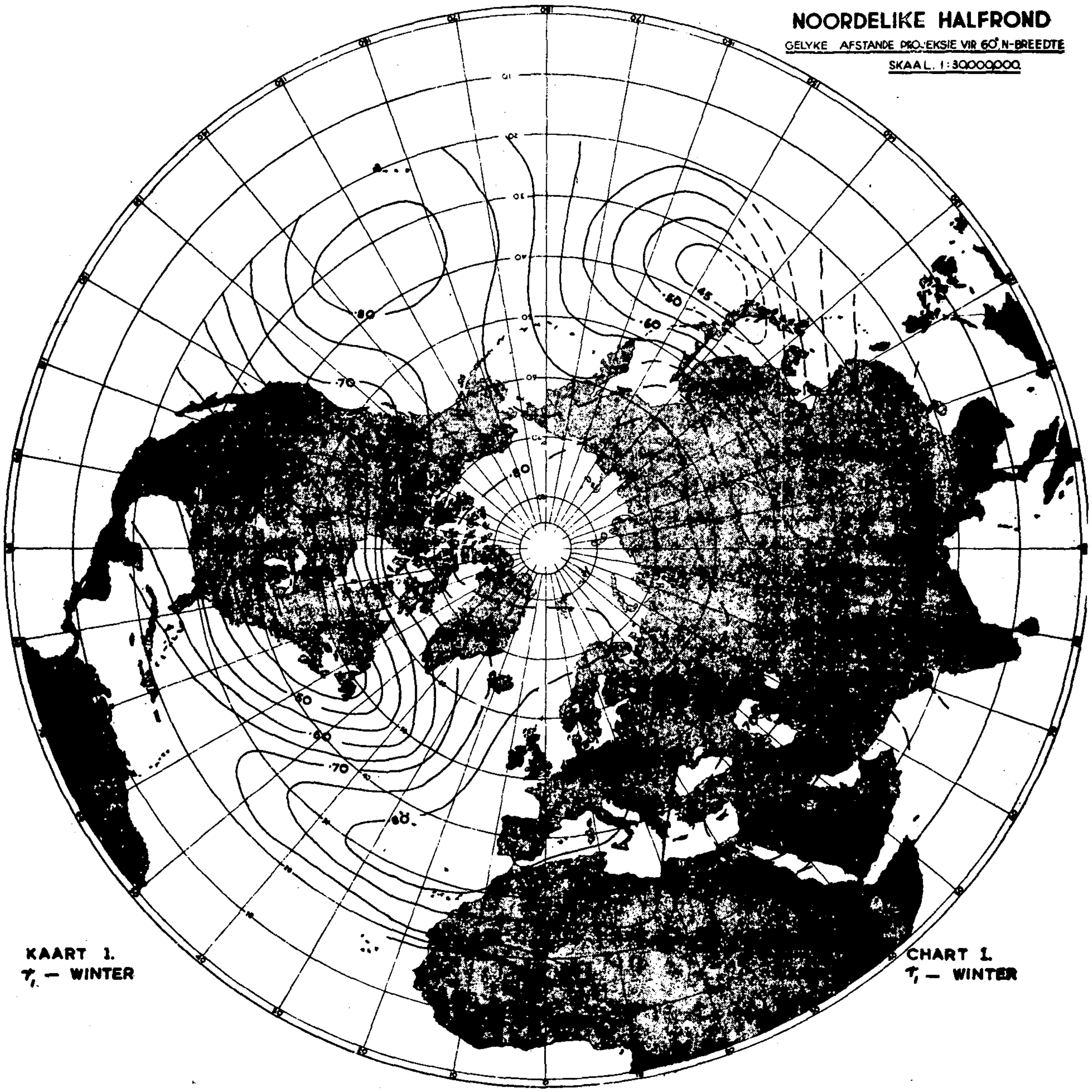

FIG. 2. The lag 1-day autocorrelation for sea level pressure in winter taken from Schumann and Van Rooy (1952). Reproduced with the kind permission of the Director of the South African Weather Bureau.

each value is independent of the next, i.e., $T_0=1$. As γ increases, persistence increases and T_0 is correspondingly larger.

Fig. 2 is the lag 1-day autocorrelation (γ) for sea level pressure for the winter season taken from Schumann and Van Rooy (1952). Fig. 3 shows the lag 1-day autocorrelation computed for winter temperatures over the United States. One can convert from γ in these two figures to T_0 for a 31-day average using Table 1. The resulting patterns of T_0, although not identical, are very similar to those already estimated from the shapes of the spectra of daily data for winter pressure (Madden, 1976) and temperature (Madden and Shea, 1978). A T_0 equal to 5 ($\gamma \approx 0.7$) implies that an average of 6 days, each 5 days apart, gives as stable an estimate of the mean as an average of the entire 30 or 31 days in a given month.

TABLE 1. T_0 determined from (2) for various values of γ with $T=31$. If γ is the lag 1-day autocorrelation, T_0 is in days.

γ	0.0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	5.0
T_0	1.0	1.1	1.2	1.3	1.5	1.6	1.7	2.0	2.3	2.5	2.9
γ	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.0	
T_0	3.3	3.8	4.4	5.2	6.2	7.7	9.9	13.4	19.5	31.0	

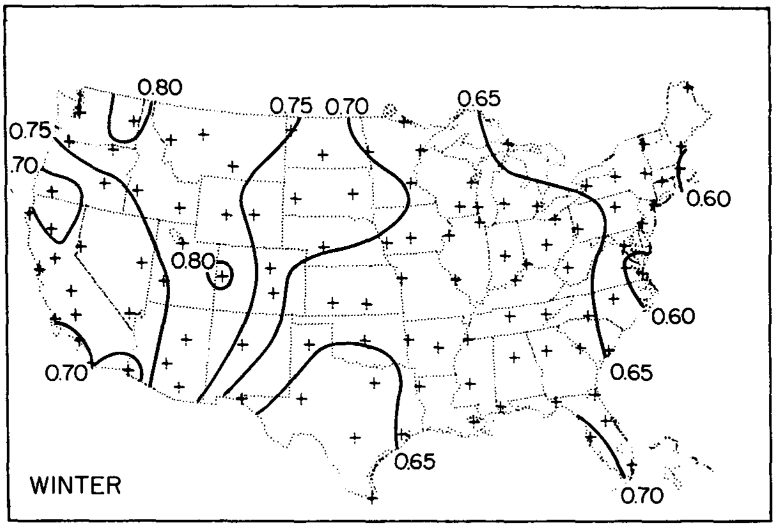

FIG. 3. The lag 1-day autocorrelation for temperature in winter at stations marked by the cross. Estimates based on data described in Madden and Shea (1978).

4. Variance of a time average

Finally from (1) and (2) we see that

$$\frac{\sigma_T^2}{\sigma^2}=\frac{T_0}{T}. \tag{3}$$

The ratio T_0/T for different γ and varying averaging times T is shown in Fig. 4. For $\gamma=0$, σ_T^2/σ^2 goes as $1/T$, the well known result for independent data. A somewhat similar graph was presented by Leith (1973).

5. Summary

A first-order autoregressive process has often been used to model meteorological data (e.g., Klein, 1951; Jenkinson, 1957; Leith, 1973). Here we give a characteristic time between independent estimates for such a model. In addition the ratio of σ_T^2/σ^2 is given for various lag 1 autocorrelations and averaging times. By computing the variance of daily data and the lag 1-day autocorrelation one can make a first approximation of the natural variability, or variance of time averages due to sampling variations. In addition it is possible to use a similar technique to estimate the natural variability of a general circulation model. Shukla (1975) has attempted this in his numerical study of the effects of changing sea surface temperatures on the Indian monsoon. Laurmann and Gates (1977) also discuss the Markov model in their considerations of the statistics of climate experiments based on general circulation models (GCM). Jones (1976) has pointed out that the degrees of freedom of such a model are uncertain and rigorous statistical tests cannot be applied. While this is true, one alternative, i.e., that of computing several GCM realizations, (Chervin and Schneider, 1976) involves considerable computer time and the simple Markov model may, in some instances, serve adequately.

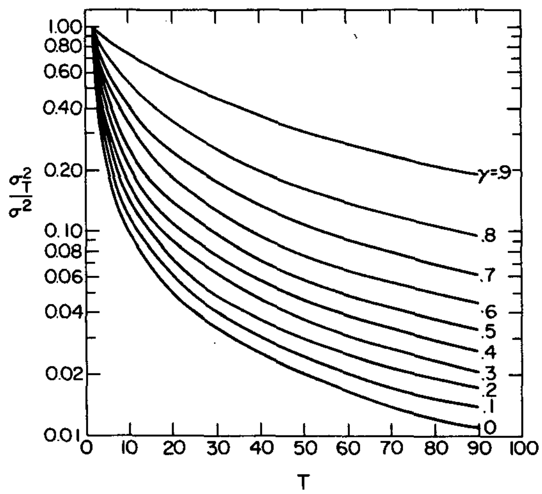

FIG. 4. Fractional reduction of variance (σ_T^2/σ^2) for various averaging times (T) and lag 1 autocorrelations (γ) for a first-order autoregressive process.

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