Measuring Heat Flux and Structure Functions of Temperature Fluctuations with an Acoustic Doppler Sodar

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ABSTRACT

The vertical velocity variance during convective activity was measured with an acoustic Doppler sounder. Here we show that this technique provides measurement of the heat flux profile in the well-mixed surface layer and gives a reference height h' related to the inversion height Z_I during the mornings.

The paper also shows that the structure function of temperature fluctuations C_T^2 can be obtained by these measurements using the Kaimal *et al.* (1976) formula.

1. Introduction

The structure function of temperature fluctuations C_T^2 can be obtained by measurement of acoustic "reflectivity" (backscattered acoustic intensity) using a monostatic vertical acoustic sounder. Measurement of C_T^2 is very useful for optical tracking and in turbulence studies and it has been extensively studied (Tsvang, 1963, 1969; Koprov, 1965; Kaimal, 1966; Warner, 1972; Coulman, 1973; Bufton, 1973; Caughey and Reading, 1974; Warhaft, 1973; Azimakopoulos, 1976; Weill *et al.*, 1976).

During convective activity, the profile of C_T^2 near the surface layer is related to the surface layer heat flux (Wyngaard *et al.*, 1971). Haugen and Kaimal (1978) have found that measurement of the profile of C_T^2 using an acoustic sounder can lead to serious inaccuracies. In this paper we first describe a method of evaluation of heat flux profile in the well-mixed layer and of heat flux of the surface layer using the vertical velocity variance measured by the Doppler shift of the acoustic sounder. Second we make a comparative evaluation of the different expressions for C_T^2 in the convective boundary layer and show that the measurement of vertical velocity variance $\overline{w'^2}$ and of the inversion height Z_I, using an acoustic sounder, can give good C_T^2 estimates. Experimental measurements made at Voves near Chartres (1977) are presented in section 4.

2. Vertical velocity variance during morning convective activity

Previously, Panofsky and McCormick (1960), Panofsky and Mazzola (1971), Panofsky *et al.* (1977), Panofsky (1978), Caughey and Readings (1974), McBean and McPherson (1976) and Yokoyama *et al.* (1977) have shown that using similarity theory

$$\sigma_w^2 = \overline{w'^2} \approx A\left[z\left(-\overline{u'w'}\,\frac{d\bar{U}}{dz} + \delta\,\frac{g}{\theta}\,\overline{w'\theta_v'}\right)\right]^{2/3}, \quad (1)$$

where

σ_w	standard deviation of vertical velocity
A,δ	universal constants
$-\overline{u'w'}\,\frac{d\bar{U}}{dz}$	local mechanical production
$\frac{d\bar{U}}{dz}$	mean wind shear
$\frac{g}{\theta}\,\overline{w'\theta_v'}$	local buoyancy production
z	height
g	gravity acceleration
$\theta'v$	virtual potential temperature perturbation
u'	longitudinal velocity perturbation
w'	vertical velocity perturbation.

In a well-mixed layer, the mechanical production is negligible and (1) can be simplified to

$$\sigma_w^2 \approx \alpha\left(z\,\frac{g}{\theta}\,\overline{w'\theta_v'}\right)^{2/3}, \quad (2)$$

where $\alpha = A\delta^{2/3} \approx 1.4$ (see Caughey and Readings, 1974; McBean and McPherson, 1976).

Accordingly, a plot of σ_w^3/z versus z gives the local heat flux

$$\frac{\sigma_w^3}{z} \approx \alpha^{3/2}\,\frac{g}{\theta}\,\overline{w'\theta_v'}. \quad (3)$$

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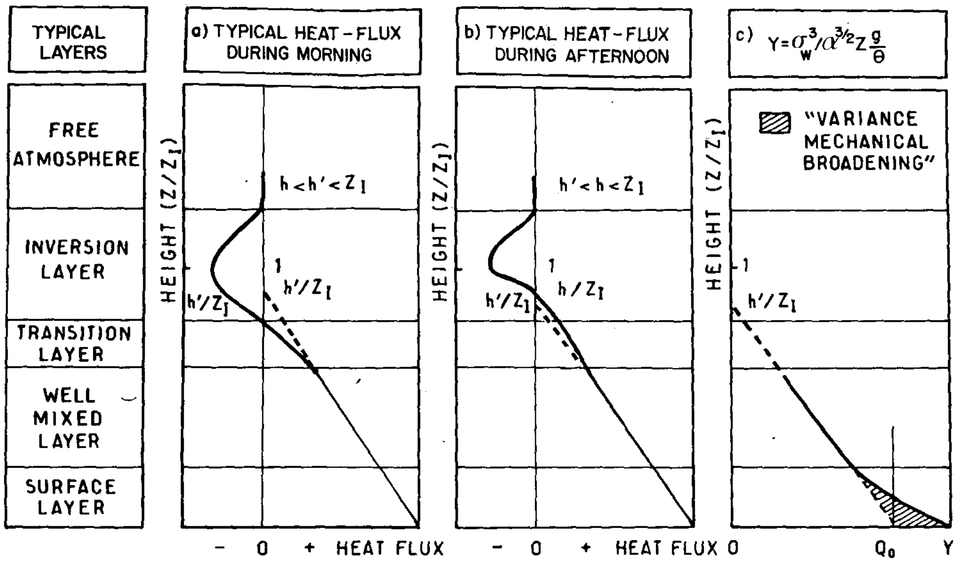

FIG. 1. Typical heat flux profiles during convective activities: (a) during morning $h < h' < Z_I$ (shallow convection), where h is the height of zero heat flux, h' is the height of zero heat flux obtained by linear extrapolation, and Z_I is the height of maximum negative heat flux; (b) during afternoon (well developed convection) $h' < h < Z_I$; (c)

$$Y = \sigma_w{}^3\left(\frac{g}{\bar{\theta}}\right)^{-1}\alpha^{-3/2}z^{-1} \approx \left(\frac{g\delta}{\bar{\theta}}\right)^{-1}\bar{U}^2\frac{d\bar{U}}{dz} + Q_0$$

giving h'/Z_I and Q_0. Note that the hatched part corresponds to a "variance broadening" due to mechanical production.

Therefore, in a dry convective layer with $\theta_v' \approx \theta'$, the use of a vertical Doppler sounder gives the vertical velocity variance profile from which the heat flux profile can be determined. Measurements are made at various heights using the monostatic Doppler sounder, a system which corresponds to nearly perfect Eulerian profiles.

In the well-mixed layer $\overline{d\theta/dz} = 0$ and $\partial\bar{\theta}/\partial t = -\partial \overline{w'\theta'}/\partial z =$ constant and the heat flux decreases linearly with height. Therefore,

$$\frac{\sigma_w{}^3}{z} = \alpha^{3/2}\frac{g}{\theta}Q_0\left(1 - \frac{z}{h'}\right), \tag{4}$$

where Q_0 is the surface temperature flux [incorrectly referred to as surface heat flux (Wyngaard, 1972)] and h' is the height at which heat flux vanishes by linear extrapolation. According to (4), σ_w exhibits a maximum for $z = 0.5\,h'$.

It has previously been shown (André *et al.*, 1978) that in a convective layer, the maximum velocity variance occurs near the height $z = Z_I/3$, where Z_I is the inversion height which corresponds to the first stable static layer and is equivalent to the layer of maximum negative heat flux. Therefore, h' must be of the order of $0.7\,Z_I$. In the present paper, Eq. (4), derived for the well-mixed adiabatic layer does not take into account entrainment effects and the divergence of the radiative flux at the top of the mixed layer which is not strictly abiabatic. For these reasons, Eq. (4) gives an height h' different from the height h where heat flux really vanishes ($h' \neq h$).

Experimental results taken from the literature and atmospheric boundary-layer simulation (André *et al.*, 1978) suggest two cases:

(i) $h < h' < Z_I$ during mornings
(ii) $h' < h < Z_I$ during afternoons.

Case (i) was observed near midday by Cattle and Weston (1975), whereas as predicted here, cases (i) and (ii) were observed by Clarke *et al.* (1971). Therefore, during shallow convection with $Z_I < 450$ m, for which case (i) is valid, the vertical velocity variance profile can give

- The heat flux profile in the well-mixed adiabatic layer
- The surface heat flux by extrapolation of the linear part of the profile to $z = 0$ (during convective situation)
- The reference height h' interesting to compare with the inversion height Z_I and the zero heat flux height h.

This is illustrated in Fig. 1. It must be recalled that in agreement with Kaimal *et al.* (1976), the value of Z_I determined by an acoustic sounder corresponds to the height of maximum backscattered intensity.

3. Structure functions of temperature fluctuations in the convective atmospheric boundary layer

Wyngaard *et al.* (1972) have shown that in the convective surface layer

$$C_T^2 \approx 4/3 T_*^2 z^{-2/3}\left(-\frac{z}{L}\right)^{-2/3}, \quad (5)$$

where

T_* scaling temperature $[=|Q_0|/u_*]$
Q_0 surface layer heat flux
u_* friction velocity
L Monin–Obukhov length $[=-u_*^3/k(g/T)Q_0]$
k von Kármán constant.

Using the inertial subrange of temperature spectral density in the convective boundary layer, Kaimal *et al.* (1976) have shown that C_T^2 can be written as

$$0.83 \times 3.2\theta_*^2 Z_I^{-2/3}\left(\frac{z}{Z_I}\right)^{-4/3}, \quad z < 0.5Z_I, \quad (6.1)$$

$$2.1 \times 3.2\theta_*^2 Z_I^{-2/3}, \quad 0.5Z_I < z < 0.7Z_I, \quad (6.2)$$

$$6.1 \times 3.2\theta_*^2 Z_I^{-2/3}\left(\frac{z}{Z_I}\right)^{3}, \quad 0.7Z_I < z < Z_I, \quad (6.3)$$

where

θ_* scaling potential temperature $[=Q_0/w_*]$
w_* convective scaling velocity $[=(Q_0 Z_I g/T)^{1/3}]$.

According to Panofsky (1978),

$$\left(\frac{u_*}{w_*}\right)^3 = -\frac{L}{Z_I}k$$

which makes Eq. (6.1) equivalent to (5). Thus C_T^2 exhibits a minimum for $0.5Z_I < z < 0.7Z_I$. This situation occurs at the height where the temperature variance is minimum in the well-mixed layer and for a constant mechanical dissipation rate (Pennel and Lemone, 1974; Kaimal *et al.*, 1976; Wyngaard, 1972; Lenschow, 1974; Weill *et al.*, 1978).

In conditions where the dissipation rate decreases with height in the well-mixed layer C_T^2 exhibits a minimum at heights $> 0.7\ Z_I$. The turbulent kinetic energy budget indicates that the height of minimum C_T^2 is determined by the minimum value of

$$C_T^2 \approx 3.2\left(\frac{g}{\theta}\overline{w'\theta'} + \frac{\partial}{\partial z}\overline{e'w'}\right)^{-1/3}\left(\frac{\partial}{\partial z}\overline{w'\theta'^2}\right), \quad (7)$$

where e' is the turbulent kinetic energy.

In accordance with the results of Kaimal *et al.* (1976), our acoustic sounder reflectivity profile results show a minimum near a height of $0.7\ Z_I$. This is illustrated in Figs. 2a–2c which show vertical reflectivity profiles computed during a morning convective activity at Chigne (France). The mean height corresponding to the minimum of acoustic reflectivity Z_m during the three time periods of measurement are 158, 201 and 326 m, respectively, which give a value of $Z_m/Z_I \approx 0.7$.

Our results would fit the equation

$$C_T^2 \approx 2.7\theta_*^2(Z_I)^{-2/3}\left(\frac{z}{Z_I}\right)^{-4/3}\left(1-\frac{z}{Z_I}\right)^{-2/3}, \quad (8)$$

which is a slightly modified form of (6.1). Eq. (8) is valid for $z < 0.9\ Z_I$ and exhibits a minimum at $z = 2/3\ Z_I$. It should be noted that in these expressions, the Z_I value is the mean Z_I value during the measuring time interval.

Hence, Eq. (8) is only valid during steady-state conditions and during time periods where the Z_I variations are not too large.

Eq. (8) is compared to Eq. (6.1) in Fig. 3. It can be seen that the discrepancy between the expressions is greatest near the nose of the curves. The discrepancy, however, is within the measuring error (Weill *et al.*, 1976).

Results obtained in 1977 indicated that the height z corresponding to the minimum C_T^2 is proportional to time and thus to $-Z_I/L$. This height increases from $0.5\ Z_I$, corresponding to a $-Z_I/L$ value of ~ 5, to $0.8\ Z_I$, near midday, corresponding to a $-Z_I/L$ value of ~ 10. A more adequate form for (8) is thus suggested:

$$C_T^2 = 2.7\theta_*^2 Z_I^{-2/3}\left(\frac{z}{Z_I}\right)^{-4/3}\left(1-a\frac{z}{Z_I}\right)^{-2/3}, \quad (9)$$

where $a = 2Z_I/3Z_m = 0.83$ during well-developed convection in late morning.

4. Experimental results

The results reported here were obtained in 1977 from a flat corn field site near Voves, located 100 km south east of Paris. On 2 July, three experiments were simultaneously conducted: the vertical Doppler sounder experiment; a tethered balloon temperature probe experiment; and a rawindsondes launching experiment. The latter experiment covered the period from 0500 to 1500 local time.

a. *Acoustic Doppler sounder measurement of the vertical velocity variance*

The main parameters of the sodar equipment used are given in Table 1. The measurement of the vertical velocity variance is severely limited by the filtering of the aerial beam and the low-frequency limit only depends on the number of pulses.

We assume an illumination function of the form

$$I(x) = \exp\left[\frac{-x^2}{2(\Delta x)^2}\right], \quad (10)$$

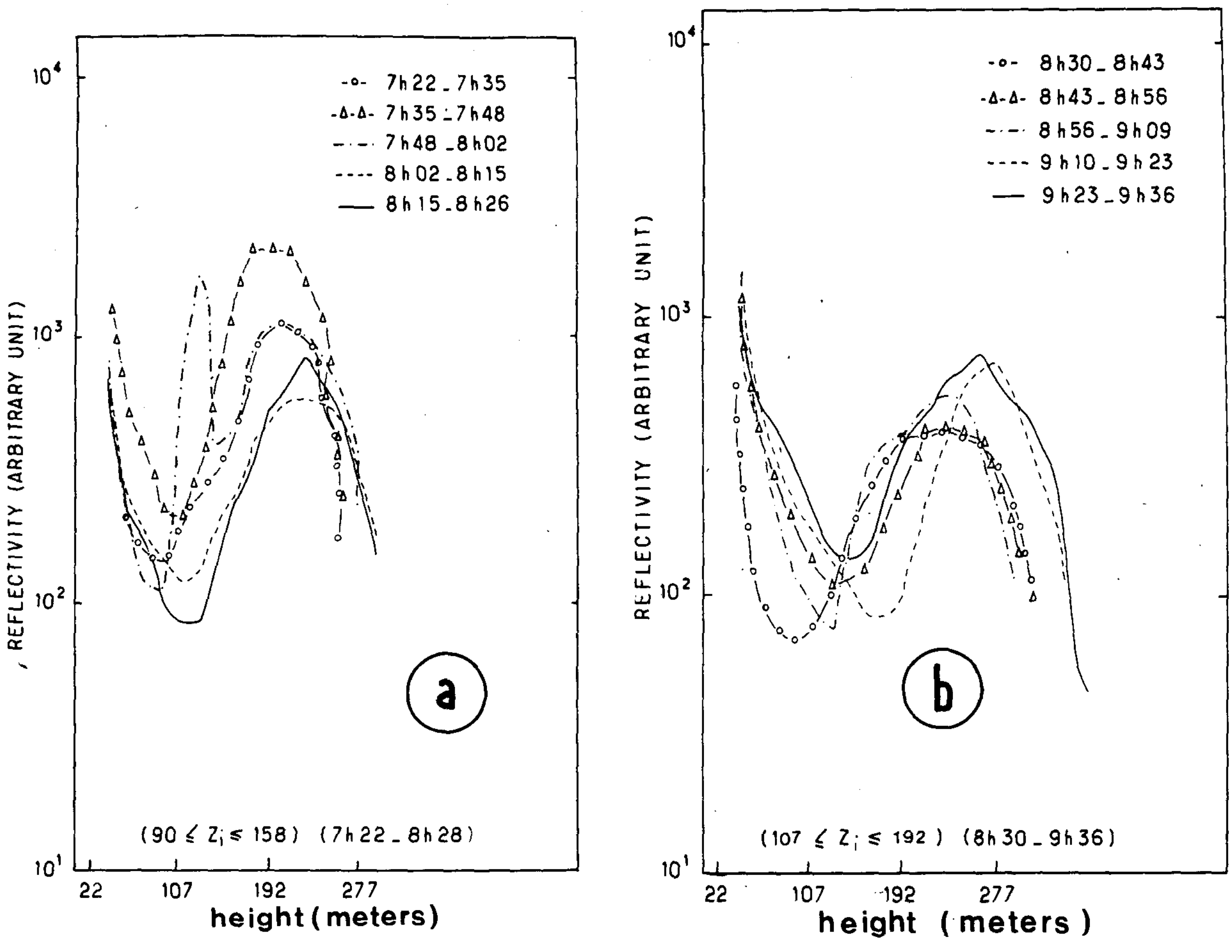

FIG. 2. Reflectivity profiles from the Chigne Experiment, June, 1975: (a) from 0722 to 0826, (b) from 0830 to 0936, (c) from 0939 to 1024. All times local standard.

where x is the horizontal scale space observed by the sodar at time $t = x/U$ (where U is the mean wind velocity) and Δx is the horizontal dimension illuminated by the aerial beam during the time interval

$$\Delta t = \Delta x/U = (z/U) \tan\alpha, \tag{11}$$

where α is the half-beam width of the antenna. The transfer function of the aerial beam is

$$S(f) = \exp\left[-\frac{1}{2}\left(\frac{f}{f_c}\right)^2\right], \tag{12}$$

where f_c is the sodar cutoff frequency

$$f_c = \frac{U}{2\pi z \tan\alpha}. \tag{13}$$

Eq. (13) gives the dimensionless cutoff frequency factor

$$n_c = \frac{f_c z}{U} = \frac{1}{2\pi \tan\alpha} \approx 1.82. \tag{14}$$

Previous results (Monji and Businger, 1972) indicate that the above limitation still permits the measurement of the vertical velocity variance during convective activity.

b. *Heat flux profile and inversion height*

Fig. 4 illustrates the variation of the inversion height with time using reflectivity and/or temperature measurements. As shown in Section 1, the data from reflectivity measurements and the use of Eq. (4) give the reference height h', the heat flux profile and the surface layer heat flux.

As shown in Fig. 4, the reference height h' is near the inversion height Z_I. This is because during the morning shallow boundary layer, the difference $Z_I - h'$ is comparable to the one gate resolution of the acoustic sounder (± 17 m). By late morning, the inversion layer is too high to be measured using the acoustic sounder, but the reference height h' is smaller than the inversion height Z_I. However, the accuracy in the measurement of h' decreases by midday because of the larger dispersion in the

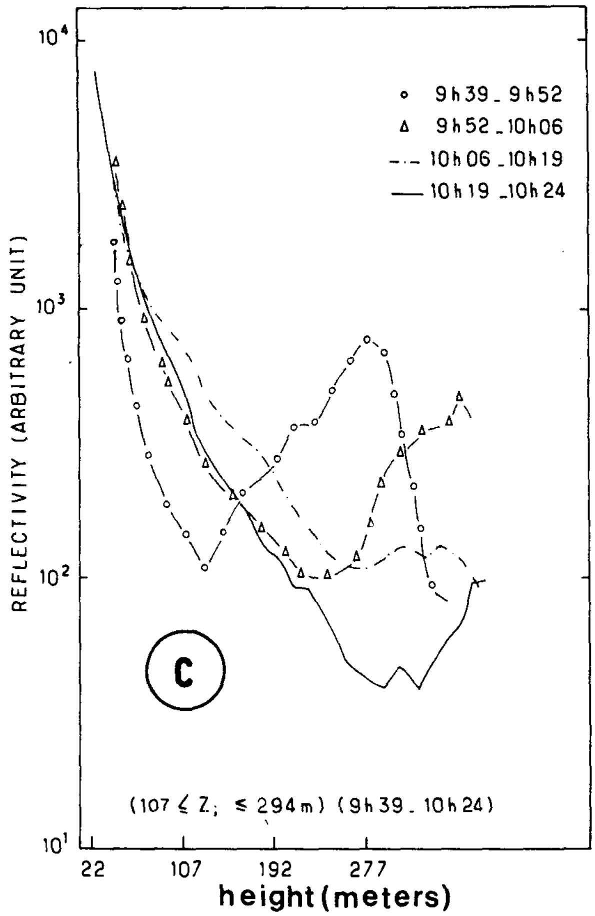

FIG. 2. (*Continued*)

$\sigma_w{}^3/\alpha^{3/2}z$ profile. The surface heat flux was computed for a period of 30 min and was found to be comparable to the results of the Institut National de la Recherche Agronomique (INRA) using a balance method of measurements for 1 h using the BEARN[1] system (Perrier *et al.*, 1976). Our method has the advantage of short measuring times necessary to compute heat flux since it permits taking account of several thermal plumes. Table 2 shows the good agreement between the two methods of surface heat flux computation and illustrates the usefulness of the method presented here to observe changes in the heat flux with time.

[1] Bilan d'Energie Albedo, Rayonnement Net.

TABLE 1. Main parameters of the acoustic Doppler sounder.

Acoustic emission frequency (Hz)	2000
Width of the aerial beam (deg)	10
Pulse duration = duration of a gate analysis (ms)	100
Pulse length = gate length (m)	17
Repetition rate (s)	4
Number of points used in a gate for the Doppler shift computation using the fast Fourier transform	32

c. Evaluation of C_T^2

Structure functions of temperature fluctuations at a height of 100 m were measured using a captive balloon temperature probe designed by the Etablissement d'Études et de Recherches Météorologiques (Heissat and Gerbier, 1973). A comparison is made using C_T^2 evaluated from acoustic sounder data using Eqs. (5) and (6.2). For the captive balloon the temperature was continuously recorded and temperature spectral analysis undertaken every 100 s. A set of 18 such measurements were necessary to calculate C_T^2. The method of calculation has previously been described (Weill *et al.*, 1976). The results given in Table 2 show good agreement between the C_T^2 value obtained by the three methods of calculation considering measuring inaccuracies of ~40%. The inaccuracies are mainly caused by the lack of precision in the determination of wind speed and heat flux. It should be noted that the C_T^2 values given by (6.2) and (8) are within ±5% of each other.

5. Conclusions

A method of heat flux measurement in the convective boundary layer is presented. This method gives the heat flux profile in the well-mixed layer and permits an evaluation of the surface heat flux. The method is supported by experimental results.

Furthermore, this method also permits the evaluation of a reference height h' where heat flux would vanish in the absence of entrainment. During

TABLE 2. Evaluation of C_T^2.

Local time	h' (m) Evaluated by the vertical velocity variance method	Z_i (m) Evaluated by sounding	Q_0 (K m s^{-1}) Evaluated by: Vertical velocity variance method	Q_0: INRA	$10^3 C_T^2$ (K^2 m$^{-2/3}$) Evaluated by: Sodar Eq. (8)	Sodar Eq. (6.2)	Balloon
0658–0728	220	200	0.047	0.052	—	—	—
0728–0758	260	260	0.067	0.052	—	—	—
1032–1102	645	660	0.11	0.09	3.2 ± 1.3	2.9 ± 1.2	1.9 ± 0.8
1102–1132	750	755	0.16	0.09	5.2 ± 2.1	4.8 ± 1.9	4.4 ± 1.8

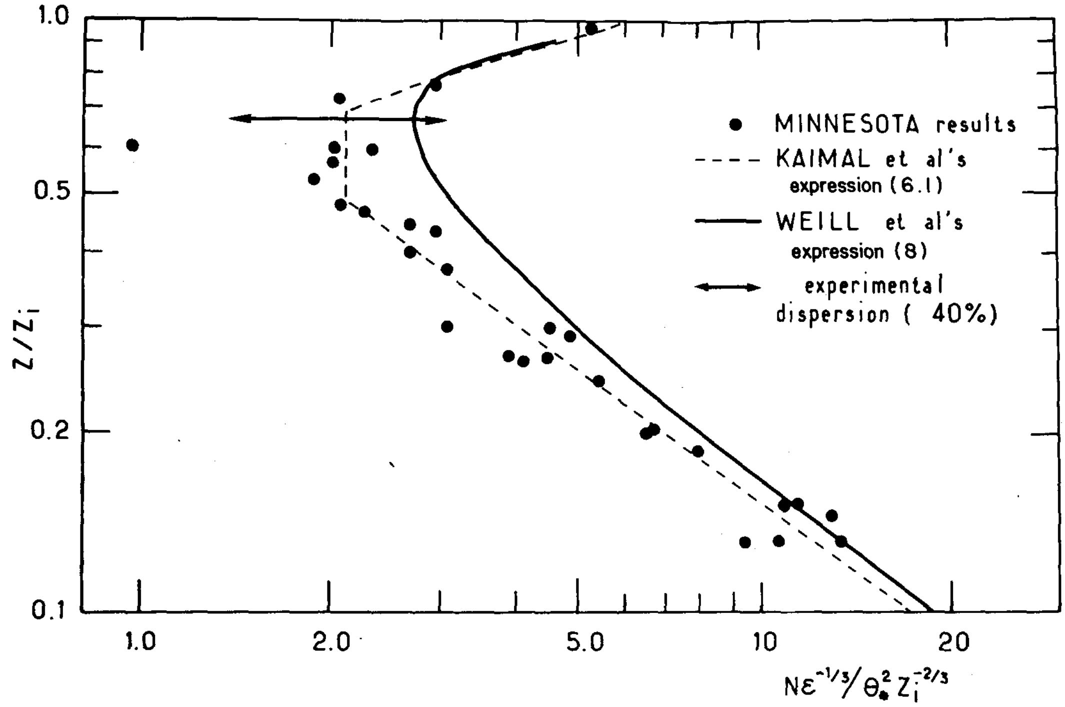

FIG. 3. Structure functions of temperature fluctuations using the results of Minnesota and Eqs. (6.1) and (8).

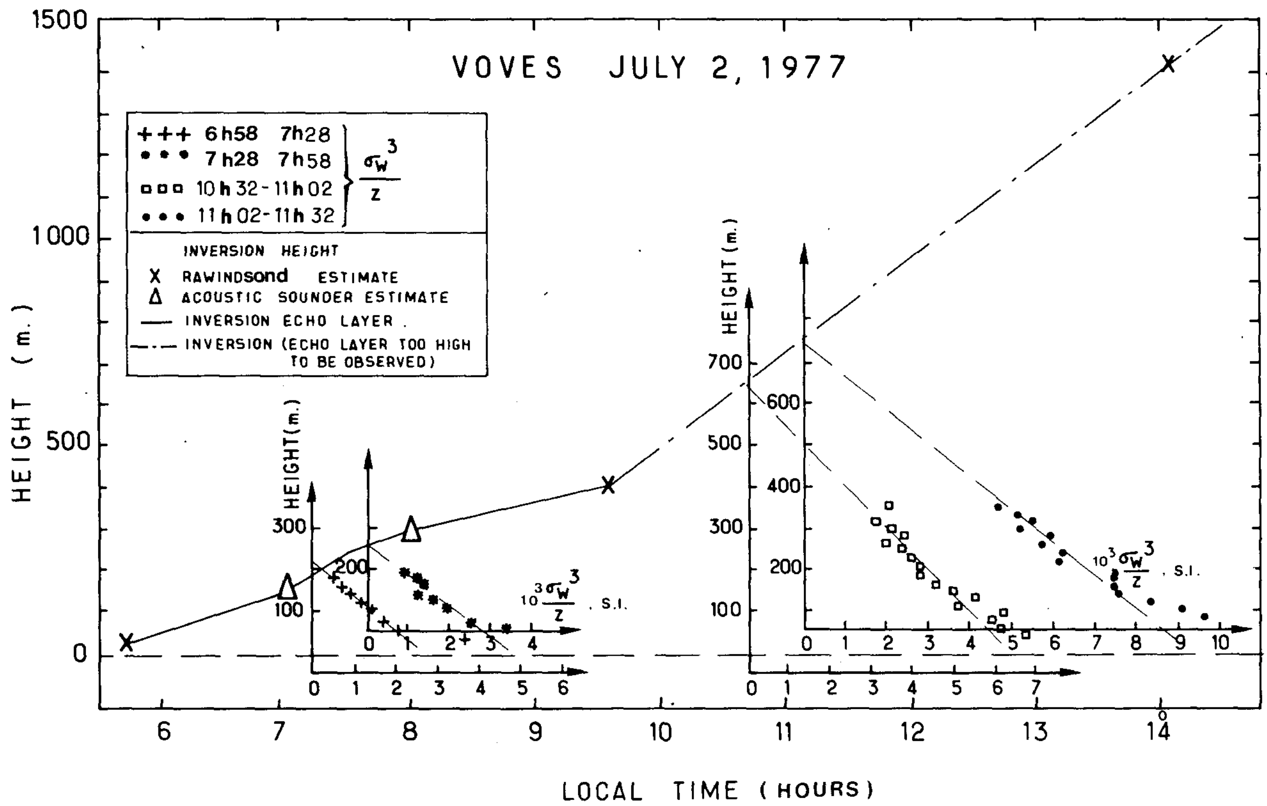

FIG. 4. Variation of inversion height and $\sigma_w{}^3/Z$ profiles, 2 July 1977 at Voves, France.

mornings, h' is near the inversion height Z_I, whereas in the afternoons $h' < Z_I$ and h' continues to be measurable and thus permits an evaluation of Z_I, which is too large to be measured by the acoustic sounder.

During mornings, the structure functions of temperature fluctuations C_T^2 can be evaluated from the knowledge of Q_0 obtained by the variance method and the knowledge of Z_I, obtained by reflectivity or maximum temperature gradient. A modified expression which permits the evaluation of C_T^2 up to $0.9\ Z_I$ is proposed. Experimental results show that the proposed expression is in good agreement with Kaimal *et al.* (1976).

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