Time, Angle and Range Averaging of Radar Echoes from Distributed Targets

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ABSTRACT

In radar meteorology, the average of the weather echo power is used in the computation of reflectivity, liquid water content, rainfall rate, etc. The uncertainty in measuring or estimating average weather echo power is then important in establishing confidence in the above computed values. To help establish a confidence level, we note that there exists a unique relationship between the weather radar echo correlation function and the receiver detected output correlation function. This unique relationship is used here to calculate the variance of the average (mean) weather echo estimates. Another measure of uncertainty related to the variance is the number (or equivalent number) of independent samples. In this work, we show the equivalent number of independent samples for average weather echoes at the output of three common radar receivers: linear, logarithmic and square law. This is shown for correlated samples of receiver output at different times, angles and ranges, Gaussian-shaped Doppler spectra and antenna patterns, a rectangular transmitted pulse, and an infinite bandwidth receiver.

1. Introduction

It is desirable to determine the accuracy of mean intensity estimates from weather echoes. When independent radar echo data are averaged, the variance of the mean intensity estimate is inversely proportional to the number of samples (Bendat and Piersol, 1968). This reduction in variance may be used as a measure of the precision of the mean estimate. On the other hand, the number of independent samples used to make the mean intensity estimate may also be used as a measure of precision of the mean estimate. When dependent radar data samples are averaged, we use as a measure of precision either the variance reduction or the equivalent number N_i of independent samples. This paper addresses the computation of N_i for the three common incoherent radar receivers and weather radar echoes. We note that N_i associated with a mean intensity estimate is defined as the ratio of the receiver signal variance to the variance of the mean intensity estimate.

Statistical dependence between adjacent data samples depends on time if time samples are taken, or on space if angular or range samples are taken. Therefore, a reduction in variance (increase in N_i) of the receiver output signal is expected by averaging over either time, antenna angle, range, or any combination of these.

The receiver detection law (square law, linear, logarithmic, etc.) modifies the statistics of the signals received by the radar (Davenport and Root, 1950). Therefore, averaging receiver detected output (video) signals yield values of N_i which depends on the particular receiver law. We generally wish to know the average received power from radar echoes. Rogers (1971) showed that computing the average power using outputs of linear or logarithmic receivers would yield biased results. Zrnic' (1975) showed that the bias of the mean intensity estimate and the variance of the mean intensity estimate would be a function of the number of independent samples. In this paper, we extend the computation of mean intensity estimate to include sampling of dependent radar data in time, angle and range, for linear, logarithmic and square-law radar receivers. Equations are derived for N_i in the case of a Gaussian-shaped Doppler spectrum, a Gaussian-shaped antenna pattern and a rectangularly shaped transmitted pulse. However, the method can be applied to any pulse, beam and spectrum shape. For all cases, an infinite bandwidth receiver, stationarity and homogeneity are assumed.

2. Weather radar signal properties

The signal received by a pulsed radar is the sum of backscattered electric fields from the scatterers in a pulse volume. The pulse volume length is $D_0(=c\tau_0/2$, where τ_0 is the transmitted pulse length).

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The pulse volume angular width depends on the antenna pattern.

The radar signal $x(\tau)$ is given in terms of the complex signal

$$x(\tau) = I(\tau)\cos(\omega_c\tau) - Q(\tau)\sin(\omega_c\tau), \quad (1)$$

where $I(\tau)$ and $Q(\tau)$ are the inphase and quadrature components and ω_c is the carrier frequency (see Davenport and Root, 1950). $I(\tau)$ and $Q(\tau)$ are narrow-band Gaussian processes with equal variances (see Marshall and Hitschfeld, 1953).

Kerr (1951) relates the normalized correlation functions of the output signal for linear, square law and logarithmic receivers[1] to the correlation function ρ of $I(\tau)$ or $Q(\tau)$:

Linear

$$\rho_{\text{lin}}(\tau) = \left[2E(\rho) - (1-\rho^2)K(\rho) - \frac{\pi}{2}\right] \Big/ \left(2 - \frac{\pi}{2}\right) \quad (2a)$$

Square law

$$\rho_{\text{sl}}(\tau) = \rho^2 \quad (2b)$$

Logarithmic

$$\rho_{\log}(\tau) = \frac{6}{\pi^2}\sum_{m=1}^{\infty}\rho^{2m}m^{-2}. \quad (2c)$$

Here $K(\rho)$ and $E(\rho)$ are the complete elliptic integrals of the first and second kind, respectively. Correlation functions given in Eq. (2) need not be restricted to explicit functions of time. They are a function of the input signal correlations which may include time, angle and range. Considering time, angle and range as independent variables, it can be shown that the normalized correlation function is

$$\rho(D,\Delta,\tau) = \rho(\tau)\rho(\Delta)\rho(D), \quad (3)$$

where $\rho(\tau)$, $\rho(\Delta)$ and $\rho(D)$ are the normalized correlation functions for time, angle and range, respectively. The radar receiver output correlation functions are obtained by substituting Eq. (3) into Eqs. (2a), (2b) or (2c).

3. Equivalent number of independent samples

The equivalent number N_i of independent samples has been defined as the ratio of the variance of a single sample to the variance of the sampled mean (Lee, 1961; Nathanson, 1969). For discrete sampling N_i is given as

$$N_i^{-1} = \frac{1}{N_q} + \frac{2}{N_q^2}\sum_{K_q=1}^{(N_q-1)}(N_q - K_q)\rho_q(K_q l_q), \quad (4a)$$

where N_q is the number of samples spaced l_q apart, where q represents time, angle or range. For continuous sampling $l_q \to 0$, and the summation becomes the integral

$$N_i^{-1} = 2\int_0^{M_q}\frac{M_q - X_q}{M_q^2}\rho(X_q)dX_q, \quad (4b)$$

where M_q is the sample size and X_q refers to time, angle or range.

To determine N_i for any receiver type, we substitute the appropriate correlation function from Eq. (3) into Eqs. (2a), (2b) or (2c) and then into Eqs. (4a) or (4b).

4. Averaging in time space

For samples averaged with a stationary antenna and obtained at a fixed range location, we have $\rho(D)\rho(\Delta) = 1$. The assumed normalized correlation function [Eq. (3)] and the Doppler power spectrum variance are then modeled, respectively, as

$$\rho(D,\Delta,\tau) = \rho(\tau) = \exp(-\tau^2/2\sigma_\tau^2),$$
$$\sigma_f^2 = (2\pi\sigma_\tau)^{-2}, \quad (5)$$

where σ_τ is the correlation time scale and σ_f^2 the Doppler power spectrum variance. Combining Eq. (5) and Eqs. (2a), (2b) and (2c) we obtain for each receiver

Linear

$$\rho_{\text{lin}}(\tau) = \frac{\pi}{(4-\pi)}\sum_{m=1}^{\infty}A_m^2 m^{-2}\exp(-m\tau^2/\sigma_\tau^2) \quad (6a)$$

Square law

$$\rho_{\text{sl}}(\tau) = \exp(-\tau^2/\sigma_\tau^2) \quad (6b)$$

Logarithmic

$$\rho_{\log}(\tau) = \frac{6}{\pi^2}\sum_{m=1}^{\infty}m^{-2}\exp(-m\tau^2/\sigma_\tau^2). \quad (6c)$$

Here $A_m = (2m-1)!!/2^m(m-1)!\,(2m-1)$, with double factorial indicating products of odd values.

Substituting Eqs. (6a), (6b) and (6c) in Eq. (4a) gives N_i for N discrete samples spaced l_τ seconds apart:

Linear

$$N_{i\,\text{lin}}^{-1} = \frac{1}{N} + \frac{2\pi}{(4-\pi)}\sum_{K=1}^{N-1}\sum_{m=1}^{\infty}(N-K)(A_m/mN)^2 \times \exp[-m(Kl_\tau/\sigma_\tau)^2], \quad (7a)$$

Square law

$$N_{i\,\text{sl}}^{-1} = \frac{1}{N} + 2\sum_{K=1}^{N-1}(N-K)N^{-2}\exp[-(Kl_\tau/\sigma_\tau)^2], \quad (7b)$$

[1] Receivers here refer to signal detection law.

Logarithmic

$$N_{i\,\log}^{-1} = \frac{1}{N} + \frac{12}{\pi^2} \times \sum_{K=1}^{N-1} \sum_{m=1}^{\infty} (N-K)(Nm)^{-2} \exp[-m(Kl_\tau/\sigma_\tau)^2]. \quad (7c)$$

The total averaging time is $T = (N-1)l_\tau$.

When $l_\tau \rightarrow 0$, the case of continuous sampling in time, N_i is obtained by substituting Eqs. (6a), (6b) or (6c) into Eq. (4b). The results are

Linear

$$N_{i\,\mathrm{lin}}^{-1} = \frac{\sigma_\tau(\pi)^{3/2}}{T(4-\pi)} \sum_{m=1}^{\infty} \left(\frac{A_m}{m}\right)\left\{m^{-1/2}\,\mathrm{erf}(m^{1/2}T/\sigma_\tau) + \frac{\sigma_\tau}{mT\pi^{1/2}}[\exp(-mT^2/\sigma_\tau^2 - 1)]\right\}, \quad (8a)$$

Square law

$$N_{i\,\mathrm{sl}}^{-1} = \frac{\pi^{1/2}\sigma_\tau}{T}\,\mathrm{erf}\left[\frac{T}{\sigma_\tau}\right] + \left[\frac{\sigma_\tau}{T}\right]^2 \{\exp[-(T/\sigma_\tau)^2 - 1]\}, \quad (8b)$$

Logarithmic

$$N_{i\,\log}^{-1} = \frac{6\sigma_\tau}{\pi^2 T} \sum_{m=1}^{\infty} \left\{\frac{\pi}{m^{3/2}}\,\mathrm{erf}\left[\frac{m^{1/2}T}{\sigma_\tau}\right] + \frac{\sigma_\tau}{Tm^4}\{\exp[-m(T/\sigma_\tau)^2 - 1]\}\right\}. \quad (8c)$$

5. Averaging in range space

A rectangular transmitted pulse shape of length D_0 is considered. For this pulse shape, Eq. (3) reduces to the normalized form ($\rho(\tau)\rho(\Delta) = 1$):

$$\rho(D,\Delta,\tau) = \begin{cases} \rho(D) = 1 - D/D_0, & 0 \leq |D| \leq D_0 \\ 0, & |D| \geq D_0. \end{cases} \quad (9)$$

Combining Eq. (9) with Eq. (2a), (2b) or (2c), the receiver correlation functions become

Linear

$$\rho_{\mathrm{lin}}(D) = \begin{cases} \frac{\pi}{4-\pi} \sum_{m=1}^{\infty} (A_m/m)^2(1 - D/D_0)^{2m}, & 0 \leq |D| \leq D_0, \\ 0, & |D| \geq D_0, \end{cases} \quad (10a)$$

Square law

$$\rho_{\mathrm{sl}}(D) = \begin{cases} (1 - D/D_0)^2, & 0 \leq |D| \leq D_0, \\ 0, & |D| \geq D_0, \end{cases} \quad (10b)$$

Logarithmic

$$\rho_{\log}(D) = \begin{cases} \frac{6}{\pi^2} \sum_{m=1}^{\infty} m^{-2}(1 - D/D_0)^{2m}, & 0 \leq |D| \leq D_0, \\ 0, & |D| \geq D_0. \end{cases} \quad (10c)$$

For N discrete range samples spaced a distance l_r, N_i is found by combining Eqs. (10a), (10b) and (10c) with Eq. (4a). The results are

Linear

$$N_{i\,\mathrm{lin}}^{-1} = \begin{cases} \frac{1}{N} + \frac{2\pi}{N^2(4-\pi)} \times \sum_{K=1}^{K_u} \sum_{m=1}^{\infty} (N-K)(A_m/m)^2(1 - Kl_r/D_0)^{2m}, & K_u \leq D_0/l_r \\ \frac{1}{N}, & l_r > D_0 \end{cases} \quad (11a)$$

Square law

$$N_{i\,\mathrm{sl}}^{-1} = \begin{cases} \frac{1}{N} + \frac{2}{N^2} \sum_{K=1}^{K_u} (N-K)(1 - Kl_r/D_0)^2, & K_u \leq D_0/l_r \\ \frac{1}{N}, & l_r > D_0, \end{cases} \quad (11b)$$

Logarithmic

$$N_{i\,\log}^{-1} = \begin{cases} \frac{1}{N} + \frac{12}{N^2\pi^2} \sum_{K=1}^{K_u} \sum_{m=1}^{\infty} (N-K)m^{-2}(1 - Kl_r/D_0)^{2m}, & K_u \leq D_0/l_r \\ \frac{1}{N}, & l_r > D_0. \end{cases} \quad (11c)$$

The total averaging range is $D_1 = (N - 1)l_r$.

In the limit $l_r \to 0$ (continuous averaging), N_i is obtained by combining Eqs. (10a), (10b) and (10c) with Eq. (4b).

Linear

$$N_{i\,\mathrm{lin}}^{-1} = \begin{cases} \dfrac{2\pi}{(4-\pi)}\left[\dfrac{D_0}{D_1}\right]^2 \sum\limits_{m=1}^{\infty}\left[\dfrac{A_m}{m}\right]^2\left[\dfrac{D_1/D_0}{2m+1} - \dfrac{1}{4m^2+6m+2}\right], & D_1 \geqslant D_0, \quad (12a) \\ \dfrac{2\pi}{4-\pi}\left(\dfrac{D_0}{D_1}\right)^2 \sum\limits_{m=1}^{\infty}\dfrac{A_m{}^2}{m^2(2m+2)(2m+1)}\left[\left(1-\dfrac{D_1}{D_0}\right)^2(m+1) + 2(m+1)\dfrac{D_1}{D_0} - 1\right], & D_1 \leqslant D_0, \quad (12b) \end{cases}$$

Square law

$$N_{i\,\mathrm{sl}}^{-1} = \begin{cases} 2D_0/3D_1 - D_0{}^2/6D_1{}^2, & D_1 \geqslant D_0 \quad (12c) \\ 1 - 2D_1/3D_0 + D_1{}^2/6D_0{}^2, & D_1 \leqslant D_0, \quad (12d) \end{cases}$$

Logarithmic

$$N_{i\,\log}^{-1} = \begin{cases} \dfrac{12D_0{}^2}{\pi^2 D_1{}^2}\sum\limits_{m=1}^{\infty} m^{-2}\left[\dfrac{D_1/D_0}{2m+1} - \dfrac{1}{4m^2+6m+2}\right], & D_1 \geqslant D_0 \quad (12e) \\ \dfrac{12D_0{}^2}{\pi^2 D_1{}^2}\sum\limits_{m=1}^{\infty}\dfrac{1}{(2m+2)(2m+1)}\left[\left(1-\dfrac{D_1}{D_0}\right)2(m+1) + 2(m+1)\dfrac{D_1}{D_0} - 1\right], & D_1 \leqslant D_0. \quad (12f) \end{cases}$$

6. Averaging in angle space

The autocorrelation coefficient of the radar echo signal can be found from an expression analogous to Eq. (9) used in range sampling [$\rho(D)\rho(\tau) = 1$]:

$$\rho(D,\Delta,\tau) = \rho(\Delta) = \int_\theta\int_\phi g(\theta,\phi)g(\theta,\phi-\Delta)d\theta d\phi, \quad (13)$$

where θ and ϕ are angular coordinates from the direction of maximum radiation.

The angle Δ is the amount the antenna axis is rotated (or angular separation between two separate antennas).

For a circular aperture, the assumed volume normalized two-way field pattern after Probert-Jones (1962) is

$$g(\theta,\phi) = \frac{\beta^2}{\pi\theta_1{}^2}\exp[-\beta^2(\theta^2/\theta_1{}^2 + \phi^2/\phi_1{}^2)]. \quad (14)$$

Here, ϕ_1 and θ_1 are the one-way minus 3 dB level beam widths and $\beta = [2\ln(2)]^{1/2}$. For an antenna pattern with a circular cross section and for antenna motion in the ϕ direction, then by substituting Eq. (14) into (13) the correlation becomes

$$\rho(\Delta) = \exp(-\beta^2\Delta^2/2\theta_1{}^2). \quad (15)$$

The angle correlation function for the three receiver laws obtained from Eq. (2) and (15) become identical to (6) with the substitution of $(\beta\Delta/\theta_1)$ for (τ/σ_τ). Therefore, N_i for angular averages, for discrete and continuous sampling is obtained from Eqs. (7) and (8).

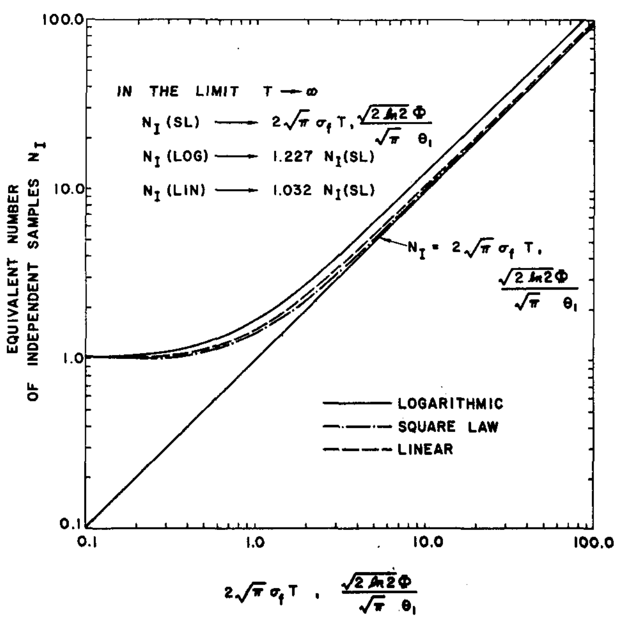

FIG. 1. Equivalent number of independent samples continuous time T or angular Δ averaging and for logarithmic, square-law and linear receivers.

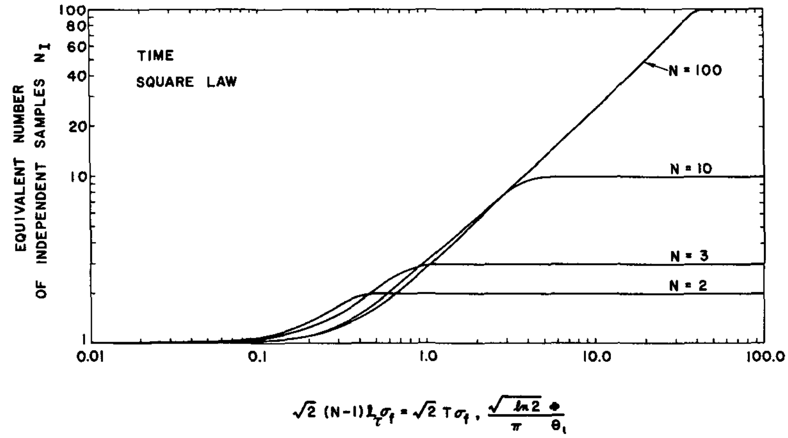

FIG. 2. Equivalent number of independent samples for N discrete samples in time T or angle Δ for the square law receiver.

7. Numerical results

The equations describing discrete and continuous sampling are analyzed using numerical techniques; the results are shown in Figs. 1–8.

a. Time

Fig. 1 shows N_i for the three receiver laws when averaging continuously in time. The second label on the abscissa is for averaging in angle (see Section 7b). For the square-law receiver, N_i has an asymptotic value

$$N_i = 2\pi^{1/2}\sigma_f T \tag{16}$$

as $\sigma_f T \rightarrow \infty$. This is a good approximation for N_i when $(2\sqrt{\pi}\sigma_f T) > 10$. Under these conditions, the linear and logarithmic curves differ from the square-law curve by the factors 1.032 and 1.227, respectively. All three receivers yield approximately one equivalent sample if $(2\sqrt{\pi}\sigma_f T) < 0.1$.

In Figs. 2 and 3, numerical results for discrete time sampling are given. The linear receiver is not included since the square-law curve (Fig. 2) closely approximates the linear receiver (see Fig. 1). An interesting feature in these graphs occurs for a small number of samples; it is possible to obtain a higher N_i value in a given sample time T with fewer samples. Otherwise, these figures are similar to Fig. 1, except N_i becomes equal to the number of samples as T increases. A direct comparison between N_i for different receiver types is given in Fig. 4 as a function of $T\sigma_f$ and 10 samples.

As an example of the utility of Figs. 1–4, consider the video of a radar with a stationary antenna sampling for 100 ms at fixed range. Then for a Doppler spectra width of 20 Hz, N_i is approximately 7.09, 7.32 and 8.70 for the square-law linear and logarithmic receivers, respectively. It could be stated that the variance of the received signal has been reduced by these amounts through averaging.

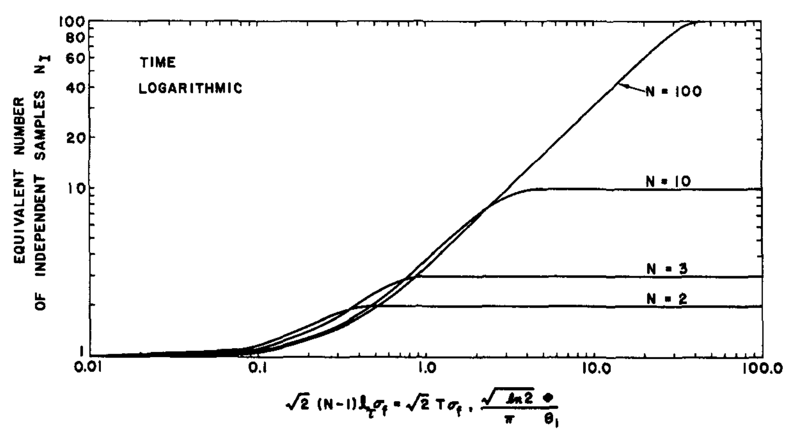

FIG. 3. Equivalent number of independent samples for logarithmic receiver and N discrete samples in time T of angle Δ.

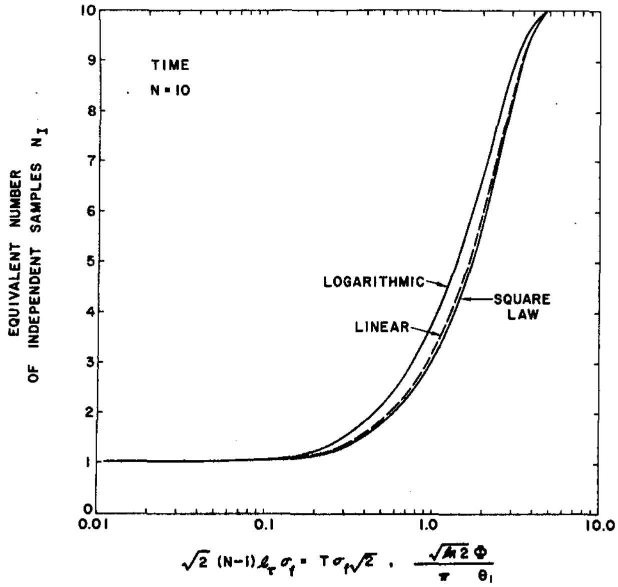

FIG. 4. Comparison of the number of independent samples from 10 discrete samples in time T or angle Δ from logarithmic, linear and square-law receivers.

b. Angle

The N_i factor for angular averaging is also shown in Figs. 1–4. The assumed form of the time and angle correlation functions are identical; thus, the results are identical. A second scale for angular averaging in terms of ϕ/θ_1 is included to read angular N_i values. The angle over which averages are made is represented by ϕ.

As an example of angular sampling, consider two radar beams with the measurements made at identical time and range. Assuming the two beams both have beamwidth (θ_1) of 4° and are separated by an angle (ϕ) of 8°, N_i is found to be approximately 1.8 for a square-law receiver.

c. Range

The equations describing range averaging are graphed in Figs. 5–8. For continuous range averaging, shown in Fig. 5, the square-law receiver curve has the limit $3D_1/2D_0$ as $D_1 \rightarrow \infty$, whereas the linear and logarithmic receivers tend asymptotically to 1.042 N_i(sl) and 1.312 N_i(sl), respectively. All three, as expected, yield only one sample as $D_1 \rightarrow 0$. The logarithmic curve gives the maximum N_i, the square-law curve gives the minimum number, with the linear approximately equal to the square law.

Figs. 6 and 7 show N_i for discrete sampling at the output of square-law and logarithmic receivers. The linear receiver yields results similar to the square law and are not shown. The case for 10 discrete samples is provided in Fig. 8 for all three receiver types. Here, N_i for all receivers is ~10 for $(D_1/D_0) > 10$ and ~1 for $(D_1/D_0) < 10^{-2}$.

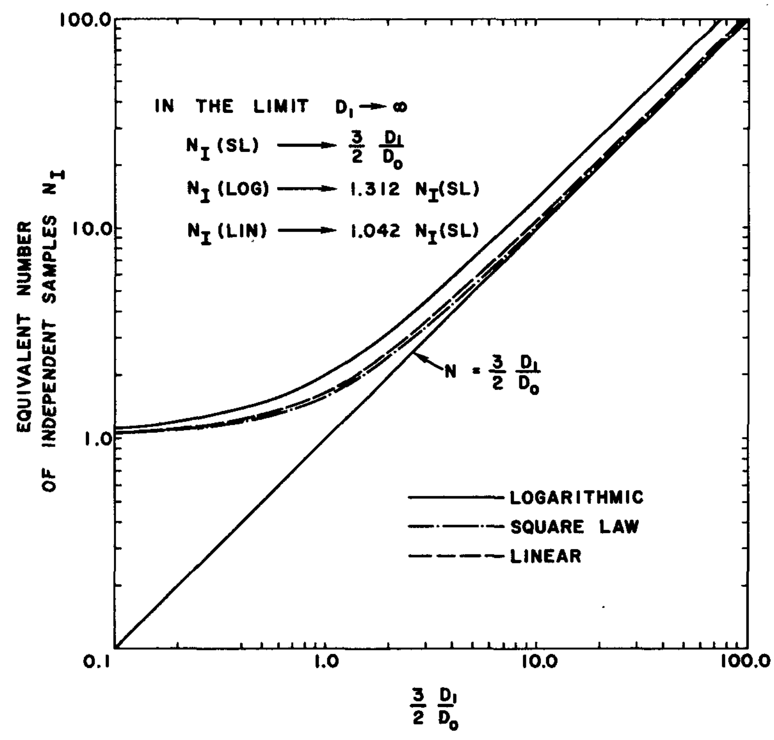

FIG. 5. Equivalent number of independent samples for continuous sampling in range D_1 for logarithmic square-law and linear receivers.

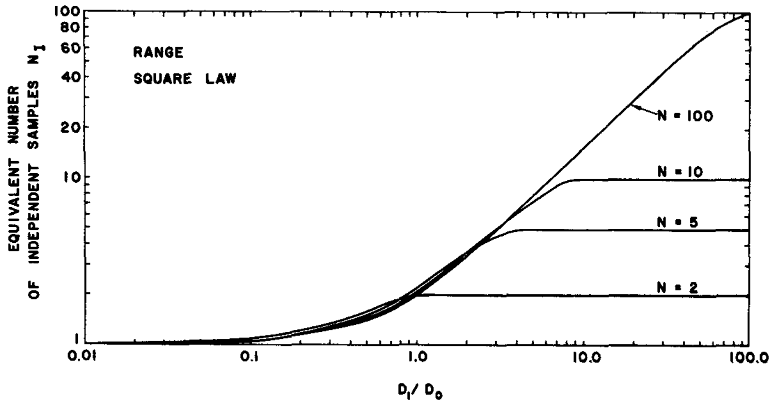

FIG. 6. Equivalent number of independent samples for N discrete samples in range D_1 for the square law receiver.

As an example of range sampling, consider that a radar has a square law receiver and a rectangular shaped 1 μs transmitted pulse (D_0 = 150 m). Then sampling the square-law receiver output continuously over a range of 300 m (D_1) will yield a value of $N_i = 3.43$.

8. Averaging in time, angle and range

In previous sections, sampling in time, angle and range independently are treated. However, in a typical pulse radar system, echo signals are sampled at fixed ranges and regularly spaced times while the antenna axis changes angular position at a constant rate. Under these conditions, one need only to consider time and range as independent variables. Angle sampling is then related to time in terms of a constant rotation rate α, i.e., $\Delta = \alpha\tau$. The correlation function for angle and time at one range is combined as follows:

$$\rho(\Delta,\tau) = [\exp(-\beta^2\Delta^2/2\theta_1^2)][\exp(-\tau^2/2\sigma_\tau^2)], \quad (17)$$

and at $\Delta = \alpha\tau$

$$\rho(\alpha,\tau) = \exp[-\tau^2/2(\sigma_\tau')^2], \quad (18)$$

where

$$(\sigma_\tau')^{-2} = (\beta\alpha/\theta_1)^2 + (\sigma_\tau)^{-2}. \quad (19)$$

The equivalent Doppler spectral width then, is given by

$$(\sigma_f')^2 = (\beta\alpha/2\pi\theta_1)^2 + \sigma_f^2. \quad (20)$$

Hence, antenna rotation increases the spectral width for each receiver type. One need only to compute values of σ_τ' or σ_f' and use the time curves shown in Figs. 1–4 to obtain N_i for combined time and angle (time-angle) sampling. To include range with the combined time and angle sampling, N_i would be the number computed (for time-angle) multiplied by the number of independent samples in range.

As an example of determining N_i when combining time and angle, as in Eq. (18), and also averaging over range, consider a radar system with a square-law receiver, an antenna beamwidth (θ_1) of 1.0 deg, an antenna rotation rate of $\alpha = 6.0$ deg s^{-1}, a

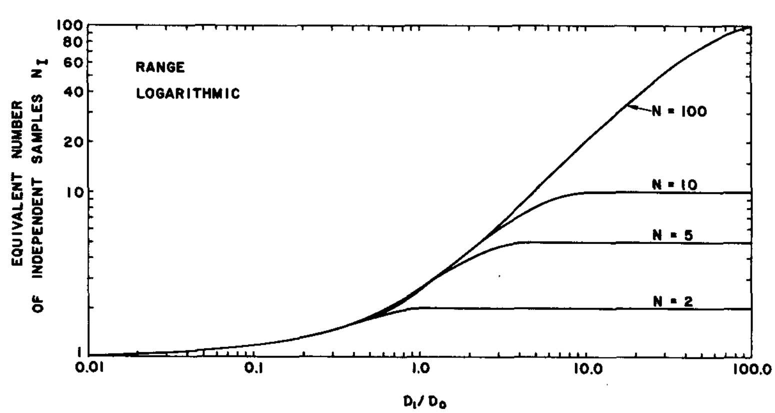

FIG. 7. As in Fig. 8 except for the logarithmic receiver.

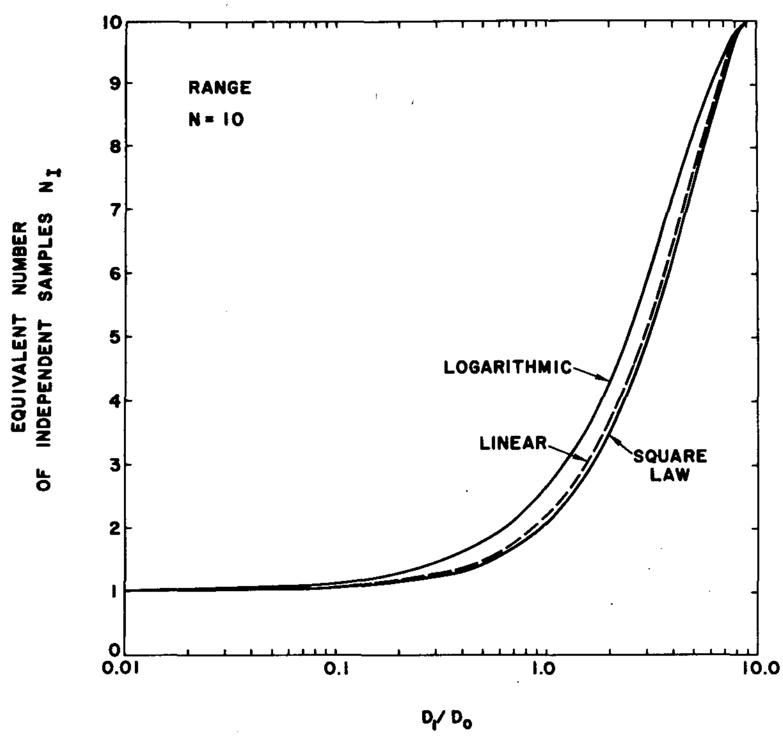

FIG. 8. Comparison of the number of independent samples obtained with 10 discrete samples in range D_1 from the logarithmic, linear or square-law receiver.

Doppler spectrum width of 20.0 Hz (σ_f), a 1.0 μs transmitted pulse (D_0 = 150 m), and whose output is sampled continuously in time for 100 ms and 30 m in range. From the range example in Section 7c, N_i for range is 3.43. N_i for time-angle is 7.09 as determined in the time example in Section 7a. The Doppler spectrum width is increased only ~0.2% by rotating the antenna and the total N_i for time, angle and range averaging then is 24.3.

9. Summary

The logarithmic receiver averaged output yields the largest value of N_i for dependently sampled data presented here. However, Sirmans and Doviak[2] have shown that when the averaged outputs of logarithmic or linear receivers are unbiased in calculating average power the square-law receiver averaged output yields the smallest variance.

When the antenna is rotated, angle sampling can be expressed as a function of time and combined with time sampling which gives an increased Doppler spectrum width and an increase in number of independent samples. The equivalent number of independent samples can be further increased by averaging in range. Averaging in time and at discrete ranges or continuously over a given range yields an N_i given by the products of N_i for range averaging and N_i for time-angle averaging. For either time-angle or range sampling, there exists a sample size at which discrete sampling yields a larger number of independent samples (variance reduction) than continuous uniformly weighted sampling. Therefore, an optimum weight, different than uniform, exists for computing mean values with smallest variance.

The assumptions used to derive the numerical values shown graphically in Figs. 1–8 should not limit their use in practical application, since they are good approximations to the physical process. The restrictions include the assumption that 1) statistical parameters are stationary throughout the time, angle and range interval sampled; 2) the Doppler spectrum is Gaussian; 3) the antenna pattern is Gaussian; 4) the transmitted pulse is rectangular; 5) the receiver band width is wide (infinite) compared to the reciprocal of transmitted pulse width; 6) no noise is added by the receivers; and 7) range samples are taken over a distance which is small compared to radar to sample volume distance.

[2] Sirmans, D., and R. J. Doviak, 1973: Meteorological radar signals intensity estimation. NOAA Tech. Memo. ERL NSSL-64, National Severe Storms Laboratory [NTIS 8Q COM-73-11923/2A8].

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