# Reynolds Stress Deflections of the Bivane Anemometer

G. CHIMONAS

*CIRES/WPL, NOAA, Boulder, CO 80302*

(Manuscript received 31 May 1979, in final form 17 November 1979)

ABSTRACT

We show that a bivane anemometer or other elevation angle sensing device records a nonzero mean angle when responding to cross-correlated fluctuations in a mean wind. Our analysis shows how this mean offset can be used to derive the wind-aligned Reynolds stress directly.

This theory is applied to a historical data set. It is first shown that the prior derivation of a mean vertical wind component is erroneous, and then that reinterpretation of results in terms of the Reynolds stress response is consistent with other aspects of the record.

## 1. Introduction

Although bivane anemometers are less sensitive than some other wind measuring devices, their ruggedness and relatively low cost determine their use in many operational situations (SethuRaman and Tuthill, 1978). However, certain peculiarities of the instrument severely restrict the usage of the data sets it produces. For instance, in field conditions where no mean vertical wind can be expected, the bivane will display a nonzero mean elevation angle, no matter how carefully it is calibrated in the wind tunnel; moreover, accurate inversion of bivane readings into wind components is often complex and indirect (Kaimal and Touart, 1967). It is not too surprising then that in studies where high accuracy is required, as in turbulence transfer investigations, the bivane has historically led to puzzling results (Barad, 1964) and is now often excluded from advanced micrometeorological systems. This is not necessarily justified. Perhaps because of its mechanical simplicity, the output from the bivane is frequently used in a simplistic fashion. This leads to arduous numerical processing, often based on approximations or averaging techniques that introduce errors comparable in magnitude with the quantity whose measurement is being attempted.

In this paper we suggest that alternative approaches are available, and that for some aspects of dynamic meteorology the bivane may be capable of providing sophisticated information in a relatively direct manner. Like any aerodynamic system, this instrument exhibits nonlinear responses to the perturbations about a mean wind field, and proper manipulation of the output allows direct access to valuable nonlinear information, specifically the flow-aligned component of the Reynolds stress.

In the subsequent presentation, the specific term bivane anemometer will be used in the text. Nevertheless, the results are more general, and may be usefully applied to any elevation-angle sensing device.

## 2. Mean response of the bivane to correlated fluctuations

Our analysis shows that appropriate manipulation of the bivane output allows direct access to significant second-order means. The results are analogous to those obtained by Rose (1962) in his investigations of hot-wire anemometers and the $x$ meter.

The technique relies on an aspect of the bivane response which we may illustrate with a very elementary example. Suppose an ideal bivane experiences a wind field

$$\mathbf{V} = \hat{\mathbf{x}}U_0 + (\hat{\mathbf{x}} + \hat{\mathbf{y}})a \sin\omega t, \qquad (1)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in the horizontal and vertical directions, respectively.

Since the bivane aligns against the instantaneous wind direction, its elevation angle $\phi$ is given by

$$\phi(t) = -\tan^{-1}\left(\frac{a \sin\omega t}{U_0 + a \sin\omega t}\right). \qquad (2)$$

This function is not symmetric about zero. Fig. 1 shows that for $U_0/a > 0$ the oscillations are weighted to the negative phase of $\sin\omega t$, and $\phi$ assumes a nonzero negative mean value. As we shall show below, the mean offset of $\phi$ is related to the cross correlation of the fluctuations.

Of course there is no intrinsic error implied here. If wind speed and angle are accurately recorded,

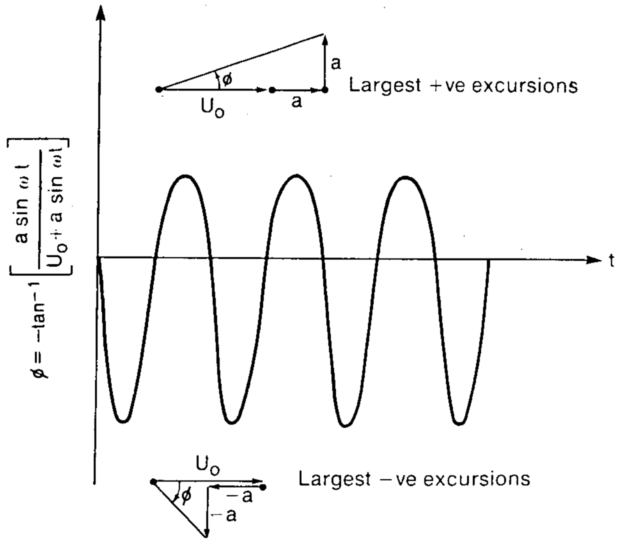


FIG. 1. Schematic showing the asymmetry in vane response to a wind system $\hat{x}(U_0 + a \sin\omega t) + \hat{y} a \sin\omega t$.

the vector wind field (1) may be recovered. But it is very tempting to simplify the analysis by discarding seemingly uninteresting portions of the data. For instance, in wave work high-frequency turbulence is often regarded as nothing but background noise to be eliminated by some form of running average. However, correlations of the high-frequency components provide a mean offset of the bivane that survives the averaging technique, and the end result is a set of low-frequency massaged data that is no longer self-consistent. Valuable information has been lost and peculiarities that cast doubt on the data sets have been introduced. We suggest that reinterpreting the offsets can restore part of the information content of the data while allowing the very real convenience of analyzing a smoothed version of it.

We will now investigate the relation between the Reynolds stress and the mean bivane deflection. The analysis makes the usual assumptions of boundary-layer theory, although in actual applications it is wise to check these for consistency.

Let the wind field be

$$\mathbf{V} = \hat{\mathbf{x}}(\bar{U} + u') + \hat{\mathbf{y}}(\bar{W} + w'), \tag{3}$$

where by definition $\bar{U}$ and $\bar{W}$ are component averages over the time interval of interest. In the long time limit we would expect $\bar{W} \to 0$ over flat terrain; but since it is often necessary to examine relatively short intervals of record, a small but nonzero $\bar{W}$ will be included in this analysis.

The elevation angle $\phi$ of the bivane is given by

$$\phi = -\tan^{-1}\left(\frac{\bar{W} + w'}{\bar{U} + u'}\right). \tag{4}$$

We now assign smallness parameters

$$\left.\begin{array}{l} |w'| \quad \text{and} \quad |u'| \approx \epsilon_1 |\bar{U}| \\ \qquad\qquad\quad |\bar{W}| \approx \epsilon_2 |\bar{U}| \end{array}\right\} \tag{5}$$

and expand Eq. (4) in Taylor's series to produce

$$\phi = -\frac{w'}{\bar{U}} + \frac{(w'u' - \overline{w'u'})}{\bar{U}^2} + \frac{u'\bar{W}}{\bar{U}^2} + \left[-\frac{\bar{W}}{\bar{U}} + \frac{\overline{w'u'}}{\bar{U}^2}\right] + O([\epsilon_1 + \epsilon_2]^3). \tag{6}$$

We have explicitly separated the right-hand side into mean and oscillating components. Averaging both sides gives

$$\bar{\phi} = -\frac{\bar{W}}{\bar{U}} + \frac{\overline{w'u'}}{\bar{U}^2} + O([\epsilon_1 + \epsilon_2]^3). \tag{7}$$

We now see how the mean angle $\bar{\phi}$ is generated. There is a contribution from any nonzero mean vertical wind component. This, and error in the instrument, are the commonly identified sources. But there are also contributions from cross correlations of the wind components. We have identified here the leading correlation, which is readily related to the Reynolds stress on the mean wind direction. As the averaging time interval $T_A$ is increased, $\bar{W}$ will normally decrease, and if we require

$$T_A: \epsilon_2 \leq \epsilon_1^3,$$

it follows that

$$\bar{\phi} \to \frac{\overline{w'u'}}{\bar{U}^2} + O(\epsilon_1^3), \tag{8}$$

or

$$\overline{w'u'} \to \bar{U}^2\bar{\phi} + O(\epsilon_1^3). \tag{9}$$

There is no reason whatsoever to suspect that the interval average of the cross correlations should vanish, so we must expect, in general, a nonzero contribution of order $\epsilon_1^2$ in (8).

It should be noted that the angle $\phi$ is always referenced to the mean wind direction. We only find out the absolute value of $\bar{U}$ when we add information about the wind azimuth. This has the following consequence. We know, from elementary considerations, that near the ground the wind is a monotonic function of height, and the mean turbulence transfer of momentum is in the negative direction of the wind gradient. Thus

$$\overline{w'u'} = -Q^2\bar{U}, \tag{10}$$

where $Q^2$ is a positive coefficient.

In absolute terms, this means that $\overline{w'u'}$ changes sign with the (cyclic) rotation of the local mean wind, and referred to an absolute frame the long-term average of $\overline{w'u'}$ may fluctuate about, and tend toward, zero. However, in the measurement of $\phi$, the direction of $\bar{U}$ is *by definition* the positive direction

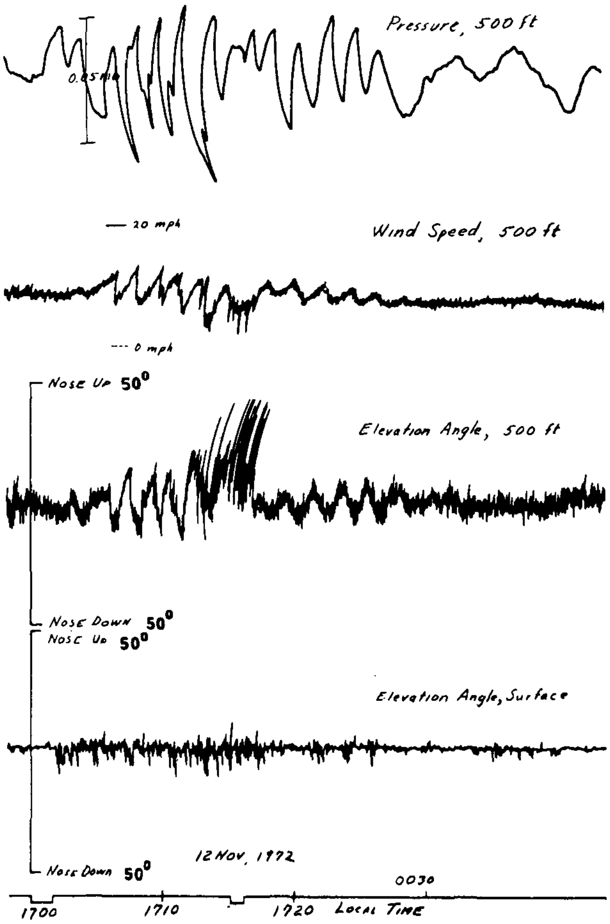


FIG. 2. Raw microbarograph and anemometer records from the top of the 150 m tower, showing the large-amplitude perturbations in pressure and horizontal and vertical velocities accompanying the wave-generation event observed by the acoustic echo sounder. The elevation angle of the surface wind recorded at a height of 2 m is also shown, to indicate that the wave generation was accompanied by an increase in the turbulence intensity of the surface boundary layer.

against which fluctuations $u'$ are registered, i.e., the local frame of reference rotates with the mean wind vector. Hence near the surface, where (10) is obeyed, $\overline{w'u'}$ is always negative in the frame of measurement and both the intermediate and long-term averages of $\phi$ will be negative and of order $\epsilon_1^2$ [cf. Eq. (8)].

The simple calculations given here show how correlated fluctuations within the response range of the instrument produce a mean deflection, but leave open the question as to whether a particular bivane anemometer will give in this way a reliable measure of the total Reynolds stress. Clearly, the instrument needs to be able to respond faithfully to the frequency ranges of the fluctuations that dominate the Reynolds stress transfer. It is also probable that both the averaging interval and the conditions of measurement will condition the validity of the analysis. Only direct field tests can tell. At the present time, the 300 m tower of the Boulder Atmospheric Observatory (operated jointly by the Wave Propagation Laboratory and NCAR) uses only sonic anemometers. The bivanes of earlier field work have all been retired in favor of these more accurate (if more expensive) devices. It is hoped that sometime in the near future a mechanical elevation-angle sensing system will be collocated on the tower with one of the sonic systems. The potential usefulness of the approach outlined above could then be put to a rigorous test.

## 3. The bivane response in a wave/turbulence event

Data preserved from a rather ancient field experiment has been found to illustrate one specific application of the theory. The measurements were taken during a field experiment conducted in 1972. A 150 m meteorological tower was available for boundary layer profiling and continuous monitoring at certain fixed levels. Gill bivane anemometers were used for the wind sensing. In the course of the experiment particularly interesting events were noted for later analysis. One such, a rather vigorous burst of wave activity, was examined and published by Hooke *et al.* (1973). The data, in both raw and processed forms, were presented in that paper. We reproduce those records in Figs. 2 and 3.

The following comments may be applied to the raw data:

1) The upper level tower data show strong wave activity between 1705 and 1720, with noticeable but reduced activity for the subsequent 10 min. The wave period is ~2 min.

2) All tower levels recorded enhanced turbulent activity over the same intervals. The turbulence recorded on the bivane has time scales of 5 s and less. Comparison of the microbarograph and anemometer records shows that there is a separation of time scales between "wave" and "turbulence" which is sufficiently great that the microbarograph filters remove the latter but pass the former.

3) To extract and analyze the wave components of the record, the raw data were selectively digitized from the analog records. This entailed reading to the "center" of the noisy band of turbulent fluctuations. In effect, then, a running average was applied to the record to eliminate high frequencies and reduce the work needed to extract the low-frequency wave motions.

One observation about the reduced data (Fig. 3) is pertinent: the vertical wind speed is seen to have a mean offset from zero over the interval of the wave activity. We have examined contiguous sections of the bivane data when atmospheric conditions were quiet. No alignment error or mean offset of the bivane was found for these intervals.

Over the first 10 min of wave activity, the mean value of $W$ in Fig. 3 is about $-0.5$ m s$^{-1}$. We know that such large sustained downdrafts are very unlikely near the ground. In fact they are inconsistent with the other data. This is easily seen as follows.

Since $\bar{W}$ is zero at ground level, a value $\bar{W} = -0.5$ m s$^{-1}$ at 150 m implies a mean gradient through the region of

$$\frac{d\bar{W}}{dz} = -3.3 \times 10^{-3}\ \mathrm{s}^{-1}. \tag{11}$$

But the continuity equation requires

$$\frac{d\bar{U}}{dx} = \frac{d\bar{W}}{dz}. \tag{12}$$

The wave packet is traveling at ~3.5–4.0 m s$^{-1}$ (Hooke *et al.*, 1973), so the 10 min interval of activity corresponds to a spatial extent $\bar{X}$ of 2.1–2.4 km for the wavepacket.

The values $\bar{W}$ *et seq*. hold over this packet space, hence the change of mean horizontal velocity over the event must be

$$\Delta\bar{U} \approx \frac{d\bar{U}}{dx}\bar{X}, \tag{13}$$

$$\approx 6.9\text{–}7.9\ \mathrm{m\ s}^{-1}. \tag{14}$$

Examination of the record of $u$ (Fig. 3) for the 150 m height shows no such change. Nor do the other records of the tower. Note that the procedure leading to (14) is a rigorous lower bounding calculation. If $d\bar{W}/dz$ were selectively distributed over the interval, larger values of $\Delta\bar{U}$ would result at corresponding heights. We can therefore conclude with confidence that the derived value of $\bar{W}$ is entirely spurious. How then did it arise? Alignment errors of the bivane have already been eliminated. It seems likely then that the offset discussed in Section 2 of this paper may have a role. We now show that the data handling technique used by Hooke *et al.* (1973) to remove the high frequencies will introduce such an offset if there are cross correlations of these components.

We now seek to choose a time interval for the average [represented by the overbars in Eq. (6)] that corresponds to the manner in which the raw data were handled. The signal in Fig. 2 has two distinct peaks in its frequency distribution, one around $8 \times 10^{-3}$ Hz corresponding to the wave, and one at (or above) 0.2 Hz, which we attribute here to turbulence. The smoothing process applied to the data consisted of fitting a locally smooth line through the mean locus of the ragged record produced by the turbulence. The aim, of course, was to eliminate the turbulence without destroying the wave. Within subjective limits, the process consists of applying a running average over a few cycles of the turbulent

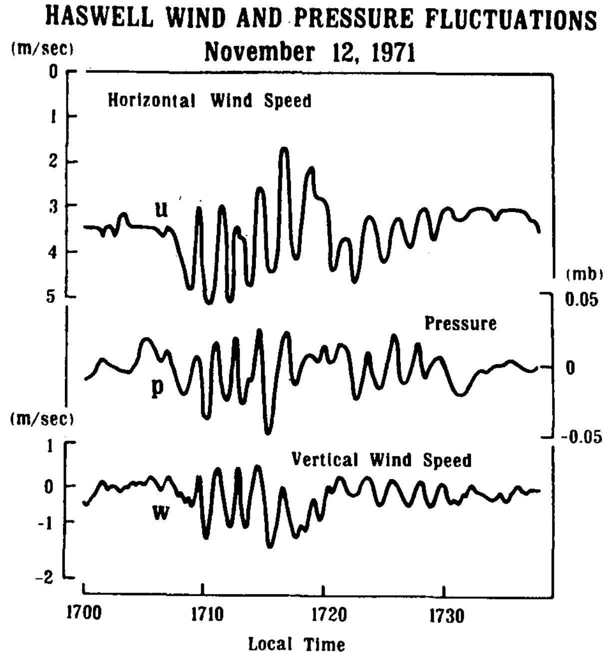


FIG. 3. Reduced forms of the data shown in Fig. 2. The anemometer records have been smoothed and analyzed into velocity components, while the pressure data have been replotted on a rectilinear scale.

fluctuations. To represent this mathematically, we separate the actual velocity field into different frequency components and apply the appropriate time interval interpretation of the overbar.

Let

$$\left.\begin{aligned} W &= W_W + W_T \\ U &= U_M + U_W + U_T \end{aligned}\right\}, \tag{15}$$

where we have separated the velocity components *of the entire record* into means, wave parts and turbulence parts. [Note: This separation is quite distinct (as yet) from anything implied in the formulation of (6). Also, we have left out any mean value of $W$ for the record interval, as we have already concluded that this result of the smoothed analysis is spurious.]

We may now define the overbar average as the time average over an interval $T$, where $T$ is a few times the characteristic period of the turbulence, i.e., $T \sim$ tens of seconds. Then, applying this to (15) we have

$$\left.\begin{aligned} \bar{W} &= W_W(t) + O(\delta^2 W_W + \epsilon W_T) \\ \bar{U} &= U_M + U_W(t) + O(\delta^2 U_W + \epsilon U_T) \end{aligned}\right\}. \tag{16}$$

In these relations $W_W(t)$ and $U_W(t)$ are the values of these functions at the time $t$ corresponding to the midpoint of the interval over which the average is applied. The correction factor $\epsilon$ is the residual from averaging a random or quasi-random function over a finite interval. It will be of the order of the correlation time of the turbulence divided by $T$, and we have arranged that this should be no more than $0.1 \approx (\frac{1}{4}\tau) \div (\text{few} \times \tau)$. The other correction factor is easily estimated as

$$\delta \approx T/\text{wave period}. \quad (17)$$

We now have the effective separation

$$\left.\begin{aligned} W &= \bar{W} + w' \\ U &= \bar{U} + u' \end{aligned}\right\}, \quad (18)$$

which should be compared with Eq. (3), where $\bar{W}$ and $\bar{U}$ are given by (16), and the departures by the differences between (15) and (16):

$$\left.\begin{aligned} w' &= W_T + O(\delta^2 W_W + \epsilon W_T) \\ u' &= U_T + O(\delta^2 U_W + \epsilon U_T) \end{aligned}\right\}. \quad (19)$$

Substituting these expressions into (6) gives

$$\phi(t) = \left[ - \frac{W_T}{(U_M + U_W(t))} + \frac{W_T U_T - \overline{W_T U_T}}{(U_M + U_W(t))^2} \right] - \frac{W_W(t)}{(U_M + U_W(t))} + \frac{\overline{W_T U_T}}{(U_M + U_W(t))^2} + O\left( \left[ \frac{U_T + W_T + U_W + W_W}{U_M} \right]^3 + \delta^2 \left[ \frac{W_W + U_W}{\bar{U}} \right] + \epsilon \left[ \frac{W_T + U_T}{\bar{U}} \right], \text{etc.} \right). \quad (20)$$

In the present case none of the correction terms is significant. Finally,

$$\bar{\phi}(t) = - \frac{W_W(t)}{(U_M + U_W(t)} + \frac{\overline{W_T U_T}}{U_M{}^2} + \cdots. \quad (21)$$

The first term on the right-hand side of (21) is exactly what the running average technique sought to achieve. It represents the deflection of the bivane that would occur in the wave plus mean velocity fields alone. But the next term is an unwanted remnant of the high-frequency oscillations. In each interval $T$ it contributes to the deflection $\bar{\phi}$, and if it is effectively constant over the event of interest, as it will be if the nature of the turbulence remains unchanged, it provides a long term offset of $\phi$.

It is easily seen that computing the vertical wind from the $T$ average of the wind speed and expression (21) for $\phi$ gives two terms. The first is the true wind component that comes from the wave motion. The second is a spurious contribution

$$W_S(t) \approx - \frac{\overline{W_T U_T}}{U_M}. \quad (22)$$

Examing Fig. 3 shows we can identify the mean vertical wind previously reported during the wave event with an approximately constant value $\overline{W_T U_T} \approx 1.8$ mks. This value is not inconsistent with the high-frequency excursions of $\phi$ seen on the raw data record. It may also be remarked that it is considerably larger than, and of the opposite sign to, the wave associated Reynolds stress deduced by Hooke *et al.* (1973).

## 4. Conclusions

We have shown theoretically that a wind-aligned Reynolds stress produces an elevation angle offset of a sufficiently sensitive bivane anemometer. It is suggested that this may provide a basis for directly monitoring this Reynolds stress component in some situations, and that the observer must always guard against misinterpreting the offset as a mean vertical component of the wind field.

We have applied the analysis to a special case, a strong boundary layer wave/turbulence event. A more simplistic analysis gave a physically unrealistic mean vertical wind during the event. The correlation offset hypothesis, on the other hand, provides a consistent and plausible interpretation of the data, which also indicates a much enhanced turbulent Reynolds stress during the interval of strong wave activity.

*Acknowledgments.* I am indebted to R. J. Hill for his criticisms of an earlier version of this paper, and to J. C. Kaimal and J. C. Wyngaard for their helpful suggestions for its improvement.

REFERENCES

Barad, M. L., 1964: The vertical transfer of momentum and heat at and near the earth's surface. *Trans. N.Y. Acad. Sci.*, **26**, 830–844.

Hooke, W. H., F. F. Hall and E. E. Gossard, 1973: Observed generation of an atmospheric gravity wave by shear instability in the mean flow of the planetary boundary layer. *Bound.-Layer Meteor.*, **5**, 29–41.

Kaimal, J. C., and C. N. Touart, 1967: Critical examination of a transformation to vector-mean coordinates. *J. Appl. Meteor.*, **6**, 583–587.

Rose, W. G., 1962: Some corrections to the linearized response of a constant-temperature hot-wire anemometer operated in a low-speed flow. Amer. Soc. Mech. Eng., Paper No. 62-WA-11.

SethuRaman, S., and W. A. Tuthill, 1978: Response characteristics of a new bidirectional vane. *Bull. Amer. Meteor. Soc.*, **59**, 1114–1118.