The Response of Superpressure Balloons to Gravity Waves

G. D. NASTROM

Fluid Mechanics Program, University of Minnesota, Minneapolis, MN 55455[1]

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ABSTRACT

Analytic solutions of a balloon's response to simultaneous variations of the vertical wind speed and the density are obtained using Fourier expansion methods. It is assumed that density and wind are in phase quadrature. The balloon's response depends on wave period, density and wind amplitudes, and static stability. When density and wind amplitudes are related as in a gravity wave the amplitude of the balloon's velocity response varies from about 75 to 105% of the wind amplitude as the lapse rate condition varies from isothermal to adiabatic, for a typical wave period (13.8 min) and air motion amplitude (30 cm s^{-1}). Further, balloon phase leads air motion phase by ~30° for adiabatic lapse, but lags the air motion by ~25° for isothermal lapse conditions at this wave period. For very long period waves the balloon asymptotically approaches a true isopycnic tracer (with some phase shift) for high static stability, but not for low static stability.

1. Introduction

Constant density balloons have generally been regarded as good tracers of horizontal air motions, but it has long been recognized that their vertical motions are only partial indicators of the vertical air motions. Angell and Pack (1962) noted that the primary balloon-height oscillations which they observed seemed related to the atmospheric static stability and the natural period of vertical air parcel oscillations, but they recognized that the balloon could be out of phase with any vertical air currents or could have a different amplitude from them. Studies of mountain wave and gravity wave activity prompted more detailed investigations of the response of a balloon to vertical air currents. Hirsch and Booker (1966), Vergeiner and Lilly (1970) and Hanna and Hoecker (1971) each used numerical integrations of the balloon's equation of vertical motion to analyze the relationship between balloon and air motions. Most studies considered a single sinusoidal wave as the air current, although Hoecker and Hanna (1971)[2] also included horizontal air motions in their computations.

The possible importance of atmospheric density variations was highlighted by Reynolds (1973) who studied temperature, pressure and balloon-height data for flights made in the lee of mountains (mountain wave conditions). He concluded that balloons closely follow isopycnic surfaces, overestimating the crests and troughs of isopycnic waves by only ~6%. Other recent analyses of balloon motions have concentrated on the neutral buoyancy oscillation (Levanon *et al.*, 1974; Levanon and Kushnir, 1976) and the response of a balloon to sinusoidal variations of the height of a balloon's neutral density surface (Massman, 1978). No studies seem to have been made which accommodate simultaneous density and vertical wind variations.

In a gravity wave (including mountain waves) both density and vertical wind speed vary with time. Further, the linear wave theory (Hines, 1974) indicates that the relative phase lag and the amplitude ratio of density and wind variations are not arbitrary, but are related by the so-called polarization relations. As there is considerable evidence that gravity waves play an important role in some tropospheric processes (Einaudi *et al.*, 1978/79) and as constant density balloons may be useful for studying them, it is desirable to predict expected balloon responses to gravity waves. It is emphasized that results from the studies listed above are not suited for this purpose because they do not include both density and wind variations. Further, predictions are most readily made from analytic solutions to the equation of motion, and a general solution to this problem has not been given before.

The purpose of this paper is to present expected balloon responses to single gravity waves of constant period and amplitude using an analytic solution to the balloon's equation of motion, including both density and vertical wind speed (w) variations.

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[1] Permanent affiliation: Research Division, Control Data Corporation, Minneapolis, MN 55440.

[2] Hoecker, W. H., and S. R. Hanna, 1971: Computed response of tetrahedral constant-density balloons to vertical sinusoidal and helical air motions. NOAA TM ERL ARL-31, U.S. Dept. of Commerce, Washington, DC, 31 pp.

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The equation of motion is linearized using routine expansion methods and the analytic solutions are obtained for balloon amplitude and phase relative to those of the atmospheric disturbances.

2. Theory

The equation of vertical motion for a balloon is

$$(M_B + \eta M_a)\frac{\partial^2 z}{\partial t^2} = (M_a + \eta M_a)\frac{\partial w_a}{\partial t} - \tfrac{1}{2}\rho_a C_d A_B\left(\frac{\partial z}{\partial t} - w_a\right)\left|\frac{\partial z}{\partial t} - w_a\right| - g(M_B - M_a), \quad (1)$$

where

- M_B mass of the balloon system (skin + gas + sonde)
- M_a mass of the displaced air ($=\rho_a V$)
- ρ_a ambient air density
- V volume of the balloon system
- z displacement of balloon from its float level
- w_a vertical air velocity
- A_B cross sectional area of balloon
- C_d form drag coefficient
- η added mass coefficient.

The terms on the right-hand side of (1) are called the dynamic, form drag and buoyancy force terms, respectively. The origin of these terms is given in many standard fluid mechanics texts (e.g., Lamb, 1932, especially Article 143; Batchelor, 1967, especially p. 409) and the interpretation of them has been discussed in several papers (Levanon *et al.*, 1974; Hanna and Hoecker, 1971; Massman, 1978) so will not be repeated here. Skin friction drag, aerodynamic lift and the influence of motions with very small space scales (including any self-induced motions) are assumed negligible. Numerical integration of (1) to obtain $z(t)$ is straightforward for specified atmospheric conditions and balloon constants, and a few such results will be presented later. Analytic solution of (1), however, does not appear possible without first using some simplification.

We assume the balloon is near its float level, i.e., near $z = 0$, where $M_B = M_a$ in an undisturbed atmosphere, and apply the Boussinesq approximation. Also we assume the balloon is spherical, so $\eta = \tfrac{1}{2}$, $A_B = \pi r^2$ and $V = 4\pi r^3/3$, where r is the balloon radius. Next, note that M_a is a function of altitude, depending upon both ρ_a and V. Constant density balloons expand slightly with increased superpressure, but over broad ranges of superpressure the relation between volume and superpressure is nearly linear for the small balloons used in the lower troposphere (Hoecker, 1975) and the relation shows negligible hysteresis at moderate superpressures (Beemer and Markhardt, 1977).[3] Thus, for small displacements from the float level, one can expand M_a, allowing for possible time dependency of the atmospheric density, as

$$M_a = \rho_a V = \bar{\rho}_a[1 + R(t)]V = \left(\rho_0 + \frac{\partial\bar{\rho}_a}{\partial z}z\right)[1 + R(t)]\left(V_0 + \frac{\partial V}{\partial z}z\right), \quad (2)$$

where ρ_0 and V_0 are the values at $z = 0$ in an undisturbed atmosphere, $\bar{\rho}_a = \bar{\rho}_a(z)$ is the undisturbed atmospheric density at z, and $R(t)$ is the percentage change with time of density.

Applying these assumptions and the approximation (2) to (1) gives

$$\frac{\partial^2 z}{\partial t^2} = -\omega^2 z + \tfrac{2}{3}gR - A\left(\frac{\partial z}{\partial t} - w_a\right)\left|\frac{\partial z}{\partial t} - w_a\right| + \frac{\partial w_a}{\partial t}, \quad (3)$$

where $\omega^2 = -2g(\partial M_a/\partial z)/3M_B$ and $A = C_d/4r$. Using the volume versus superpressure curves given by Beemer and Markhardt (1977) or Hoecker (1975) and realistic density profiles, it is found that ω^2 changes by less than 1% $(100\text{ m})^{-1}$ in the lower troposphere. Thus, ω^2 will be treated here as a constant. [The results quoted by Levanon *et al.* (1974) suggest that ω^2 is nearly constant for the TWERLE balloons used in the lower stratosphere, hence generalizing this analysis.]

The influence of the assumptions used in (3) on its integrity can be checked in Table 1 where the results of numerical integrations of (1) and (3) are given. Details of the integration procedure and the balloon constants used are given in the Appendix. In Table 1, the differences between the results for (1) and (3), called the complete and abridged models, respectively, are very small for the amplitude of all harmonics. The phases of the first harmonic, which has much larger amplitude than other harmonics, are also nearly equal. It is concluded that (3) retains the essential features of (1).

Analytic solutions of (3) are sought for the case of constant amplitude, sinusoidal air motions and density variations which have identical wave periods. In a gravity wave with vertical wavelength, period and vertical motion amplitude on the order of 1 km, 20 min and 100 cm s^{-1}, respectively, the polarization relations indicate the phase difference

[3] Beemer, J. D., and T. W. Markhardt, 1977: Tetroon evaluation program. NASA-CR-150437, Washington, DC, 32 pp.

Table 1. Comparison of numerical integration results for Eq. (1) (complete model) and for Eq. (3) (abridged model) for balloon velocity response. Air motion is prescribed by $W_a = w \sin\nu t$ and density variations by $\rho_a = \bar{\rho}(z)(1 - \zeta \cos\nu t)$. Phase is the time of maximum balloon velocity.

Model	Period (s)	w (cm s^{-1})	ζ (density change)	Harmonic analysis of model output W_B: Amplitude of harmonic (cm s^{-1}) 1	2	3	4	5	Phase of the first harmonic (deg)
Complete	832	30	0	25.2	0.0	1.0	0.0	0.2	42.6
Abridged				22.2	0.0	1.1	0.0	0.2	35.8
Complete	832	30	0.002	25.1	0.6	0.8	0.1	0.3	89.0
Abridged				24.4	0.0	0.7	0.0	0.2	93.6
Complete	832	150	0.002	144.7	0.1	6.7	0.3	3.1	77.2
Abridged				139.0	0.0	7.8	0.0	3.4	74.5
Complete	2496	30	0.002	12.3	0.2	2.7	0.0	1.3	58.4
Abridged				10.9	0.0	2.4	0.0	1.1	63.7

between wind and density variations is very near 90°. Even if one chooses not to assume that wind and density are related by the polarization relations, the relative phase lag must still be very near 90° or a substantial vertical mass flux will result. Such a locally unstable situation will be avoided by assuming that wind and density are 90° out of phase, although no assumption regarding their relative amplitudes will be made yet.

When the atmospheric disturbance is defined by $w_a = w \sin\nu t$ and $R = -\zeta \cos\nu t$, where $\nu = 2\pi/$period, Eq. (3) gives

$$\frac{\partial^2 z}{\partial t^2} = -\omega^2 z - \tfrac{2}{3} g\zeta \cos\nu t - A\left(\frac{\partial z}{\partial t} - w \sin\nu t\right) \times \left|\frac{\partial z}{\partial t} - w \sin\nu t\right| + w\nu \cos\nu t. \quad (4)$$

Even though (4) is much more simple than (1), it is still nonlinear. An important clue for linearizing (4) is found in Table 1 where only the odd harmonics have noticeable amplitude for the abridged model, and where the amplitude of the first harmonic is much larger than any other. By the principle of harmonic balance, this suggests that the form drag term in (3) can very well be approximated by a single sinusoidal term having the same period as the atmospheric disturbance. Further encouragement to approximate the nonlinear term by a sinusoid comes from the Fourier expansion of the function

$$F(x) = \sin x \,|\sin x|,$$

which is

$$F(x) = \frac{8}{3\pi}\sin x - \frac{8}{15\pi}\sin 3x + \cdots + \frac{8}{n\pi(4 - n^2)}\sin nx.$$

Note that only the odd harmonics are present in this expansion and that the first harmonic's amplitude is much greater than those which follow. This is similar to the situation in Table 1.

Proceeding, we assume a solution of (4) of the form

$$z = -\frac{C}{\nu}\cos(\nu t - \theta) - \frac{D}{2\nu}\cos(2\nu t - \phi) - \cdots$$

which we approximate by its first harmonic and expand the nonlinear term of (4) in a Fourier series. Applying the principle of harmonic balance, slight rearrangement gives for the first harmonic

$$\cos\nu t\left[\nu C\left(1 - \frac{a^2}{\nu^2}\right)\cos\theta - \nu w\right] + \sin\nu t\left[\nu C\left(1 - \frac{\omega^2}{\nu^2}\right)\sin\theta\right]$$
$$= \cos\nu t\left[\frac{8A}{3\pi} C(C^2 + w^2 - 2Cw\cos\theta)^{1/2}\sin\theta - \frac{2g\zeta}{3}\right] + \sin\nu t\left[-\frac{8A}{3\pi}(C^2 + w^2 - 2Cw\cos\theta)^{1/2}(C\cos\theta - w)\right]. \quad (5)$$

TABLE 2. Comparison of the first harmonic of the numerical integration results and the analytic solution for Eq. (3). Air motion and density same as described in Table 1.

Period (s)	w (cm s^{-1})	ζ (density change)	Numerical integration Amplitude (cm s^{-1})	Numerical integration Phase (deg)	Analytic solution Amplitude (cm s^{-1})	Analytic solution Phase (deg)
832	30	0.002	24.4	93.6	24.5	86.8
	30	0	22.2	35.8	22.0	37.4
	0	0.002	22.0	136.1	21.4	133.0
832	150	0	141.3	72.1	140.8	65.9
2496	30	0.002	10.9	63.7	11.0	59.7
	30	0	9.7	20.8	10.2	17.3
	0	0.002	8.2	97.0	8.4	96.1
*832	30	0.002	38.0	134.9	38.4	126.7

* For this case only an adiabatic lapse rate was used.

Equating the amplitude and phase on each side of (5) yields

$$\theta = \cos^{-1}\left\{\frac{\nu[C^2a + w^2] - \hat{\zeta}w}{C\nu w(1+a) - \hat{\zeta}C}\right\}, \qquad (6)$$

$$C = \pm\left\{\frac{-\beta \pm (\beta^2 - 4\alpha\delta)^{1/2}}{2\alpha}\right\}^{1/2}, \qquad (7)$$

where

$$a = 1 - \frac{\omega^2}{\nu^2}, \quad \hat{\zeta} = \frac{2g\zeta}{3},$$

$$\alpha = \left\{\frac{8A}{3\pi}\left[\frac{\nu w(1-a) - \hat{\zeta}}{\nu w(1+a) - \hat{\zeta}}\right]\right\}^2,$$

$$\beta = \nu^2 a^2 \frac{\nu w(1-a) + \hat{\zeta}}{\nu w(1+a) - \hat{\zeta}} - 2\left(\frac{8A}{3\pi}w\right)^2 \frac{[\nu w(1-a) - \hat{\zeta}][\nu w(1-a) + \hat{\zeta}]}{[\nu w(1+a) - \hat{\zeta}]^2},$$

$$\delta = \left[\frac{8A}{3\pi} w \frac{\nu w(1-a) + \hat{\zeta}}{\nu w(1+a) - \hat{\zeta}}\right]^2 - (\nu w - \hat{\zeta})^2\left\{\frac{\nu w(1-a) - \hat{\zeta}}{\nu w(1+a) - \hat{\zeta}}\right\}.$$

The proper sign for C is that which gives $|\cos\theta| \leq 1$, taking into account the sign of the denominator and numerator when evaluating (6).

Similar expressions can be found for the higher harmonics of the balloon's motion if desired, but according to Table 1, the higher harmonics are usually of little interest.

The integrity of (6) and (7) as solutions to (3) can be judged from Table 2, where the analytic and numerical integration solutions are compared for several period, air motion, density and static stability combinations. In general, the comparison is good.

3. Results and discussion

The only assumed relationship between density and wind variations so far has been that they are in phase quadrature. The atmospheric conditions specified in Table 1, for example, assumed no other relationship. Thus, (6) and (7) are fairly general solutions for balloon response. If it were desired to study the response to only wind (w) or only density (ζ) variations, one could simply take $w = 0$ or $\zeta = 0$. For the case $\zeta = 0$, the phase response from (6) agrees very well with the numerical integration results of Hanna and Hoecker (1971).

In gravity waves, the amplitudes of the wind and density variations are related according to the polarization relations approximately by

$$\zeta \approx \frac{g(\gamma - 1)}{\gamma R_a T}\frac{w}{\nu} = \frac{N^2}{g}\frac{w}{\nu}, \qquad (8)$$

where T is a representative air temperature, γ the ratio of the specific heats, R_a the gas constant for air, and N^2 the Brunt-Väisälä frequency. Applying (8) in (7) one obtains $\alpha = 0$ when $\omega^2 = 2gN^2/3g = 2N^2/3$ in which case (7) is not defined. But, for this case (4) has the direct solution $C = w$ and $\theta = 0$. In summary, it can be shown that solutions for the balloon's response exist for all values of ν, w, ω^2 and ζ.

The static stability of the atmosphere is reflected by ω^2, which is related to the lapse of temperature and the Brunt-Väisälä frequency N^2 by

$$\omega^2 = -\frac{2g}{3}\frac{\partial \ln v}{\partial z} + \frac{2g}{3T}\left(\frac{g}{R_a} + \frac{\partial T}{\partial z}\right) \qquad (9)$$

and

$$\omega^2 = -\frac{2g}{3}\frac{\partial \ln v}{\partial z} - \frac{2g^2}{3\gamma R_a T} + \frac{2}{3}N^2. \qquad (10)$$

At $T = 278$ K, and for the balloon constants defined in the Appendix, an isothermal atmosphere has ω^2

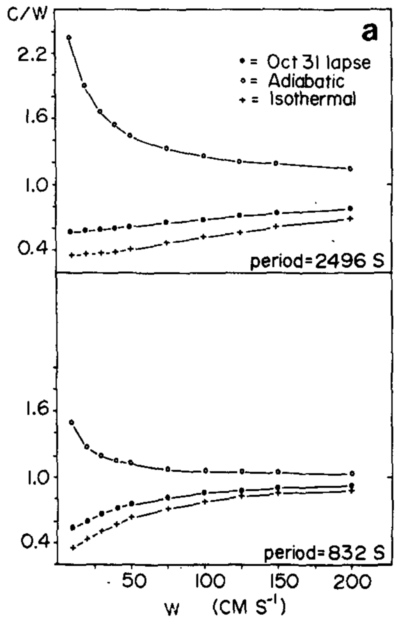

FIG. 1a. Balloon amplitude response from Eq. (7) as a function of vertical air velocity w and density (related by $\zeta = 3.51 \times 10^{-7}\, w\nu^{-1}$) amplitudes for typical wave periods and static stabilities, expressed as gain (Cw^{-1}).

$= 6.44 \times 10^{-4}\ s^{-2}$ and an adiabatic lapse rate gives $\omega^2 = 0.69 \times 10^{-4}\ s^{-2}$. The results will be given here for these two extreme cases, and for the lapse rate used in the numerical examples (see Appendix), called 31 October, ($\omega^2 = 3.95 \times 10^{-4}\ s^{-2}$) to illustrate the effect of changing lapse rates.

Fig. 1 shows the amplitude and phase response from Eqs. (7) and (6) for various lapse rates as a function of w and ζ at two typical wave periods. Here the temperature was taken to be 278 K so $\zeta = 3.51 \times 10^{-7}\ w/\nu$. For small w (and hence small ζ), the ratio C/w depends heavily on w and on static stability. For large w (and hence large ζ), the ratio C/w approaches unity. For $w = 200$ cm s^{-1} for the adiabatic case, it is 1.05 for the period = 832 s and 1.25 for period = 2496 s.

The phase given in Fig. 1b is the time of maximum balloon velocity and can be compared with the phase of air velocity (90°) or density (180°) changes. At large w, the relation between phase and w is linear with a slope near 0. But, perhaps more importantly, note the high dependence of phase on static stability. This is discussed more later.

Fig. 2 shows the amplitude response from (7) as a function of period for selected values of w and various lapse rates. For large values of ω^2, the response decreases with increasing period and asymptotically approaches the value appropriate to the case $w = 0$. This is because w is always multiplied by ν in (7) and for very small ν (very large period) the product approaches a negligible value and the density terms

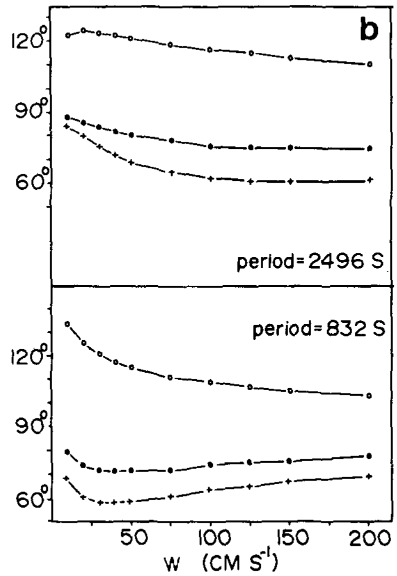

FIG. 1b. Balloon phase response from Eq. (6). Phase is the time of maximum balloon vertical velocity. (The phase of air velocity is 90°, and phase of density is 180°.)

dominate. At $\omega^2 = 2N^2/3$, regardless of ν, $C = w$, and for smaller values of ω^2 the response increases with period. But it is not proper to assert that the response has an asymptotic limit as $\nu \rightarrow 0$ for small ω^2, because for very large periods the assumptions used in deriving (7) may not be valid. Thus, for long wave periods the balloon asymptotically approaches

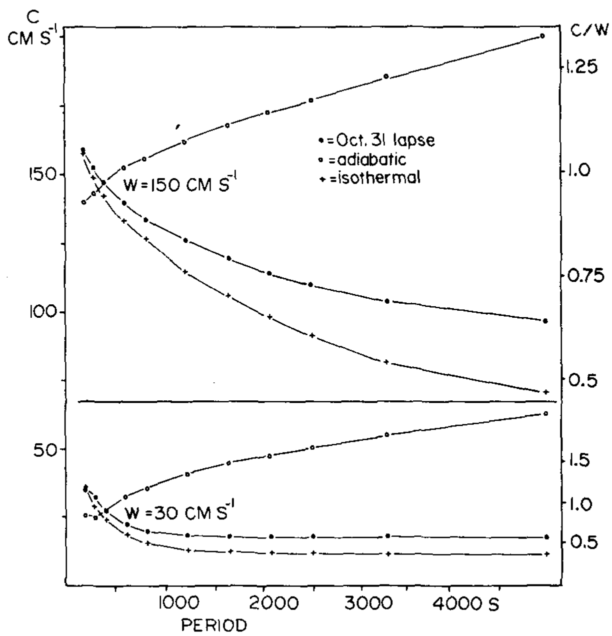

FIG. 2. Balloon amplitude response from Eq. (7) as a function of wave period, for typical vertical air velocity and density (related by $\zeta = 3.51 \times 10^{-7}\, w\nu^{-1}$) amplitudes and static stabilities.

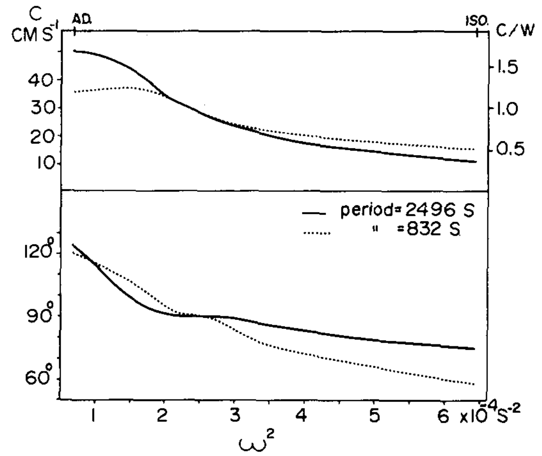

FIG. 3. Amplitude (top) and phase (bottom) response of balloon from Eqs. (6) and (7), with w = 30 cm s^{-1} and $\zeta = 3.51 \times 10^{-7}$ $w\nu^{-1}$ for two typical wave periods as a function of the static stability parameter ω^2. End points of the curves are for the adiabatic and isothermal lapse conditions.

a true isopycnic tracer for high static stability, but not for low static stability.

Fig. 3 shows the amplitude and phase response of a balloon as a function of static stability for the case w = 30 cm s^{-1} (and $\zeta = 3.51 \times 10^{-7}$ w/ν) for two typical wave periods. For $\omega^2 < 2N^2/3$, the ratio C/w is greater than 1 and the phase is greater than 90° (i.e., the balloon's maximum velocity occurs after the air's maximum vertical velocity), but for higher static stability the ratio C/w is less than 1 and phase is less than 90°.

4. Conclusions

The equation of vertical motion for a balloon can be linearized by a routine expansion method and yet retain its essential features. The linearized equation can be solved analytically for steady wave conditions to render estimates of the amplitude and phase of a balloon relative to sinusoidal vertical wind (w) and density variations, assuming density and wind are in phase-quadrature.

The solutions (6) and (7) are not dependent on any particular balloon characteristics. In fact, there is reason to suggest they hold equally well for small boundary-layer balloons and for large stratospheric balloons. The assumptions used in deriving these solutions should be good for most conditions encountered by real balloons. One tacit assumption which could be violated remains to be stated: the phase of the atmospheric disturbance was assumed constant with height. In a vertically propagating gravity wave the phase changes with height and a real balloon in a real gravity wave will encounter ever-changing phases as it oscillates in the vertical. But for the vertical gravity wavelengths considered here (~1 km) or for nonpropagating waves this phenomenon is unimportant.

When density and wind variations are related as in a gravity wave, the amplitude response of a balloon increases linearly with vertical air motion amplitude with response approaching unity for all lapse rate conditions for large waves. For long-period gravity waves, the balloon asymptotically approaches a true isopycnic tracer (with some phase shift) for high static stability, and for low static stability this is not true.

The results presented here for balloon response as a function of the vertical air velocity could easily be rearranged to give air velocity as a function of balloon speed. If temperature and pressure were accurately and simultaneously measured along with balloon height, there would be no reason to assume a relationship between density and wind variations (e.g., as from the polarization relations). In fact, one could then compute atmospheric density from the data and could check on the validity of the polarization relations. Thus, it is desirable that all constant density balloon experiments include temperature and pressure, along with height, as basic variables to be measured.

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APPENDIX

Numerical Methods

The numerical integration of (1) and (3) was made using a standard Runga Kutta method with 0.25 s time steps. The balloon was started from rest at its float level and at time $t = 0$ an atmospheric wave was switched on. This procedure initially produces transients in the solution, but they decay very quickly as shown in Table A1. All other results given in this paper are for the fourth cycle of the

TABLE A1. Decay of transients for numerical integration of Eq. (3) for the case w = 20 cm s^{-1}, period = 832 s, and $\zeta = 0$. The atmospheric disturbance is switched on with $W_a = w \sin\nu t$ at $t = 0$.

Time segment (s)	Harmonic analysis of model output W_B: Amplitude of harmonic (cm s^{-1}) 1	2	3	4	5	Phase of harmonic (deg) 1	3	5
0–832	12.1	0.1	1.1	0.7	0.3	30	7	1
832–1664	12.8	0.0	0.8	0.0	0.2	32	56	176
1664–2496	12.8	0.0	0.8	0.0	0.2	32	56	176
2496–3328	12.8	0.0	0.8	0.0	0.2	32	56	176

atmospheric wave. Several balloons were flown at St. Cloud, Minnesota, on 31 October 1978, and a later paper will deal with the data from that experiment. The balloon constants chosen here apply to one of those balloons, and may be considered typical for balloons flown in the upper boundary layer. For the integrations the temperature sounding at St. Cloud on the afternoon of 31 October was used. The balloon float level was taken to be 770.7 m, and conditions at float level in the undisturbed atmosphere were as follows:

T = 278.14 K
p = 905.2 mb
Δp = superpressure = 50.3 mb
$\rho_0 = 1.134 \times 10^{-2}$ g cm^{-3}.

The balloon constants used were:

M_B = 1220 g
C_d = 0.6
$V_0 = 1.076 \times 10^6$ cm^3
r = 65 cm
MW = molecular weight of balloon gas = 12.4 g mol^{-1}
M_G = mass of gas inside balloon = 414.5 g.

The volume versus superpressure curves of Beemer and Markhardt (1977)[3] were used, and the pressure inside the balloon was recomputed at each step, allowing for adiabatic heating of the gas and heat exchange with the air. Heat exchange was assumed to proceed with a time constant of 5 min.

One interesting side-product of the integrations is the time history of the magnitude of the three acceleration terms in (1). These are given in Fig. A1 for a typical case. Note that the form drag term is very nearly sinusoidal (~95% of its variance is contained in the first harmonic), lending support to the approximation used in (5). Also note that as the form drag and buoyancy accelerations are strongly opposed the net acceleration of the balloon is due here largely to the dynamic acceleration. For longer period oscillations, where ν becomes very small, the dynamic acceleration term becomes less significant, i.e., $w\nu \cos\nu t \rightarrow 0$ as $\nu \rightarrow 0$.

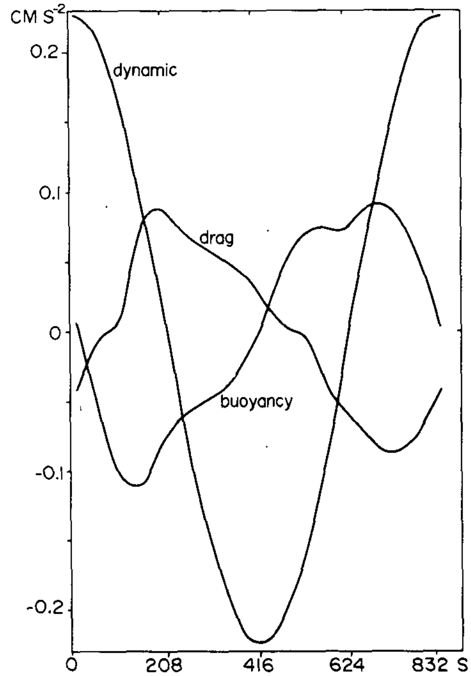

FIG. A1. Time history of the acceleration terms from Eq. (1) for the case $W_a = 30 \sin\nu t$, $\rho_a = \bar{\rho}_a(1 - 0.002 \cos\nu t)$, and period = 832 s.

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