On the Prediction and Variability of Water Vapor

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(Manuscript received 9 October 1962)

ABSTRACT

Twelve-hour predictions of the mixing ratio and the per cent change in mixing ratio for several pressure surfaces are generated by a trajectory procedure. Inter-level comparisons of the forecasts are made. The mixing ratio predictions are compared with simple persistence.

Statistical analyses, including use of spectral analysis, are used to examine inter-level relationships of the mixing ratio, and to reveal characteristics about the spatial and temporal variability of the mixing ratio.

1. Introduction

A knowledge of the horizontal and vertical distribution of water vapor is of obvious importance in the prediction of clouds, precipitation, visibility and other weather parameters. Equally significant is the effect of water vapor on microwave radio propagation, infrared sensing, satellite and rocket tracking.

Research in atmospheric moisture has been directed largely toward studying the monthly, seasonal, or annual fluxes, and the moisture balances over large regions. For example, we may point to studies by Benton and Estoque (1954), Hutchings (1957) and Starr and Peixoto.[1] Only a few studies have been concerned with the predictability of water vapor, and these have, in general, directed their problem to some particular level in the atmosphere. There has been no investigation of moisture predictability as a function of height. Lewis (1957) has described a trajectory technique for predicting the 700-mb dewpoint depression. His results indicated that 12-hr forecasts based upon two-dimensional trajectories were no more accurate than those obtained from persistence. A significant improvement, however, was obtained from the use of three-dimensional trajectories. On the other hand, Vederman (1961), in using Lewis' technique to develop cloud and precipitation forecasts, found that the forecasts of 700-mb dewpoint spread were unsatisfactory, and he chose to consider the average of the dewpoint spreads from 700 and 850 mb. Miyakoda (1956), in a study of precipitation forecasting, investigated 24-hr prediction of the mixing ratio tendency at 1000 and 850 mb. His data sample was small at 1000 mb and even smaller at 850 mb.

The purposes of our study are: (1) to investigate water vapor predictability for a 12-hr period at 1000, 850, 700 and 500 mb, and for the layers 850–700 mb and 700–500 mb; (2) to determine the interrelationship between moisture at the various surfaces; and (3) to examine spatial and temporal characteristics of moisture. The simpler synoptic situations are treated in this paper and we plan to go on to more complex cases later, if this seems advisable.

2. Prediction method

The local change in mixing ratio can be approximated by:

$$\frac{\partial Q}{\partial t} = -\mathbf{V}\cdot\nabla Q - \omega\frac{\partial Q}{\partial P} + E - C, \qquad (1)$$

where Q is the mixing ratio, t the time, $\mathbf{V}$ the horizontal vector wind, ω the vertical velocity in pressure units (dp/dt), E the rate of evaporation, and C the rate of condensation. Q is in g per kg throughout the paper.

We have chosen to apply a simplified form of Eq (1) where the third and fourth terms from the right hand side are neglected. The last term is omitted on the basis that we have selected situations where cloudiness and precipitation were infrequent over an interval of three successive 12-hr periods. The contribution from evaporation is a more uncertain quantity; however, for the selected data there should be little surface water available for evaporation. The importance of the horizontal and vertical advection terms is evaluated, and the forecasts are compared with persistence (i.e., $\partial Q/\partial t = 0$). The prediction equation can be evaluated from either the Eulerian method or from the Lagrangian (trajectory) approach. After comparing both approaches on a small sub-set of the data, the remaining experiments are

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[1] Starr, V. P., and J. P. Peixoto, 1959: Note on the zonal flux of water vapor in the Northern Hemisphere. *Final Report, Studies of the atmospheric general circulation III*, Contract AF19(604)–2242, Mass. Institute of Technology, 41–49.

carried out using the trajectory method. Backward trajectories are computed from a forecast point using 6-hr time steps except in the Eulerian-Lagrangian comparison where 3-hr time steps are also used.

In order to isolate the forecast errors, observed winds and observed pressure heights are utilized. The large-scale vertical velocities are obtained from a 10-level diagnostic model described by Sanders *et al.*[2] The vertical displacement along a trajectory is obtained by applying the average of the vertical velocities at the two end points. The vertical gradient of Q is computed at the beginning point of a trajectory and measured between the level of advection and the next higher or lower mandatory level, depending upon the sign of the vertical motion.

3. Results of predictions

The prediction data consisted of two sets, each with three 12-hr time periods. One set covered the period from 00 GMT 13 January to 12 GMT 14 January 1959, and the second set was from the period 12 GMT 22 March to 00 GMT 24 March 1962. Thirty-six radiosonde stations in the eastern part of the United States were the prediction points. A comparison of the Eulerian and Lagrangian procedures at 700 mb was carried out for one 12-hr period. In the Eulerian method, a grid of 165 km and centered differences were used. Twelve-hour forecasts were made using 3-hr and 6-hr time steps. Use was made of the observed winds at 0-, 6- and 12-hr points, while linear-interpolated winds were used at the 3- and 9-hr points. The comparison does not include the vertical motion term, and the forecast correlations are shown in Table 1. The correlations are slightly larger for the trajectory method and for the smaller time step; however, the differences between the correlations were found not to be statistically significant. Because of the large amount of work involved in carrying out the computations by hand, we did not consider it advisable to further explore the Eulerian approach. Hence, all of the remaining experiments are based on the trajectory method, use a 6-hr time step, and evaluation of the results is expressed through use of the standard error of estimate, S_E. The standard error of estimate is computed from the relationship

$$S_E = \sigma_y (1 - R_{xy}^2)^{1/2},$$

where σ_y is standard deviation of the predictand, and R_{xy} is the correlation between x and y.

TABLE 1. 12-hour prediction correlations (R) of 700-mb mixing ratio. F_T refers to the trajectory method and F_E refer to the Eulerian method.

	F_T	F_E
R (3-hr time step)	.43	.37
R (6-hr time step)	.31	.23

Table 2 presents the standard error for persistence and prediction for the four pressure surfaces. Three-dimensional predictions were computed only on set 1 of the data. The small improvement with the inclusion of the vertical motion term is consistent with the type of synoptic situations selected for this study. In view of these results, the contribution from the vertical advection term was not included in set 2. In comparing the forecast results with those from persistence, we find that the forecast error is smaller, except at 1000 mb. The improvement over persistence increases with height, so that at 500 mb the forecast error is only 60 per cent of the persistence error. The errors themselves decrease with height, with the largest decrease between 700 and 500 mb. This vertical variation in the errors is due largely to the fact that the variance of water vapor, as will be shown later, falls off sharply with elevation in the free atmosphere.

Since the moisture distribution in the vertical at times appears to be quite irregular, it was felt that the average moisture between pressure surfaces would be more predictable. Forecasts of the average Q between 850 and 700 mb, and between 700 and 500 mb were generated in two ways. First, the forecasts of the mean Q in a layer were obtained by simply averaging the forecast values for the appropriate levels. In an alternative manner, the flow only at 700 mb was used to obtain trajectories of the moisture at 850, 700, and 500 mb, and the appropriate averages were then computed. These predictions of the vertically averaged Q were made only on data set 2, and the results are presented in Table 3. First, substitution of the 700-mb flow for the flow at the other two surfaces results in a small increase in the errors. In comparing the errors with those in Table 2 (set 2), we see that layered moisture is both more persistent and more predictable.

For some purposes it may be important to consider moisture prediction in terms of the per cent change in Q, i.e., $\Delta Q/Q$. Tables 4 and 5 present the errors for predictions of $\Delta Q/Q$ in format similar to those of Tables 2 and 3, except that no persistence values are given. It is noteworthy that the errors, in contrast to those for Q, now increase with height. At 500 mb, an error of near unity shows that the errors in the prediction of Q are about as large as Q itself. From Table 5 we see that substitution of the 700-mb flow for the flow at the other two surfaces results in much poorer predictions for the 850–700-mb layer, and slightly poorer results for the upper layer. Also, in comparing results with those in Table 4, we see that the layered values are considerably more predictable.

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[2] Sanders, F., J. A. Wagner and T. N. Carlson, 1960: Specification of cloudiness and precipitation by multi-level dynamical models. *Sci. Rep. No. 1*, Contract AF19(604)–5491, Mass. Institute of Technology, 111 pp.

TABLE 2. Standard errors of estimate for 12-hour prediction of mixing ratio by persistence and by the trajectory method.

Pressure surface (mb)	Persistence			2-D Forecast			3-D Forecast
	Set 1	Set 2	Average of Sets 1 and 2	Set 1	Set 2	Average of Sets 1 and 2	Set 1
1000	1.07	1.13	1.10	1.25	.93	1.09	
850	1.19	.86	1.03	.93	.74	.84	.93
700	.81	.86	.84	.70	.62	.66	.73
500	.35	.54	.45	.23	.30	.27	.27

TABLE 3. Standard errors of estimate (set 2) for 12-hr prediction of layer averaged mixing ratio by persistence and by the trajectory method; one procedure uses the appropriate winds from each surface and the other procedure uses *only* the 700-mb winds.

Layer average	Persistence	$V_{850\text{ mb}}+V_{700\text{ mb}}$	$V_{700\text{ mb}}+V_{500\text{ mb}}$	$V_{700\text{ mb}}$
Q (850 to 700)	.72	.51		.60
Q (700 to 500)	.56		.35	.41

TABLE 4. Similar to Table 2 but applied to percentage change in mixing ratio ($\Delta Q/Q$), and no persistence values shown.

Pressure surface (mb)	Set 1	Set 2	Average of Sets 1 and 2
1000	.31	.23	.27
850	.49	.42	.46
700	.79	.67	.73
500	.85	1.06	.93

TABLE 5. Similar to Table 3 but applied to $\Delta Q/Q$, and no persistence values shown.

Layer average	$V_{850\text{ mb}}+V_{700\text{ mb}}$	$V_{700\text{ mb}}+V_{500\text{ mb}}$	$V_{700\text{ mb}}$
$\frac{\Delta Q}{Q}$ (850 to 700)	.28		.72
$\frac{Q\Delta}{Q}$ (700 to 500)		.51	.60

4. Variability of mixing ratio

For many reasons it was felt that it would be desirable to examine the spatial and temporal variability of the mixing ratio. The predictability of moisture will depend to some extent upon the lapse rate of moisture; forecasts for levels where the variation is great are likely to suffer. Table 6 presents the average $\Delta Q/\Delta P$, where $\Delta P=50$ mb, based on 40 synoptic radiosonde observations; the averages were computed with and without regard to the sign of $\Delta Q/\Delta P$. For our purpose, the absolute changes are more meaningful and we find that the largest rate of change occurs in the 850–800-mb layer. Aside from this layer, the changes are generally about 0.7 up to 750 mb, and then decrease with each succeeding layer. Relationships in the vertical were further examined by computing correlations between Q at 700 mb and Q at other pressure surfaces, and these are given in Table 7. Within 50 mb of the 700-mb surface, the correlation is already down to near .80; or from another view, less than .65 of variability at 700 mb is revealed by the nearby surfaces. The relationships with the moisture at 850 mb and 500 mb indicate that they contain little information about the 700-mb moisture. Obviously then, forecasts of Q in one pressure surface will not yield reliable forecasts of Q in the other two surfaces.

It is interesting to look at the spatial and temporal variability of water vapor, and these are shown in Table 8 for the various surfaces. The spatial variances are obtained from sets 1 and 2 of the data, while the temporal variance is derived from 168 hourly observations at Hanscom Field, Mass. (Court and Salmela, 1961). Temporal variability seems to be much greater than the spatial variability, and one might attribute that to the fact that the same scales of motion are not being compared. This suggestion is not supported when, as is shown later, spectral analysis of the Hanscom Field data revealed that only a small fraction of the variance was due to periodicities of 6 hr or less.

Temporal characteristics are generally best revealed by obtaining the autocorrelations function and the variance (power) spectrum analysis of a time series of data. A unique series of quasi-hourly rawinsonde runs taken at Hanscom Field over seven successive days provided 168 observational points for each surface. We assumed that the time interval between observations was constant (1 hr), although in a few cases the interval was off by one-quarter to one-half hour. The analyses were performed on the mixing ratio at 1000, 850 and 700 mb. The extent of missing and questionable data at 500 mb precluded use of this surface. The few missing observations in the other surfaces were obtained by time and space interpolation. Interpolation, or even a short extrapolation, in the vertical was felt to be very reliable because the rawinsonde data were available in 50-mb steps.

The autocorrelation function of the mixing ratio is shown in Fig. 1, and the curves for the three surfaces

Table 6. Average change in mixing ratio (g/kg) per 50 mb based upon 40 raobs of 12 GMT, 13 Jan. 1959.

Pressure interval	1000–950	950–900	900–850	850–800	800–750	750–700	700–650	650–600	600–550	550–500	500–450
$\frac{\Delta Q}{\Delta P}$	−.13	.43	.44	.82	.56	.31	.40	.40	.33	.28	.25
$\left\lvert\frac{\Delta Q}{\Delta P}\right\rvert$	.72	.69	.74	.95	.74	.63	.55	.48	.34	.30	.26

Table 7. Inter-level correlations of Q based upon the 40 raobs used in Table 6.

$R_{700,\,850}$	.41
$R_{700,\,750}$	.78
$R_{700,\,650}$	.80
$R_{700,\,500}$	.48

Table 8. Variance of mixing ratio in $(\mathrm{g/kg})^2$.

	Set 1	Set 2	Special Hanscom Field data
1000 mb	4.0	2.9	5.1
850 mb	3.2	1.5	6.6
700 mb	.69	.87	3.4
500 mb	.14	.31	.56

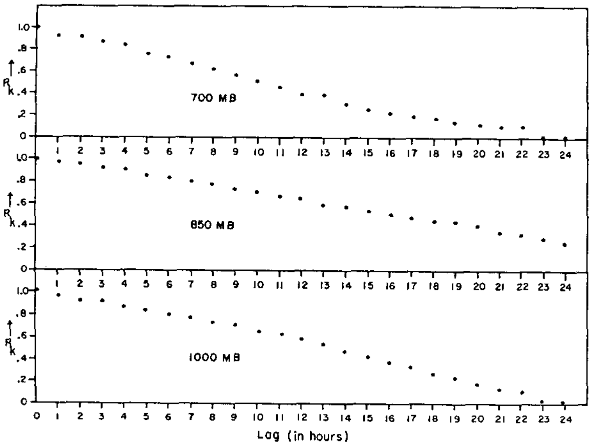

Fig. 1. Autocorrelation function (R_k) of mixing ratio, hourly values, 1–7 April 1960, Bedford, Mass.

are, in general, similar. The correlation between successive observations, one hour lag, is high, .96 for the two lower surfaces and .94 at 700 mb. This indicates a greater reliability in the moisture measurements than is often assumed. The rate of change of correlation is rather small in the three surfaces, and the correlation exceeds 0.8 out to the 4-hr lag. The most rapid change occurs in the 4- to 15-hr interval at 700 mb and in the last 12-hr interval at 1000 mb. The slope of the 850-mb curve is more nearly uniform.

Spectral estimates are obtained by taking the Fourier cosine transform of the autocorrelation (R_k). The line powers are then expressed by

$$B_0=\frac{1}{2M}(R_0+R_M)+\frac{1}{M}\sum_{k=0}^{M-1} R_k$$

$$B_h=\frac{1}{M}(R_0+(-1)^h R_M)+\frac{2}{M}\sum_{k=1}^{M-1} R_k \cos k\frac{h}{M}$$

for $k=1, 2, 3, 4\cdots M-1$

$$B_M=\frac{1}{2M}(R_0+(-1)^M R_M)+\frac{1}{M}\sum_{k=1}^{M-1} (-1)^k R_k,$$

where M = number of lags used in computing the autocorrelation. Smooth spectral estimates, U, are obtained by setting

$$U_0=0.54B_0+0.46B_1$$
$$U_h=0.54B_h+0.23(B_{h-1}+B_{h+1})$$
$$U_M=0.54B_M+0.46B_{M-1}.$$

Normalized spectra (area = 1) are shown in Fig. 2, the ordinate being per cent of variance (log scale), abscissa the frequency (linear scale). The abscissa also represents the period of oscillation where

$$\text{Period (in hours)}=\frac{1}{\text{Frequency (cy/day)}}\times 24.$$

The general character of the spectra in Fig. 2 is quite similar. The variance (or energy) is concentrated in the low frequencies (long periods) and falls off very rapidly out to the 8-hr period. This low frequency end of the spectra contains 94 per cent of variance at 1000 mb, 90 per cent at 850 mb and 95 per cent at 700 mb. The fact that very little power is contained in the high frequency implies that instrumental errors and errors due to "off-time" releases are not important relative to the variability due to the longer period influences.

5. Conclusion and summary

This study has shown that the standard error of estimate of the 12-hr prediction of water vapor decreases sharply with height. In the free atmosphere the predictions are an improvement over persistence, and this advantage increases with height. There is no apparent improvement over persistence at 1000 mb. In considering the moisture variable $\Delta Q/Q$, we find that the error, in contrast to the results with Q, increases with height to near unity at 500 mb. Errors are smaller when the mean of layers between pressure surfaces is considered.

Statistical analyses revealed that the 700-mb mixing ratio was poorly correlated with the mixing ratio at 850 and 500 mb. Also, from a time series of hourly observations it was shown that random errors and high frequency oscillations constituted only a small fraction of the total variability of the mixing ratio.

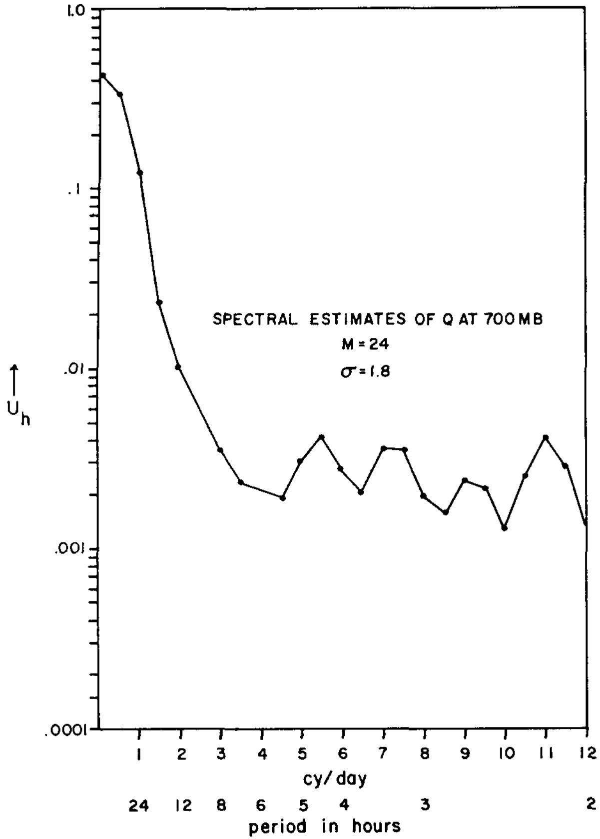

FIG. 2. Spectral estimates normalized (U_h) of mixing ratio, hourly values, 1–7 April 1960, Bedford, Mass.

REFERENCES

Benton, G. S., and M. A. Estoque, 1954: Water vapor transfer over the North American continent. *J. Meteor.*, **11**, 462–477.

Court, A., and H. A. Salmela, 1961: Hourly rawinsondes for a week. *GRD Research Notes No. 60*, U. S. Air Force Cambridge Research Laboratories, Bedford, Mass., 310–350.

Hutchings, J. W., 1957: Water vapour flux and flux-divergence over southern England, summer 1954. *Quart. J. R. meteor. Soc.*, **83**, 30–48.

Lewis, W., 1957: Forecasting 700-mb dew point trajectory by a three-dimensional trajectory technique. *Mon. Wea. Rev.*, **85**, 297–301.

Miyakoda, K., 1956: Forecasting formula of precipitation and the problem of conveyance of water vapor. *J. meteor. Soc. Japan*, **34**, 36–49.

Vederman, J., 1961: Forecasting precipitation with the aid of a high-speed electronic computer. *Mon. Wea. Rev.*, **89**, 243–250.