Note on Range Normalization of Digitized Radar Data[1]

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The problem discussed here is one example of a type that arises wherever it is necessary to process discrete or truncated representations of continuous variables. Consider the spatial distributions of radar-echo intensities associated with precipitation occurrences. These distributions are very complicated and are not readily represented as functions of continuous variables. Studies of the development and displacement of the echo distributions are necessary to define the proper role of radar data in weather forecasting, and are being based increasingly on discretized representations of the echo distributions. Discretized radar-echo intensities can be derived from the displays generated by repeated antenna scans with stepwise variation of the over-all sensitivity of the radar system; echoes that appear on the plan-position indicator (PPI) when the radar sensitivity is set low are the more intense of those that appear when the system sensitivity is set high. The intensities are "digitized" when their values are indicated by integers. Discretization in space is given by dividing the region seen by the radar by an orthogonal grid and assigning an intensity integer to each address.

The intensity of precipitation-caused radar echoes increases with precipitation rate and is approximately proportional to the inverse square of the distance (range). Any study of the changing distribution of echoes must therefore consider the effects of range, and this is done by range normalization. The principal finding of this note is that radar-echo intensity data developed by range-normalizing parent digitized data are considerably less accurate than the parent data. The over-all loss of accuracy may be taken as ± 1 integral step when the logarithm of the interval of intensity defined by each integer is constant. When the intensity factors vary, the uncertainty of the normalized integers is closely related to that of the least certain integer. The escalation of uncertainties caused by processing digitized data can be avoided by using a sensitivity-time control (STC) circuit to range-normalize the radar video signals before they appear on the PPI scope and are digitized. The bases for these findings are contained in the following examples.

Suppose, as shown in columns 1 and 2 of Table 1, that integers 0 to 9 denote steps of the over-all sensitivity of a radar system and the corresponding intensities of weather echoes that appear on the PPI. On the digitized, unnormalized PPI display of signal intensities, all these numerals may appear at all ranges, although it is obvious that when the distribution of precipitation rates is independent of range, the average rank of integers will be an inverse function of range.

An observed signal intensity, S, can be range-normalized by taking the product $S(R/RN)^2$, where R

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Table 1. Illustrative relationship between steps of radar system sensitivity and echo intensity.

Integer	Echo signal intensity limits*	Logarithmic mean of echo intensity*	10 log (lower limit); i.e., lower limit, db	Intensity interval associated with each integer, db
0	$S<1$	—	—	—
1	$1 \leq S < 4$	2	0	6
2	$4 \leq S < 16$	8	6	6
3	$16 \leq S < 64$	32	12	6
4	$64 \leq S < 256$	128	18	6
5	$256 \leq S < 1{,}024$	512	24	6
6	$1{,}024 \leq S < 4{,}096$	2,048	30	6
7	$4{,}096 \leq S < 40{,}960$	13,100	36	10
8	$40{,}960 \leq S < 163{,}840$	81,920	46	6
9	$163{,}840 \leq S$	—	52	—

* Arbitrary units. Measured radar echo intensities commonly range through values differing by 10^6, and a logarithmic scale as illustrated here is customary.

is the range at which S is observed, and RN is a constant. For a uniform target that fills the radar beam, this product is the signal intensity that would be observed if the target at range R occurred at range RN (Battan, 1959, for example).

Let $RN = 50$ mi, and consider a target denoted by the integer 2 at 86 mi. Then $(R/RN)^2 = 3$, and, given the limits corresponding to the integer 2 shown in Table 1, the normalized intensity limits are $12 \leq S < 48$. As shown by column 2 of Table 1 and by Fig. 1, the normalized integer can be denoted by a 2 *or* a 3. Only if a particular intensity value is assigned to the unnormalized integer, can it always be replaced by a particular normalized integer. Suppose that the standard represented by columns 1 and 3 of Table 1 is adopted for normalizing. Then in the case just cited, a 2 becomes a 3. The normalized integer would be a 3, but we must acknowledge the possibility that it is 2.

At other ranges, consider unnormalized 3's. There is an annulus enclosing a range of R's greater than RN wherein 3's will remain 3's after normalization although they might be 4's.[2] Thus the normalized display contains 3's that might truly be 4's and, at other ranges, 3's that might truly be 2's. Of course, it also contains 3's that are truly 3's. Unless the prior mathematics is investigated in each case; however, the normalized integers are considerably less certain than the original integers.

Another important consideration enters when the intensity range defined by integers is variable, in other words, when some integers define the echo intensity less precisely than others. The integers resulting from normalization is at least as uncertain as its corresponding unnormalized integer. For example, integer 7 in Table 1 is associated with an uncertainty of 10 db while the others are associated with 6-db uncertainties. With

[2] The relationship

$$\frac{(RN)^2 L_i}{M_k} \leq R_k^2 < \frac{(RN)^2 U_i}{M_k},$$

where L_i is the lower limit and U_i is the upper limit of signal intensity corresponding to the integer i, and R_k and M_k are the range and mean intensity corresponding to the integer k, defines the limits of R within which an integer k will be normalized to an integer i. On the other hand,

$$\frac{(RN)^2 L_j}{U_k} \leq R_k^2 < \frac{(RN)^2 U_j}{L_k}$$

represents the limiting ranges within which the integer k *might* be correctly represented by the normalized integer j. The annulus of overlap between these two sets of limits is the space in which a normalized integer i would in some cases be more accurately denoted by a j.

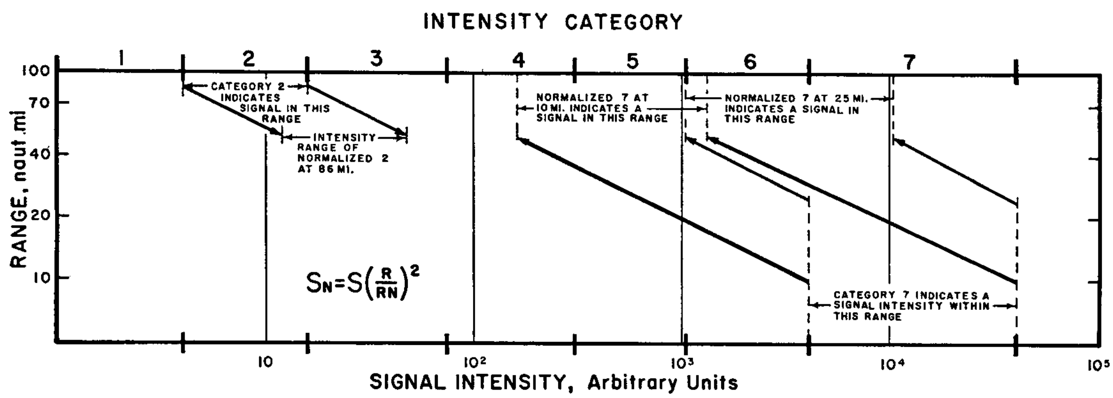

Fig. 1. Graphic illustration of range normalization of digitized radar data. The echo intensity normalized to 50 mi is given by $S_N = S(R/50)^2$, where S is the unnormalized intensity and R is the range at which the unnormalized value is observed. Note that the intensity limits of a 2 at 86 mi, normalized to 50 mi, brackets portions of categories 2 and 3; a normalized 7 at 25 mi brackets portions of categories 6 and 7; and a normalized 7 at 10 mi ranges over categories 4, 5 and 6.

$RN=50$ mi, a 7 at 25 mi would be normalized to a 6 that might truly be a 6 or a 7; and a 7 at 10 mi would be normalized to a 5 that might truly be a 4, 5 or 6. This is shown also in Fig. 1. In other words, some of the particular integers resulting from normalization of 7's may be truly represented by any one of three integers, while those resulting from normalization of other integers in the parent data may not be truly represented by more than two integers. Since the over-all uncertainty in the normalized data is closely related to the least certain step, routine collection of unnormalized radar data should always employ equally spaced steps of the receiver sensitivity.

The use of a calibrated STC to range-normalize the radar data before digitization avoids the increase in uncertainties caused by range-normalizing them after digitization. In other words, use of the STC permits truncation of the continuous echo-intensity distribution after application of the continuous range-normalizing function rather than before. An alternative means of reducing uncertainties is to digitize in finer steps of signal intensity, but this requires either proportionally more time for collecting data over any given intensity range or more complex equipment for collecting unnormalized radar data at many intensity steps simultaneously.

REFERENCE

Battan, L. J., 1959: *Radar meteorology*. Chicago, The University of Chicago Press, p. 59.