Mesospheric Density Variability Based on Recent Meteorological Rocket Measurements

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ABSTRACT

Seasonal and latitudinal variability of density between 30 and 68 km is presented. These data have been derived from direct temperature and height measurements made with meteorological rockets fired at White Sands Missile Range, N. Mex., and Fort Churchill, Canada, during 1960–1962 Meteorological Rocket Network and associated research and development activities. Some of the more significant features are the wider seasonal range at northern latitudes, the absence of significant latitudinal variation during the summer, the greatest variation in the fall and winter at northern latitudes and the overall variation that is indicated in the general region of 50 to 60 km, as much as 50 per cent in some cases. The need for both seasonal and latitudinal standard atmospheres where standards are required is clearly demonstrated. Also, the feasibility of providing timely density measurements for direct application is evident.

1. Introduction

Until recently, most of the techniques for determining high-altitude density, pressure and temperature employed expensive and complicated rocket systems. Obviously, such systems have not been feasible for investigating the high atmosphere with a sufficient quantity of soundings to permit a comprehensive variability study. However, with the advent of the Meteorological Rocket Network in 1959, using small meteorological rockets described by Webb and Jenkins (1959), an increasing volume of data is becoming available for a more detailed investigation of the meteorological aspects of the region above balloon altitudes to approximately 68 km.

The primary system used in the Meteorological Rocket Network at White Sands Missile Range and Fort Churchill, Canada, for measurements other than wind consists of a single-channel telemetering unit[1] utilizing a 10-mil ceramic bead thermistor as the sensor. This instrumentation is carried aloft in an Arcas sounding rocket and expelled on a parachute at apogee. The altitude range varies from approximately 68 km for sea-level launches to approximately 75 km for a launch elevation of 4000 ft such as at White Sands Missile Range.

The temperature is sensed during descent and telemetered to standard AN/GMD-1 (Ground Meteorological Detector) radiosonde equipment. Temperature data are recorded and reduced in much the same manner as balloon-borne radiosonde data, and pressure and density are calculated using a "tie-on" pressure from a compatible balloon-borne radiosonde. The metalized parachute is tracked by radar for position data which also provide wind data.

2. Discussion of data

This paper is concerned with data obtained from Arcas meteorological rockets fired at White Sands Missile Range, N. Mex., and Fort Churchill, Canada, in connection with Meteorological Rocket Network activities and research and development efforts. A reasonable sample of data was acquired during the seasons of Summer 1960, Fall 1960, Winter 1960–61, Winter 1961–1962 and (for White Sands only) Spring 1961. A total of 66 meteorological rocket temperature measurements were obtained from the two stations during this period.

Emphasis is placed on seasonal and latitudinal variability at corresponding geometric altitudes. For the spring season, data are available only at White Sands Missile Range. However, "quick-look" density data recently (March and April, 1962, 10 samples) obtained at Fort Greely, Alaska, indicate that the spring data are very similar to northern latitude early fall data.

Figs. 1 and 2 for summer at White Sands Missile Range and Fort Churchill show that both maximum and minimum range at each station and latitudinal variation is small during this season. With the exception of a small departure above 56 km of the White Sands summer minimum, the range of variation remains higher (more dense) than the reference 1959 Air Research and Development Command model atmos-

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[1] Clark, G. Q., and J. G. McCoy, 1962: Meteorological rocket thermometry. Missile Meteorology Division Technical Report MM-460, U. S. Army Electronics Research and Development Activity, White Sands Missile Range, N. Mex.

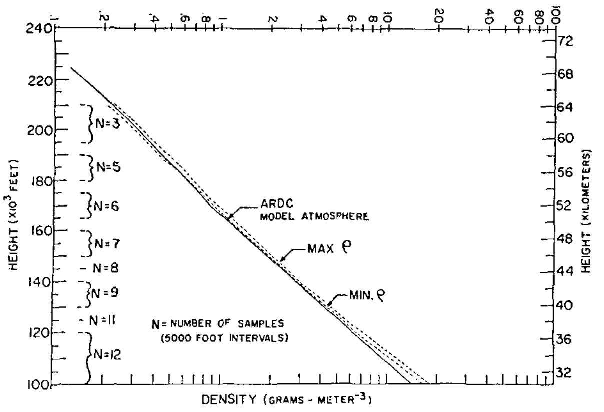

FIG. 1. Profiles of maximum and minimum density for White Sands Missile Range, N. Mex., latitude 32 deg (summer 1960) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

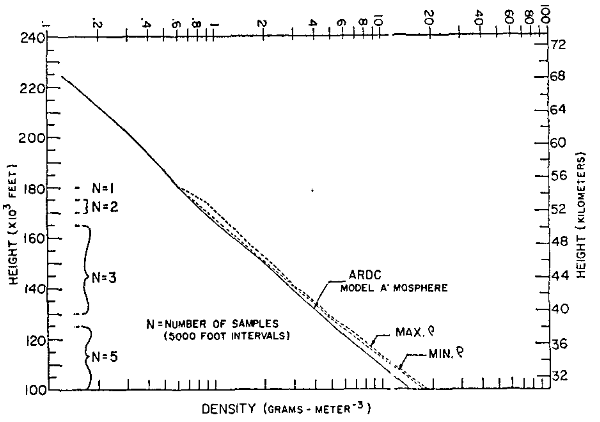

FIG. 2. Profiles of maximum and minimum density for Fort Churchill, Canada, latitude 58 deg (summer 1960) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

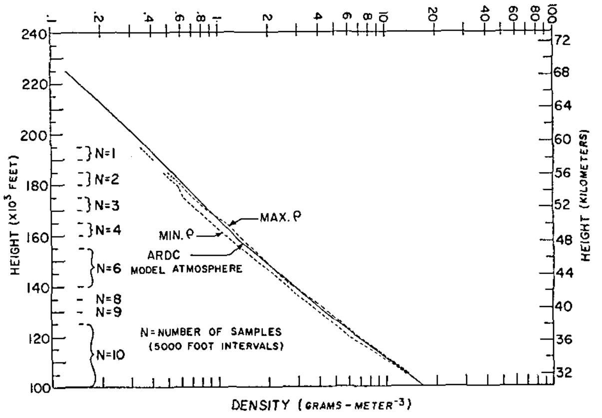

FIG. 3. Profiles of maximum and minimum density for White Sands Missile Range, N. Mex., latitude 32 deg (fall 1960) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

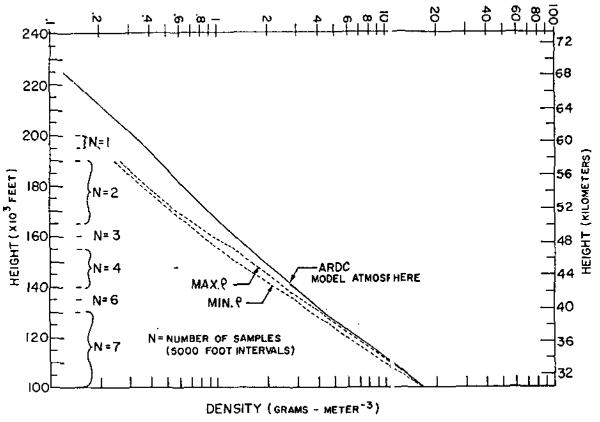

FIG. 4. Profiles of maximum and minimum density for Fort Churchill, Canada, latitude 58 deg (fall 1960) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

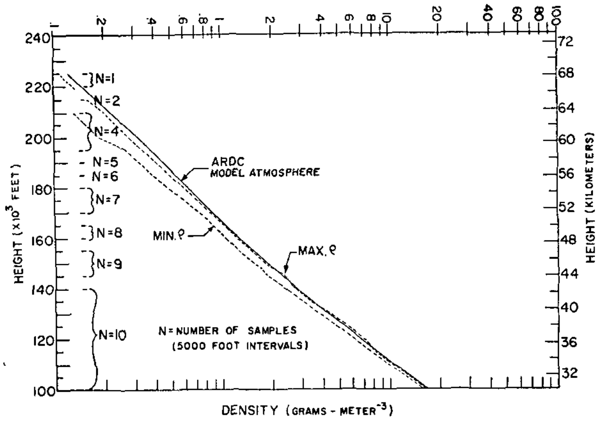

FIG. 5. Profiles of maximum and minimum density for White Sands Missile Range, N. Mex., latitude 32 deg (winter 1960–1961) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

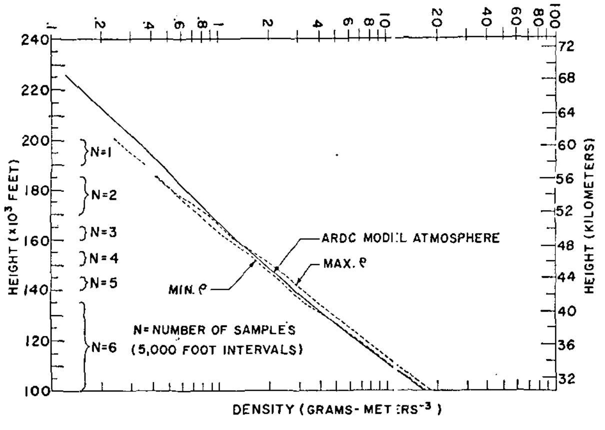

FIG. 6. Profiles of maximum and minimum density for Fort Churchill, Canada, latitude 58 deg (winter 1961–1962) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

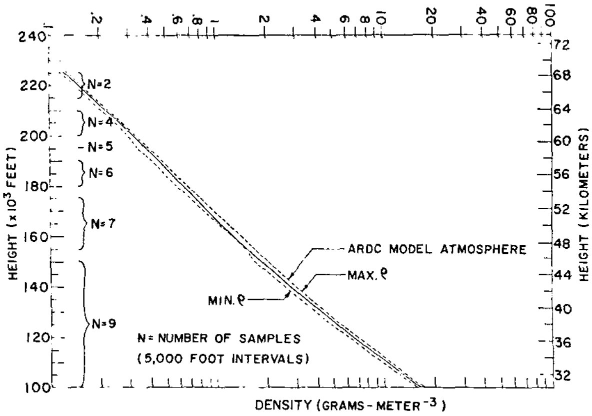

FIG. 7. Profiles of maximum and minimum density for White Sands Missile Range, N. Mex., latitude 32 deg (spring 1961) and 1959 Air Research and Development Command (ARDC) model atmosphere for density.

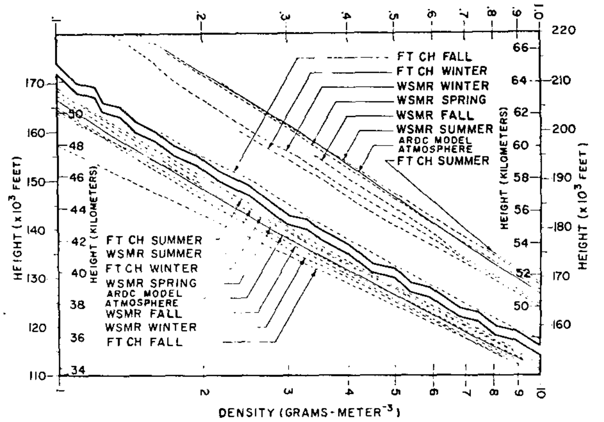

FIG. 8. Density profiles of seasonal means for White Sands Missile Range, N. Mex. (WSMR) and Fort Churchill, Canada (FT CH) and the 1959 Air Research and Development Command (ARDC) model atmosphere for density.

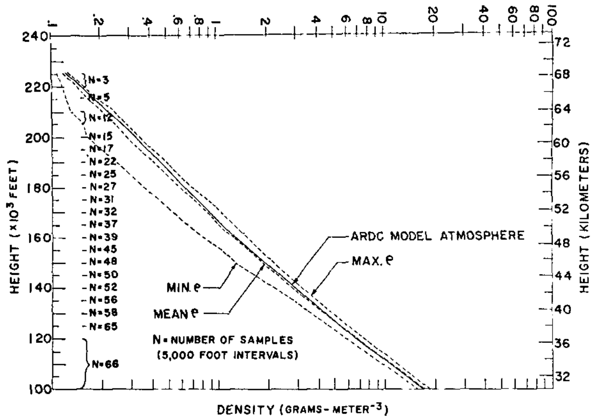

FIG. 9. Profiles of maximum, minimum and mean density for all data from White Sands Missile Range, N. Mex., and Fort Churchill, Canada, and the 1959 Air Research and Development Command (ARDC) model atmosphere for density.

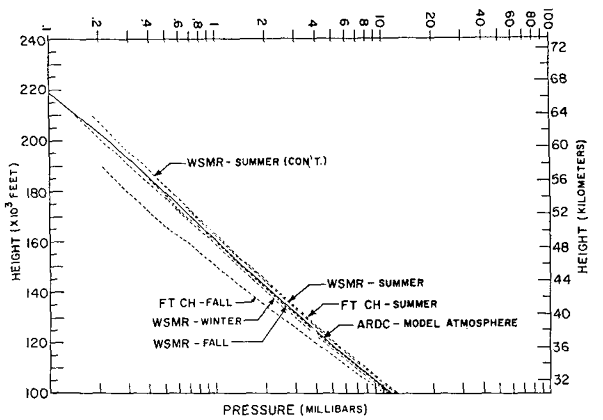

FIG. 10. Some pressure profiles of seasonal means for White Sands Missile Range, New Mexico (WSMR) and Fort Churchill, Canada (FT CH) and the 1959 Air Research and Development Command (ARDC) model atmosphere for density.

phere for density. The fall data, Figs. 3 and 4, also show that the maximum and minimum range during the particular season is small at each station; however, the latitudinal variation between the two stations begins to increase considerably. Figs. 5 and 6, containing the winter data, show a somewhat larger maximum and minimum range at each station particularly at White Sands, but the latitudinal variation is again less than that of the fall. As pointed out earlier, the only spring data available are for White Sands, Fig. 7. The maximum and minimum range is slightly less than that of winter, and the trend is toward the summer regime of increased density for mid-latitudes.

The mean density profiles in Fig. 8 show quite clearly the progressive decrease in density from summer to fall or winter at both stations. There is some uncertainty at this point concerning the significant period of minimum density at northern latitudes because of the lack of data in December and early January. The lowest values of density were obtained at Fort Churchill between the middle and end of November. Again referring to some recent Fort Greely, Alaska, "quick-look" data, minimum density occurred during the latter part of November with values as low or slightly lower than the Fort Churchill fall values. It is therefore, suggested that a more representative "season" for minimum values of density in the mesosphere at northern latitudes is perhaps from 15 November to 15 January. The low density occurring in the late fall is always associated with low pressure between the 50- and 10-mb levels, with the pressure remaining low throughout the measurement. Again referring to Fig. 8, it is evident that the seasonal variability is small at White Sands and quite large at Fort Churchill. This consolidated representation again points out the near absence of latitudinal variability in the summer and the considerable increase in latitudinal variability in the fall.

Fig. 9 presents the maximum and minimum as well as the mean of all the data. The overall variation begins to be quite large above 45 km varying between 40 and 50 per cent up to at least 60 km. The latitudinal density gradient becomes approximately 2 per cent per degree of latitude in the region of 50 per cent variation which corresponds to results obtained by Jones (1959). The mean curve in Fig. 9 agrees quite well with the 1959 Air Research and Development Command (ARDC) standard atmosphere for density profile. However, an atmospheric density slightly less than the standard ARDC model is indicated above about 40 km. It should be pointed out that in this, as well as the other figures, the extreme high end of the data is not fully representative because of the relatively few samples of data at those altitudes.

Since pressure values are a necessary by-product of the density derivations, a limited presentation of associated mean pressure profiles is included as Fig. 10. The seasonal and latitudinal variation of pressure follows essentially the same pattern as does the density variability and is, therefore, not discussed in detail.

3. Method of reduction

With the reduced temperature data, density and pressure values are calculated using the hydrostatic equation and the equation of state. The computations can be made quickly for immediate application.

The hydrostatic equation,

$$dp = -\rho g dz,$$

is modified by substituting from the equation of state,

$$dp = -\frac{pmg}{RT'}dz,$$

where T' is the virtual temperature. The equation is then integrated from the highest pressure measurement p_0 at height z_0 obtained from the balloon-borne radiosonde taken in conjunction with the rocket firing, to the height z, having the pressure p_1, as follows:

$$\int_{p_0}^{p_1}\frac{dp}{p} = -\frac{mg}{R}\int_{z_0}^{z_1}\frac{1}{T'}dz = \ln\frac{p_1}{p_0}$$

$$p_1 = p_0 \exp\left\{-\frac{mg}{R}\int_{z_0}^{z_1}\frac{1}{T'}dz\right\}$$

or as further solved and reduced for computer use

$$p_1 = p_0 \exp\left\{\frac{-g(z_1-z_0)}{R'\bar{T}'}\right\}$$

where $R'=R/m$, the gas constant for dry air, and $\bar{T}'$ is the mean virtual temperature in degrees Kelvin through the layer (z_1-z_0). As used here, $\bar{T}'$ is just the equally weighted mean temperature through the layer, since variations in composition at these altitudes are considered negligible.

The heights are geometric and are known quite accurately from radar data. An arbitrary layer thickness of 5000 ft is used for these calculations and g is the gravity value at the mean height of the layer.

The computation is repeated, with each succeeding calculated pressure (p_1) and height (z_1) at the top of the layer becoming the new p_0 and z_0, to the maximum height z_n of the temperature measurement.

Having computed the pressures at the top of each layer, the density at each of these levels is calculated using the equation of state for dry air,

$$\rho = \frac{p}{R'T},$$

where T is the measured temperature in degrees Kelvin at the particular level.

The complete problem has been programmed for use with a high speed digital computer. The input data consist of the initial pressure p_0 (dynes per square centimeter), initial height z_0 (feet MSL), maximum height z_n (feet MSL), $\bar{T}'$ (deg) values for each layer and T (deg) values at the level representing the top of each corresponding layer. Gravity values, the height increment (5000 ft), the gas constant for dry air, and conversion factors are pre-programmed. Table 1 is a sample of the output data.

It should be pointed out here that while the density values present a specific calculation at each level, some detail evidenced between levels is omitted. The method of calculation outlined above was used for convenience at the time. However, a system based on the significant levels of temperature has recently been programmed which results in density information in as much detail as is available from the temperature data.

4. Error analysis

Although the primary intent of this paper is to present and discuss some of the data derived from Arcas meteorological rockets, it necessarily follows that at least some attention should be given to possible sources of error, as well as the probable range of known errors.

Of primary importance with the method of calculation used here, is the availability of a reasonably accurate "tie-on" pressure obtained from an associated balloon-borne radiosonde measurement. The time and space variability of such a supporting measurement is neccessarily inherent and cannot be accurately resolved. Every effort is made to obtain a pressure measurement at 30 km within 30 min of the firing. In support of the

Table 1. Sample of computer output of derived density and pressure.

ARCAS meteorological rocket density calculation rocket fired at WSMR, New Mexico, on 14 Feb. '61, 1139 MST

Height ft	Height km	P dynes cm^{-2}	T deg C	Density gm cm^{-3}	Density slugs ft^{-3}
100,000	30.480	10,700.00	−46.4	1.6436 E-5	3.1890 E-5
105,000	32.004	8,548.59	−40.4	1.2793 E-5	2.4821 E-5
110,000	33.528	6,861.20	−35.8	1.0069 E-5	1.9536 E-5
115,000	35.052	5,532.73	−28.2	0.7867 E-5	1.5264 E-5
120,000	36.576	4,505.68	−19.8	0.6194 E-5	1.2019 E-5
125,000	38.100	3,680.87	−15.1	0.4968 E-5	0.9640 E-5
130,000	39.624	3,019.50	−11.1	0.4013 E-5	0.7787 E-5
135,000	41.148	2,480.77	−8.9	0.3270 E-5	0.6344 E-5
140,000	42.672	2,047.31	−0.7	0.2617 E-5	0.5078 E-5
145,000	44.196	1,699.27	7.2	0.2113 E-5	0.4096 E-5
150,000	45.720	1,416.03	8.9	0.1748 E-5	0.3393 E-5
155,000	47.244	1,179.87	9.7	0.1453 E-5	0.2819 E-5
160,000	48.768	983.62	8.6	0.1216 E-5	0.2359 E-5
165,000	50.292	819.03	6.4	0.1020 E-5	0.1980 E-5
170,000	51.816	681.09	4.2	0.0855 E-5	0.1659 E-5
175,000	53.340	565.57	2.0	0.0715 E-5	0.1389 E-5
180,000	54.864	469.08	0.3	0.0597 E-5	0.1159 E-5
185,000	56.388	388.69	−0.5	0.0496 E-5	0.0963 E-5
190,000	57.912	322.26	1.7	0.0408 E-5	0.0792 E-5
195,000	59.436	267.62	3.9	0.0336 E-5	0.0652 E-5
200,000	60.960	222.61	6.0	0.0277 E-5	0.0538 E-5
205,000	62.484	185.43	8.2	0.0229 E-5	0.0445 E-5
210,000	64.008	154.68	10.3	0.0190 E-5	0.0368 E-5
215,000	65.532	129.21	12.4	0.0157 E-5	0.0305 E-5
220,000	67.056	108.10	14.7	0.0130 E-5	0.0253 E-5
225,000	68.580	90.55	16.0	0.0109 E-5	0.0211 E-5

data contained in this paper, approximately one-third of the measurements at balloon-burst altitude were within 30 min, two-thirds within an hour, and all within 2 hours.

While the support radiosondes are released in the vicinity of the rocket launching area, the space variability at burst can be considerable. However, it could normally be expected to be within 50 mi with a probable maximum of 100 mi.

Several radiosonde measurements are made daily from different locations at White Sands Missile Range, and a close inspection of several sets of these data (a set is all of the measurements on a given day at the same altitude) revealed differences of only two- or three-tenths of a millibar in the vicinity of 30 km.

Errors due to time and space variability, when using compatible radiosonde data, are assumed to be insignificant. The overall accuracies of radiosonde measurements in general are discussed at considerable length in the literature. Whenever possible, the pressure measurement is obtained with a hypsometer type radiosonde which is generally acknowledged to be the most accurate system now available for field use.

The pressure values are determined as closely as possible to a tenth of a millibar for application in the computations. A quick inspection of the data will show that in the region of 30 km, a one-tenth millibar error is approximately one per cent which, of course, produces a one per cent error in the density calculation. The percentage of error produced by pressure errors remains constant throughout the derivation.

To evaluate the derivation of pressure, several computations were made to obtain pressure with the barometric equation using the temperature data from hypsometer radiosondes beginning at 25 km and ending at approximately 35 km. The variation between the calculated pressures and the measured pressures at the terminating end averaged one per cent with a maximum value of two per cent. Since the gas laws characteristically approach greatest accuracy at lower pressures, limited, of course, by dissociation above about 90 km, the results indicate that the gas constant for dry air as used appears to be sufficiently valid.

Next, it is necessary to consider the only other significant source of error, the temperature. Theoretical studies by Jehn and others[2] indicate a temperature accuracy of at least ±2C below 45 km, and to within ±5C up to 56 km. Above these altitudes, there are uncertainties that have not been resolved. Preliminary environmental studies, however, indicate that under ideal circumstances, temperature measurements to within ±5C are possible to as high as 68 km.

The effect of small temperature errors relative to the computed density is not large. A one per cent temperature error (approximately $2\frac{1}{2}$ deg) will result in a density error of one per cent.

Up to an altitude of 56 km, the computed densities are considered to be at least as accurate as density data

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[2] Jehn, K. H., N. K. Wagner, J. R. Gerhardt and D. R. Haragan, 1961: Wind and temperature in the atmosphere between 30 and 80 km. The University of Texas Electrical Engineering Research Laboratory Third Quarterly Technical Report Under U. S. Army Contract DA-23-072-ORD-1564, 55 pp.

derived from balloon-borne radiosondes. Although accuracies within two per cent are probable for more than half of the density data presented, all data are considered accurate to within 5 per cent.

Of general significance is the uniformity of a particular group of data, implying that any possible errors are at least systematic, and do not detract materially from the overall value of the results from a comparison standpoint.

5. Summary and conclusions

By using the method previously described, density and pressure data can be obtained from 30 km to at least 60 km economically and on a timely basis for direct application to space programs, forecasting, etc. Accuracies within 2 per cent are presently attainable under ideal circumstances. However, the data presented here are considered to be accurate within 5 per cent up to an altitude of approximately 68 km.

These data tend to verify many of the significant features of the atmosphere between 30 and 60 km that have been evident from previous experimental data, Jones (1959) and Quiroz (1961), such as the wider seasonal range at northern latitudes, the greater variation in winter at both stations, and the overall variations, as much as 50 per cent. The need for both seasonal and latitudinal standard atmospheres where standards are necessary is clearly demonstrated. For an overall standard, the total data indicate that both density and pressure are slightly lower, from 40 to 60 km, than the ARDC model. Doubtlessly, the increasing volume of data becoming available through the Meteorological Rocket Network will more accurately define these parameters in the upper atmosphere.

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