Temperature and Humidity Effects on Refractive Index Fluctuations in Upper Regions of the Convective Boundary Layer

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ABSTRACT

Here we illustrate a method which readily permits determination of the relative contributions of the individual temperature-humidity structure terms to total C_n^2 within the uppermost region of the clear, convective boundary layer. The relative contributions of terms involving C_T^2, C_{Tq} and C_q^2 to acoustic, optical and microwave C_n^2 are shown to be functions primarily of the ratio, $\Delta q/\Delta\theta_v$, of humidity to virtual potential temperature jump across the inversion. A graphical procedure is illustrated for quickly determining the expected degree of error if C_T^2 or C_q^2 are directly inferred from C_n^2.

1. Introduction

Often in remote sensing applications it is desirable to infer individual temperature and humidity structure parameter values (C_T^2, C_{Tq} and C_q^2) from the intensity of the return signal. The return signal, however, is proportional to the refractive index structure parameter C_n^2 and generally an assumption must be introduced in order to infer C_T^2 and C_q^2 from the measured C_n^2 value. For example, temperature fluctuations are often taken to be the sole contributor to acoustic C_n^2, and thus C_T^2 is inferred directly from an acoustic sounder return (Neff, 1975; Asimakopoulos *et al.*, 1976). As another example, moisture fluctuations are often assumed to be the sole contributor to microwave C_n^2, and thus clear-air radar returns are used to infer C_q^2.

A simple method of quickly determining the extent of validity of the above-mentioned inferences appears desirable. Here we describe such a method for the interfacial layer of a clear, convective boundary layer. We combine the layer-averaged expressions for individual structure parameters developed by Wyngaard and LeMone (1980) with the C_n^2 relationships formulated by Wesely (1976).

2. Temperature-humidity contributions to C_n^2

Interfacial layer-averaged structure parameters are written by Wyngaard and LeMone (hereafter WL) as[1]

$$\langle C_T^2\rangle = \frac{T_i\theta_{v*}}{Z_i^{2/3}}, \tag{1}$$

$$\langle C_{Tq}\rangle = \frac{q_i\theta_{v*}}{Z_i^{2/3}}, \tag{2}$$

$$\langle C_q^2\rangle = \frac{3.9(\rho\Delta q)^2\theta_{v*}}{Z_i^{2/3}\Delta\Theta_v}, \tag{3}$$

where

$$q_i = \rho\Delta q\left(2.2 - 2.4T\frac{\Delta q}{\Delta\Theta_v}\right),$$

$$T_i = \Delta\Theta_v\left[0.5 - 2.6T\frac{\Delta q}{\Delta\Theta_v} + 1.4\left(T\frac{\Delta q}{\Delta\Theta_v}\right)^2\right].$$

When combined with the Wesely (1976) relationships, we can evaluate the percent contribution of individual structure parameter terms to total C_n^2. Wesely writes[2]

$$(C_n^2)_a = (C_T^2/4T^2)\alpha_a^2, \tag{4}$$

$$(C_n^2)_o = \left[\left(\frac{A_1p}{T^2}\right)^2 C_T^2\right]\alpha_o^2, \tag{5}$$

$$(C_n^2)_m = \left[\left(\frac{Cp}{\epsilon T^2}\right)^2 C_q^2\right]\alpha_m^2, \tag{6}$$

where the subscripts a, o and m refer to acoustic, optical and microwave, respectively, and

$$\alpha_a^2 = 1 + \frac{2DT}{\epsilon}\frac{C_{Tq}}{C_T^2} + \left(\frac{DT}{\epsilon}\right)^2\frac{C_q^2}{C_T^2}, \tag{7}$$

$$\alpha_o^2 = 1 + \frac{2(1 - A_2/A_1)T}{\epsilon}\frac{C_{Tq}}{C_T^2} + \frac{(1 - A_2/A_1)^2T^2}{\epsilon^2}\frac{C_q^2}{C_T^2}, \tag{8}$$

$$\alpha_m^2 = 1 - \left[\frac{2A\epsilon}{C} + \frac{4e\epsilon}{pT}\right]\frac{C_{Tq}}{C_q^2} + \left[\frac{A\epsilon}{C} + \frac{2e\epsilon}{pT}\right]^2\frac{C_T^2}{C_q^2}. \tag{9}$$

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[1] We choose to use specific humidity q rather than absolute humidity Q, but otherwise the notation is the same as in WL (see the Appendix for a list of symbols).

[2] Eq. (6) is a corrected version of Wesely's expression (see Burk, 1980, Appendix). We write Wesely's expressions in terms of C_q^2 rather than C_e^2, which introduces the constant $\epsilon = 0.622$; otherwise our notation is identical.

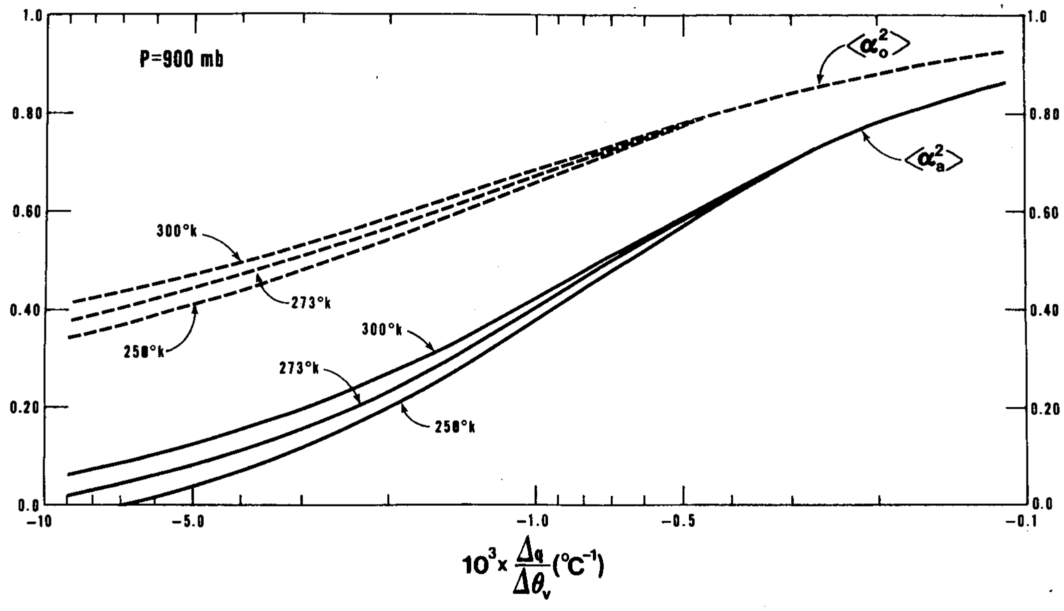

FIG. 1. Evaluation of the interfacial layer acoustic correction factor $\langle\alpha_a^2\rangle$ and optical correction factor $\langle\alpha_o^2\rangle$ as a function of $r = \Delta q/\Delta\theta_v$. Here Eqs. (10) and (11) are evaluated with $p = 900$ mb for several indicated temperatures.

The assumption frequently used to infer temperature or humidity structure parameters from remote sensor measurements is that these various α^2 values equal unity. In that event, optical and acoustic C_n^2 are solely dependent on C_T^2, and microwave C_n^2 is solely dependent on C_q^2. Here we wish to quantitatively evaluate the extent of validity of such assumptions concerning α^2 values. Further, in cases where C_n^2 is not simply dependent solely on a single structure parameter, we seek a handy method of quickly evaluating the relative contribution of each temperature-humidity structure parameter to total C_n^2. Thus, we now focus more closely on the individual α^2 values.

If we take interfacial averages of Eqs. (7)–(9), and utilize the WL expressions [Eqs. (1)–(3)], we find

$$\langle\alpha_a^2\rangle = 1 + \frac{2DT}{\epsilon}\,\frac{\rho r[2.2 - 2.4Tr]}{0.5 - 2.6Tr + 1.4(Tr)^2} + \left(\frac{DT}{\epsilon}\right)^2\left[\frac{3.9(\rho r)^2}{0.5 - 2.6Tr + 1.4(Tr)^2}\right], \quad (10)$$

$$\langle\alpha_o^2\rangle = 1 + \frac{2(1 - A_2/A_1)T}{\epsilon} \times \frac{\rho r[2.2 - 2.4Tr]}{0.5 - 2.6Tr + 1.4(Tr)^2} + \left[\frac{(1 - A_2/A_1)T}{\epsilon}\right]^2 \times \left[\frac{3.9(\rho r)^2}{0.5 - 2.6Tr + 1.4(Tr)^2}\right], \quad (11)$$

$$\langle\alpha_m^2\rangle = 1 - \left[\frac{2A\epsilon}{C} + \frac{4\epsilon e}{pT}\right]\left[\frac{2.2 - 2.4Tr}{3.9\rho r}\right] + \left[\frac{A\epsilon}{C} + \frac{2\epsilon e}{pT}\right]^2\left[\frac{0.5 - 2.6Tr + 1.4(Tr)^2}{3.9(\rho r)^2}\right], \quad (12)$$

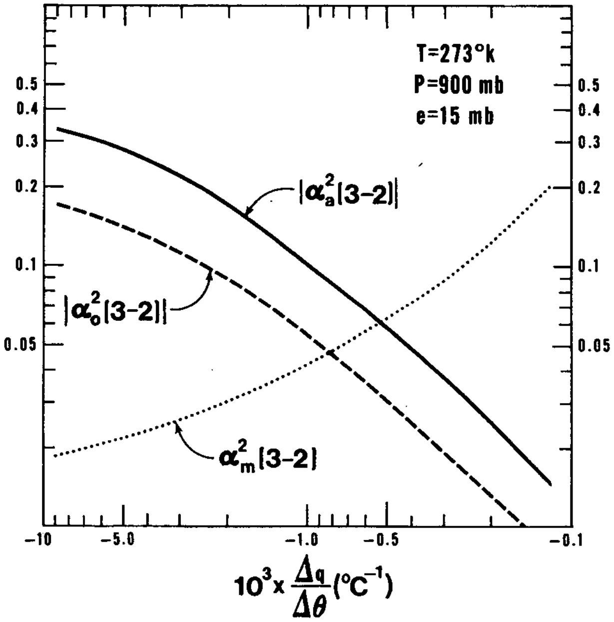

FIG. 2. Ratios of third term to second term in Eqs. (10)–(12) with $|\alpha_a^2(3-2)|$ being the absolute magnitude of this ratio in Eq. (10), $|\alpha_o^2(3-2)|$ for Eq. (11), and $\alpha_m^2(3-2)$ for Eq. (12). Dominance of second term involving C_{Tq} is indicated in each case. Selected mean temperature, pressure and vapor pressure indicated on figure.

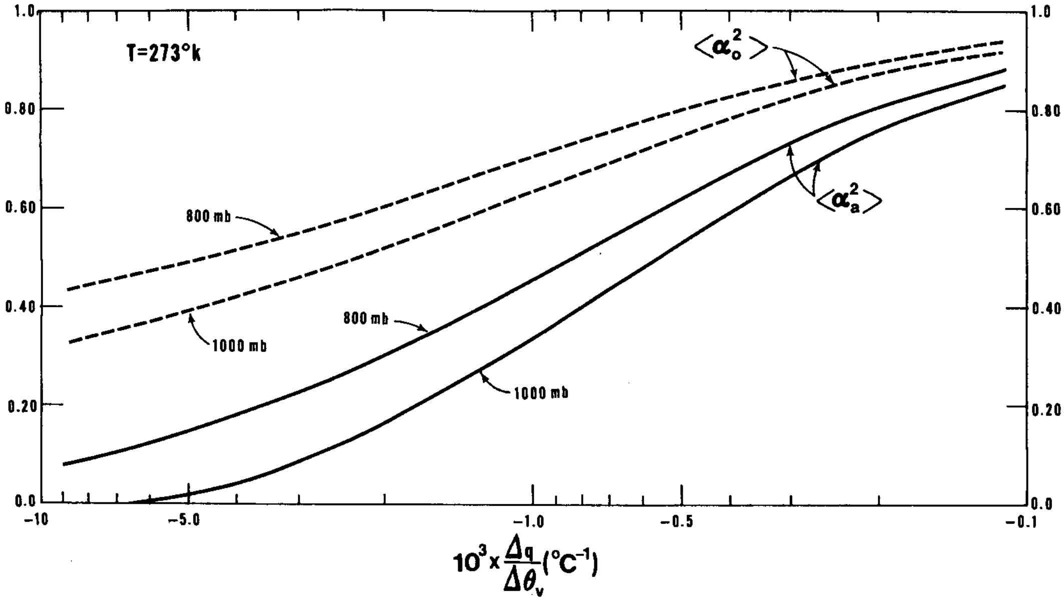

FIG. 3. Interfacial layer acoustic and optical correction factors at several indicated pressures with T = 273 K.

where we have taken $r = \Delta q/\Delta\Theta_v$. The striking (and useful) feature of these expressions is that the dependence on inversion height z_i and surface virtual temperature scale θ_{v*} has canceled out. Furthermore, we will show graphically that these $\langle\alpha^2\rangle$ expressions are functions primarily of r, with only relatively weak dependencies on temperature, pressure and, in the case of $\langle\alpha_m{}^2\rangle$, vapor pressure. Thus, for selected values of T, p and e, we may evaluate Eqs. (10)–(12) for a wide range of interfacial layer conditions by varying the ratio r.

Figs. 1–4 present plots showing the nature of these $\langle\alpha^2\rangle$ dependencies. We examine only the typical case for a convective boundary layer in which Δq is negative, $\Delta\Theta_v$ positive, and therefore, r negative.

Fig. 1 shows that the dependence on mean temperature of $\langle\alpha_a{}^2\rangle$ and $\langle\alpha_o{}^2\rangle$ is not strong. This figure also shows that when the humidity jump is small and the virtual potential temperature jump is large across the interfacial layer, $\langle\alpha_a{}^2\rangle$ and $\langle\alpha_o{}^2\rangle$ approach unity. This is reasonable since under such

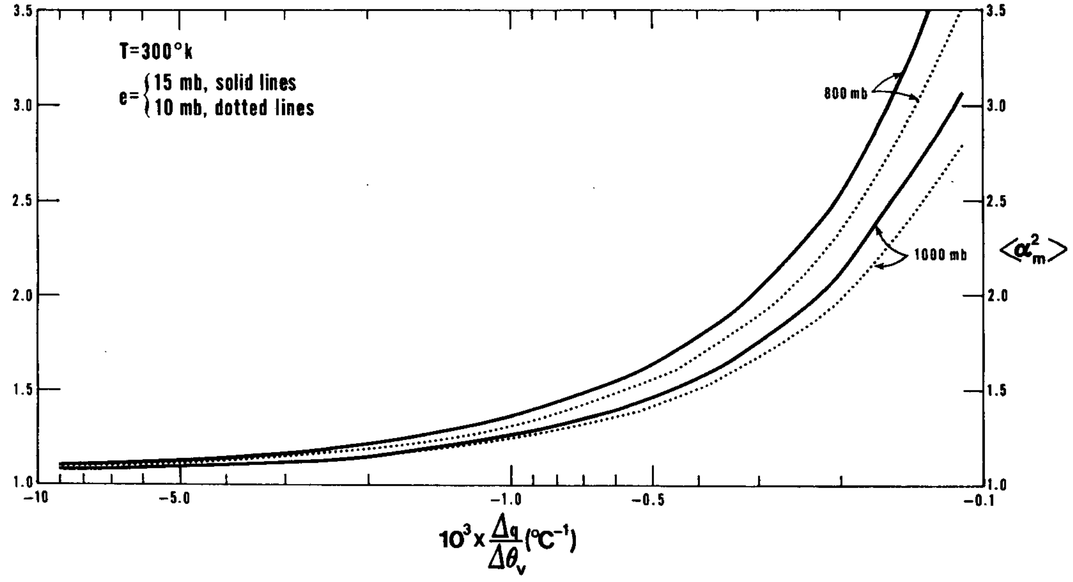

FIG. 4. Interfacial layer microwave correction factor for several indicated pressures and vapor pressures with T = 300 K.

conditions the humidity fluctuations would tend to be small, while the temperature fluctuations would be large, making $(C_n^2)_a$ and $(C_n^2)_o$ primarily dependent on C_T^2. At the other extreme of large Δq and small $\Delta\Theta_v$, Fig. 1 indicates the importance of accounting for the terms involving C_{Tq} and C_q^2 when making the transformation from C_T^2 to acoustic or optical C_n^2.

It may appear odd that the α^2 values in Eqs. (4) and (5) turn out to be less than unity. The term involving C_{Tq}, however, is negative when r is negative and, further, this term tends to be larger in absolute magnitude than the positive C_q^2 term. The dominance of the C_{Tq} term over the C_q^2 term in Eqs. (10)–(11) is displayed in Fig. 2. Plotted are the absolute magnitudes of the ratios of the third to second term in Eq. (10) [labeled $|\alpha_a^2(3-2)|$] and in Eq. (11) [labeled $|\alpha_o^2(3-2)|$].

These results are in qualitative agreement with the computations of structure parameters by Burk (1980) using a numerical boundary-layer model. (See Burk, Figs. 6, 8 and 12 for examples of clear, convective boundary layers in which there is considerable interfacial layer cancellation of the C_T^2 contribution to acoustic C_n^2 by the C_{Tq} term.) A more complete comparison of the WL formulations with predictions of a numerical turbulence closure model are presented in Burk (1981).

Fig. 3 illustrates the magnitude of the pressure dependence of $\langle\alpha_a^2\rangle$ and $\langle\alpha_o^2\rangle$. In Fig. 4 the behavior of $\langle\alpha_m^2\rangle$ is displayed, showing that $\langle\alpha_m^2\rangle$ deviates significantly from unity when $|r| < 1$. Thus, when the temperature jump is large and the humidity jump small, the C_{Tq} and C_T^2 terms make important contributions to total microwave C_n^2. In fact, the C_{Tq} term dominates the C_T^2 term, as can be seen in Fig. 2. The ratio of the third term in Eq. (12) to the second term is labeled as $\alpha_m^2(3-2)$ in Fig. 2.

3. Concluding remarks

The transformations between individual temperature-humidity structure parameters and C_n^2 shown in Eqs. (4)–(6) require knowledge of the correction factors α_a^2, α_o^2 and α_m^2. Using formulations developed by WL, we show that these α^2 factors have simple dependencies on bulk properties within the interfacial layer.

As an example, consider a sounding in which $\Delta\Theta_v = 4°\text{C}$, $\Delta q = -5 \times 10^{-3}$, $p = 900$ mb, $e = 10$ mb and $T = 285$ K. From Fig. 1 we find that the contribution of C_T^2 to optical C_n^2 is reduced by about 35% due to the terms involving C_{Tq} and C_q^2 (and Fig. 2 shows that in this example the optical C_{Tq} term is about 16 times larger in absolute magnitude than the C_q^2 term). Also, if we were to assume that acoustic C_n^2 was solely dependent on C_T^2 when making the transformation indicated in Eq. (4), Fig. 1 shows that we would be in error by nearly a factor of 3. In this example the correction factor for microwaves, $\langle\alpha_m^2\rangle$, is about 1.25 according to Fig. 4.

It should be reiterated that use of Eqs. (10)–(12), or Figs. 1–4, should be restricted to the conditions discussed in WL; *viz.*, the interfacial layer of clear, convectively driven boundary layers.

As noted by a reviewer, a word of caution is warranted concerning our treatment of C_{Tq}. The Wyngaard and LeMone (1980) expression for $\langle C_{Tq}\rangle$ has not been verified in the inertial subrange at the highest frequencies where wave-scattering phenomena occurs. The aircraft data from which C_{Tq} is computed are generally sampled at a considerably lower frequency than that responsible for the scattering. Direct testing using optical and other remote sensing techniques in conjunction with conventional aircraft sampling appears necessary to resolve remaining ambiguities.

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APPENDIX

List of Symbols[3]

Roman symbols

A	coefficient in microwave refractivity ($=77.6 \times 10^{-6}$ K mb^{-1})
A_1, A_2	coefficients in optical refractivity ($A_1 = 78.7 \times 10^{-6}$ K mb^{-1}, $A_2 = 66.3 \times 10^{-6}$ K mb^{-1})
C	coefficient in microwave refractivity ($=0.375$ K^2 mb^{-1})
C_n^2, C_q^2, C_T^2	structure parameters for refractive index, specific humidity and temperature, respectively
C_{Tq}	joint temperature-humidity structure parameter
D	constant in acoustic refractivity ($=0.307$)
e	vapor pressure
p	atmospheric pressure
q	specific humidity
q_i	interfacial layer humidity scale [Eq. (2)]
r	ratio of humidity to virtual potential temperature jump across interfacial layer [$=\Delta q/\Delta\Theta_v$]
T	absolute temperature
T_i	interfacial layer temperature scale, Eq. (1)
Z_i	inversion height

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[3] Expressions for acoustic, optical and microwave refractivity appear in Wesely (1976).

Greek symbols

$\alpha_a^2\ \alpha_o^2\ \alpha_m^2$	correction factors in transformations of acoustic, optical and microwave C_n^2 [Eqs. (4)–(6)]
ϵ	constant appearing in vapor pressure to specific humidity conversion (=0.622)
ρ	atmospheric density
θ_*	mixed-layer temperature scale
Θ_v	virtual potential temperature

Other symbols

$(\ )_r$	virtual
$\Delta(\ )$	bulk difference across interfacial layer
$\langle\ \rangle$	interfacial-layer average.

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