Determination of the Power-Law Wind Profile Exponent on a Tropical Coast

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ABSTRACT

Hourly measurements of temperature and wind speed (at 10 and 33 m) for a one-year period were made on the flat south coast of St. Croix, U.S. Virgin Islands. Values of the exponent p used in the power-law wind profile were obtained by applying criteria of Golder's (1972) nomogram, the U.S. Nuclear Regulatory Commission's (1974) temperature difference method, and Sedefian's (1980) formulation. It was found that p ranges from 0.15 for stability class A to >0.40 for class F. Thus the familiar $^1\!/_7$ power law will not be useful for a tropical coast. Since expected values of p as determined from Sedefian's nomogram were in good agreement with the observations, it is recommended for use in a tropical environment.

1. Introduction

In the atmospheric boundary layer the vertical distribution of wind speed may be represented by the power law (see, e.g., Blackadar, 1960; Munn, 1966; Sedefian, 1980)

$$u_2 = u_1(z_2/z_1)^p, \tag{1}$$

where u is the wind speed, z is the height, p is the exponent, and subscripts 1 and 2 represent measurements made at heights 1 and 2. p is a function of roughness z_0 and stability z/L, where L is the Monin-Obukhov length. Sedefian (1980) has constructed a nomogram to determine p versus $\bar{z}/z_0$ and $\bar{z}/L$ [where $\bar{z} = (z_1z_2)^{1/2}$, the geometric mean height between z_1 and z_2].

z_0 may be determined by the logarithmic wind profile under near-neutral conditions (see, e.g., Munn, 1966),

$$u = (u_*/\kappa)\ln(z/z_0), \tag{2}$$

where u_* is the friction velocity and κ is the von Kármán constant. If Eq. (2) is valid, wind speeds at two heights may be plotted on log-linear paper to determine z_0. Golder's graph (Golder, 1972) can then be used to obtain $1/L$ for various stability classes of Pasquill (1961). Thus the exponent p may be determined.

Values of p have been obtained by many investigators (cf. Blackadar, 1966; Sedefian, 1980). Most observations, however, have been conducted in middle and high latitudes. The value $p = {}^1\!/_7$ has been used widely under neutral conditions (e.g., Touma, 1977; Peterson and Hennessey, 1978). On tropical coasts, land- and sea-breezes are the most familiar local circulations (e.g., Atkinson, 1971). This diurnal alternation of offshore and onshore wind components may have some influence on values of p even under roughness and stability conditions that are similar to those found in midlatitude areas.

The main purpose of this paper is to determine values of p for various stability classes as used in middle and high latitudes. Specifically, Sedefian's nomogram is to be verified so that it can also be used in the tropics. This should be useful for studies such as atmospheric dispersion and wind-energy assessments on tropical coasts.

2. Field measurements and data analysis

Hourly temperature and wind measurements made in 1980 at 10 and 33 m above ground level were used in this study. The measurement site was located on the flat southern coast of St. Croix, U.S. Virgin Islands. Details concerning the site, instrumentation and data listings are available in Zankel *et al.* (1979), Palmerino *et al.* (1980) and McGirr *et al.* (1980, 1981). Meteorological measurements have been reviewed for adequacy by U.S. EPA Region II and were an integral part of an SO_2 monitoring program in St. Croix (Zankel *et al.*, 1979).

The procedures used in the analysis are:

1) Adapting the temperature difference method

TABLE 1. Classification of atmospheric stability from tower measurements on St. Croix.

Stability class		Vertical temperature difference* (°C m^{-1})	$T_{33} - T_{10}$ for St. Croix** [°C (23 m)$^{-1}$]
Extremely unstable	A	< −0.0190	< −0.437
Moderately unstable	B	−0.0190 to −0.0170	−0.437 to −0.391
Slightly unstable	C	−0.0170 to −0.0150	−0.391 to −0.345
Neutral	D	−0.0150 to −0.0050	−0.345 to −0.115
Slightly stable	E	−0.0050 to 0.0150	−0.115 to 0.345
Moderately stable	F	0.0150 to 0.0400	0.345 to 0.920
Extremely stable	G	>0.0400	>0.920

* Converted from ΔT method as given in U.S. Nuclear Regulatory Commission (1974).

** Values were obtained by multiplying 23 into the column on the left.

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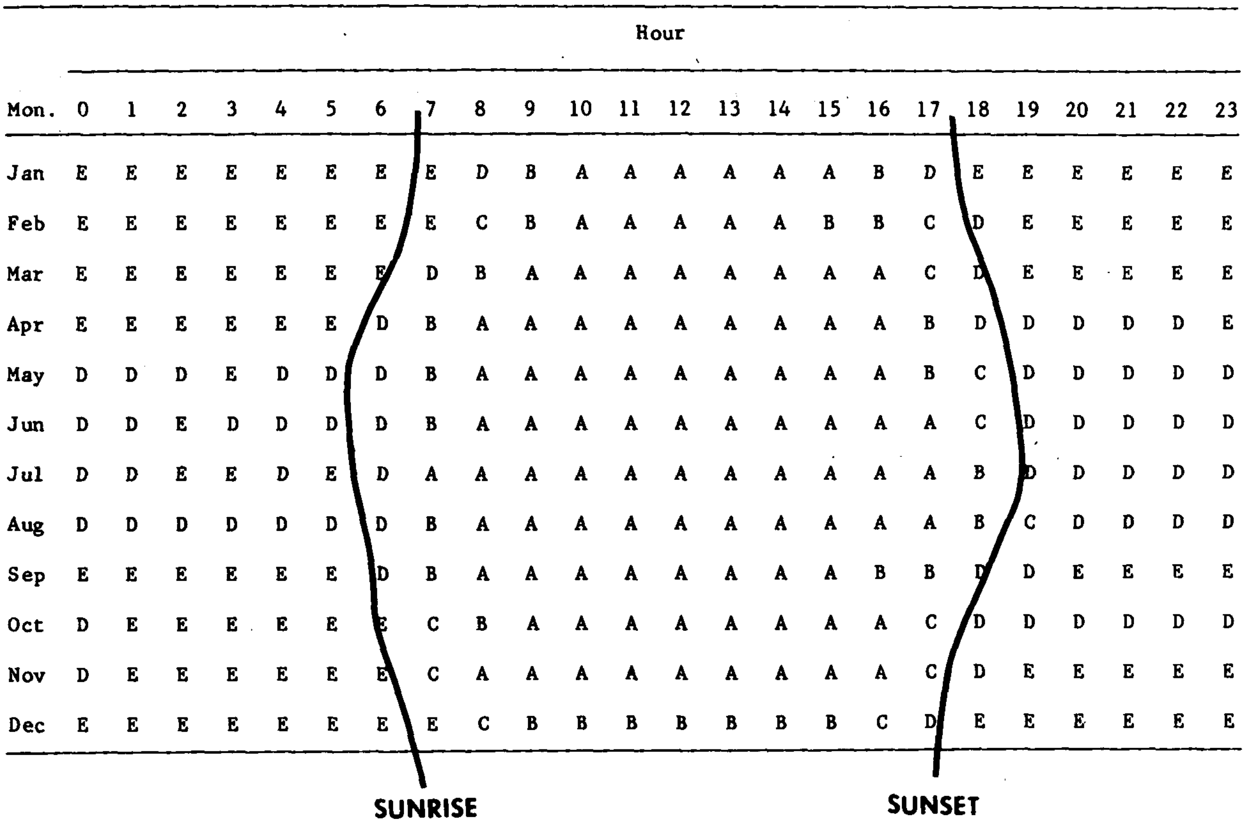

	Hour																							
Mon.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Jan	E	E	E	E	E	E	E	E	D	B	A	A	A	A	A	A	B	D	E	E	E	E	E	E
Feb	E	E	E	E	E	E	E	E	C	B	A	A	A	A	A	B	B	C	D	E	E	E	E	E
Mar	E	E	E	E	E	E	E	D	B	A	A	A	A	A	A	A	A	C	D	E	E	E	E	E
Apr	E	E	E	E	E	E	D	B	A	A	A	A	A	A	A	A	A	B	D	D	D	D	D	E
May	D	D	D	E	D	D	D	B	A	A	A	A	A	A	A	A	A	B	C	D	D	D	D	D
Jun	D	D	E	D	D	D	D	B	A	A	A	A	A	A	A	A	A	A	C	D	D	D	D	D
Jul	D	D	E	E	D	E	D	A	A	A	A	A	A	A	A	A	A	A	B	D	D	D	D	D
Aug	D	D	D	D	D	D	D	B	A	A	A	A	A	A	A	A	A	A	B	C	D	D	D	D
Sep	E	E	E	E	E	E	D	B	A	A	A	A	A	A	A	A	B	B	D	D	E	E	E	E
Oct	D	E	E	E	E	E	E	C	B	A	A	A	A	A	A	A	A	C	D	D	D	D	D	D
Nov	D	E	E	E	E	E	E	C	A	A	A	A	A	A	A	A	A	C	D	E	E	E	E	E
Dec	E	E	E	E	E	E	E	E	C	B	B	B	B	B	B	B	C	D	E	E	E	E	E	E

FIG. 1. Hourly atmospheric stability for each month in 1980 on St. Croix, U.S. Virgin Islands.

of the U.S. Nuclear Regulatory Commission (1974), use hourly listings of the temperature difference at 33 and 10 m, i.e., $T_{33} - T_{10}$, to obtain ΔT, then convert into stability classes A through G (see Table 1). Note that the use of temperature difference to define stability has recently been de-emphasized (Hoffnagle *et al.*, 1981).

2) Determine the day for each month that had the longest consecutive period of stability class D (neutral) conditions. Obtain arithmetic means from hourly u_{33} and u_{10} during this period. Perform linear regression for the logarithmic wind profile on log-linear paper to determine z_0 for each month and then take the average for the entire year to get the mean z_0 for the year, which happened to be 0.24 for the site under consideration. Note that values of u_*, which can be obtained by the slope of this linear regression (2), are not needed here.

3) Apply this z_0 (0.24 m) value in Golder's nomogram to obtain $1/L$ (m^{-1}) for each stability class.

4) From $z_1 = 10$ m and $z_2 = 33$ m, obtain $\bar{z} = (z_1z_2)^{1/2} = 18.2$ m. Then $\bar{z}/z_0 = 75.7$. Place this line onto Sedefian's nomogram to get the following values of the exponent for stability class: A, $p < 0.165$; B, $p = 0.165$; C, $p = 0.190$; D, $p = 0.235$, E, $p = 0.400$; and F, $p > 0.400$.

These p values may be considered the expected or theoretical values. Since our purpose is to verify the values by field measurements, we analyze the 33 m meteorological tower data by the following additional steps:

5) From (1), group all available hourly u_{33} and u_{10} values for each month with the same stability class and obtain average u_{33} and u_{10}. Since $z_2 = 33$

TABLE 2. Percent of occurrence for various stability classes for each month in 1980 on St. Croix.

	Stability class						
Month	A	B	C	D	E	F	G
Jan	23	4	3	19	35	11	6
Feb	27	3	5	21	33	9	2
Mar	33	2	3	15	36	9	2
Apr	39	2	3	32	19	4	1
May	40	4	7	36	12	2	0
Jun	43	3	4	37	13	1	0
Jul	39	5	3	34	20	0	0
Aug	45	5	5	31	13	0	0
Sep	40	3	4	23	23	7	0
Oct	38	3	4	27	24	4	0
Nov	39	2	3	23	23	8	3
Dec	16	3	3	24	23	23	9
Year	35	3	4	27	23	6	2

TABLE 3. Monthly variation of wind speed (m s^{-1}) at 33 and 10 m for various stability classes in 1980 on St. Croix.

Stability		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
A:	u_{33}	6.7	6.8	6.8	6.5	5.9	6.8	7.6	7.8	6.3	5.9	6.6	6.7	6.7
	u_{10}	5.8	5.7	5.7	5.6	5.1	5.6	6.1	6.3	5.3	4.8	5.5	5.5	5.6
B:	u_{33}	6.6	5.5	4.8	5.7	5.1	6.6	7.5	7.0	5.6	4.2	5.2	6.1	5.8
	u_{10}	5.8	4.6	4.1	4.5	4.3	5.3	5.8	5.4	4.4	3.4	4.1	4.9	4.7
C:	u_{33}	6.0	5.7	6.3	5.7	5.1	6.3	6.8	7.0	5.1	4.9	5.3	5.9	5.8
	u_{10}	5.2	4.5	5.4	4.6	4.3	4.9	5.4	5.1	3.9	3.9	3.9	4.8	4.7
D:	u_{33}	5.6	5.5	5.1	5.3	4.7	6.0	5.9	6.6	4.6	4.6	4.9	5.3	5.3
	u_{10}	4.7	4.2	4.1	4.1	3.6	4.5	4.5	4.8	3.3	3.4	3.7	4.0	4.1
E:	u_{33}	5.2	4.6	5.4	4.6	3.9	4.8	5.1	5.2	4.3	4.3	4.9	4.5	4.7
	u_{10}	4.1	3.2	4.0	3.1	2.7	3.4	3.7	3.4	2.7	2.8	3.2	3.4	3.3
F:	u_{33}	4.4	4.5	5.0	4.7	4.1	3.2	2.2	4.9	3.9	4.0	5.0	3.3	4.1
	u_{10}	3.0	2.7	3.1	2.8	2.4	2.1	0.4	2.9	2.5	2.4	3.1	2.4	2.5
G:	u_{33}	4.5	4.8	4.4	5.2	—	—	—	—	—	—	4.9	2.1	4.3
	u_{10}	2.9	2.8	2.3	2.9	—	—	—	—	—	—	2.8	1.3	2.5

m and $z_1 = 10$ m are known, determine the value of p for each stability class by taking logarithms on both sides of Eq. (1).

6) Compare the observed p value obtained in (5) to the expected values derived in (4) for each stability class.

3. Results

The diurnal variation of stability classes for each month is shown in Fig. 1. For reference purposes, the sunrise and sunset curves are delineated. As expected, during the day the atmosphere is in the unstable mode and is stable at night. Stability classes B and C occur more often near dawn and dusk. During summer months more A and D classes are noted than during winter months, in which class E prevails most often during nighttime (Fig. 1 and Table 2). In the course of a year, class A occurs 35% of the time; D, 27%; and E, 23% (Table 2). Note that variations of B and C are small for each month, but F and G classes (extremely stable) occur more often from November through March than during other months.

The monthly distribution in wind speed at 33 m, u_{33}, and at 10 m, u_{10}, for different stability is shown in Table 3. It can be seen that u_{10} for A and B is higher than 4 m s^{-1}. This is because the sea breeze is usually pronounced during the afternoon on tropical coasts; this local circulation, plus the large-scale

TABLE 4. Values of the exponent p as shown in Eq. (1) for each month as a function of stability class in 1980 on St. Croix.*

Class	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
A:	0.117 (142)	0.144 (181)	0.139 (152)	0.131 (240)	0.123 (294)	0.167 (299)	0.183 (226)	0.181 (360)	0.153 (207)	0.160 (246)	0.156 (175)	0.155 (57)	0.151
B:	0.110 (23)	0.149 (23)	0.132 (9)	0.202 (12)	0.141 (28)	0.179 (19)	0.217 (28)	0.220 (40)	0.206 (16)	0.181 (19)	0.212 (8)	0.182 (9)	0.178
C:	0.116 (17)	0.199 (31)	0.136 (16)	0.166 (18)	0.154 (50)	0.212 (30)	0.205 (15)	0.259 (42)	0.225 (20)	0.186 (23)	0.251 (12)	0.164 (9)	0.189
D:	0.160 (114)	0.219 (137)	0.185 (72)	0.216 (194)	0.207 (269)	0.231 (256)	0.221 (196)	0.268 (246)	0.260 (120)	0.248 (174)	0.242 (103)	0.234 (86)	0.224
E:	0.203 (212)	0.309 (216)	0.262 (168)	0.342 (116)	0.325 (86)	0.305 (91)	0.261 (116)	0.359 (106)	0.395 (121)	0.347 (158)	0.354 (103)	0.249 (83)	0.309
F:	0.336 (66)	0.444 (59)	0.380 (40)	0.417 (27)	0.440 (13)	0.354 (4)	1.348 (1)	0.441 (2)	0.374 (34)	0.440 (29)	0.417 (37)	0.289 (83)	0.473
G:	0.353 (37)	0.451 (14)	0.518 (8)	0.487 (8)	— (0)	— (0)	— (0)	— (0)	— (0)	— (0)	0.469 (14)	0.379 (32)	0.443

* Numbers in parentheses are those hours during the month in which the hourly observations were available.

TABLE 5. Comparison between expected (theoretical) value of p and observed values in 1980 on St. Croix.

Stability class	p expected	p observed
A	<0.165	0.151
B	0.165	0.178
C	0.190	0.189
D	0.235	0.224
E	0.400	0.309
F	>0.400	0.473
G		0.443

trade wind system, will certainly increase the wind speed to >4 m s^{-1}. Furthermore, in the tropics, the daytime insolation is strong, particularly during summer months. The fact that the atmosphere in the afternoon, during sea breeze conditions, is in the free convection mode has been considered elsewhere by Hsu (1973). It is clear, therefore, that stability classes A and B, due to strong insolation, can exist under conditions of wind speeds that are >4 m s^{-1}.

The procedures outlined in the previous section are used to obtain the exponent p, as shown in Eq. (1) for each month for each stability class (Table 4). Although month-to-month variation is large, the averages for the year indicate that p increases from 0.15 to >0.40. These values can now be compared with expected (or theoretical) values (Table 5).

Considering the limitations used in the procedures, as well as the uneven distribution of measurement sets [e.g., we have only one set of observations for class F in July (see Table 4), which showed higher values than other months], Table 5 substantiates the method advanced by Sedefian. This point is important because the $^1/_7$ power-law method, in which $p = 0.143$, widely used for average conditions, is not applicable to tropical coasts. As expected in Sedefian's formulation, variations of p are rather small in the unstable classes, A, B, and C. Table 5 shows that expected p values as obtained from the Sedefian nomogram are in good agreement with the observations.

4. Conclusions

It has been demonstrated that, since values of p on a tropical coast range from 0.15 for stability class A to >0.40 for F, the familiar $^1/_7$ power law cannot be applied to tropical coasts. Because Sedefian's nomogram for determining the exponent p is in good agreement with observations, it is recommended for engineering applications on other tropical coasts so long as the assumptions and limitations that were used to develop the nomogram are not exceeded.

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REFERENCES

Atkinson, G. D., 1971: Forecasters' guide to tropical meteorology. Tech. Rep. 240, Air Weather Service, U.S. Air Force, 4-26–4-32.

Blackadar, A. K., 1960: A survey of wind characteristics below 1500 ft. *Meteor. Monogr.*, No. 22, Amer. Meteor. Soc., 3–11.

Golder, D., 1972: Relations among stability parameters in the surface layer. *Bound.-Layer Meteor.*, **3,** 46–58.

Hoffnagle, G. F., M. E. Smith, T. V. Crawford and T. J. Lockhart, 1981: On-site meteorological instrumentation requirements to characterize diffusion from point source—a workshop, 15–17 January 1980, Raleigh, N.C. *Bull. Amer. Meteor. Soc.*, **62,** 255–261.

Hsu, S. A., 1973: Dynamics of the sea breeze in the atmospheric boundary layer: A case study of the free convection regime. *Mon. Wea. Rev.*, **101,** 187–194.

McGirr, R., *et al.*, 1980, 1981: SO_2 monitoring program in St. Croix, U.S. Virgin Islands. Data 1 April 1980–31 December 1980. Martin Marietta Environmental Center, Baltimore, MD.

Munn, R. E., 1966: *Descriptive Micrometeorology.* Academic Press, 245 pp.

Palmerino, M., *et al.*, 1980: SO_2 monitoring program in St. Croix, U.S. Virgin Islands. Data 1 January 1980–31 March 1980. Martin Marietta Environmental Center, Baltimore, MD.

Pasquill, F., 1961: The estimation of the dispersion of wind-borne material. *Meteor. Mag.*, **90,** 33–49.

Peterson, E. W., and J. P. Hennessey, Jr., 1978: On the use of power laws for estimates of wind power potential. *J. Appl. Meteor.*, **17,** 390–394.

Sedefian, L., 1980: On the vertical extrapolation of mean wind power density. *J. Appl. Meteor.*, **19,** 488–493.

Touma, J. S., 1977: Dependence of the wind profile power law on stability for various locations. *J. Air Pollut. Control Assoc.*, **27,** 863–866.

U.S. Nuclear Regulatory Commission, 1974: Regulatory Guide 1.23, Onside Meteorological Programs, 6 pp.

Zankel, K., *et al.*, 1979: SO_2 monitoring program in St. Croix, U.S. Virgin Islands—1978. Vol. I, Test. Martin Marietta Environmental Center, Baltimore, MD.