Reorientation of Hydrometeors in Aircraft Accelerated Flow

K. V. BEARD

Illinois State Water Survey,[1] University of Illinois, Champaign, IL 61820

5 February 1983 and 25 July 1983

ABSTRACT

The drag force, instantaneous canting angle and response characteristics are discussed for large hydrometeors in horizontally accelerated airflow. Comparisons of calculated canting angles are made with data from an aircraft precipitation spectrometer (Bringi) by assuming an airflow disturbance based on probe geometry. It is concluded that the accelerated airflow is the likely cause of an intrinsic canting angle for raindrops of about 15° but that the force is too weak to appreciably affect the axis ratio. No intrinsic canting was found for graupel.

1. Introduction

Measurements of precipitation particles by Bringi *et al.* (1984) using an aircraft mounted spectrometer (PMS Inc.) with the optical axis horizontal rather than vertical have resulted in images of oblate raindrops and conical graupel. The authors found canting angles of 25–35° for raindrops, and an average (systematic) orientation for graupel of 15° (see Fig. 1, courtesy of the authors and the Research Aviation Facility of NCAR). The reason suggested for the tilted images is a shape transformation that occurs because large hydrometeors move at an appreciable angle through the diode array. This "angle of motion effect" leads to an artificial canting for graupel consistent with the observations. In contrast the estimate for raindrops, using their formula based on an elliptical cross section, yields artificial canting angles of 22° to 14° for their aircraft speed (85 m/s) and for the appropriate range of drop sizes (2.5 to 4.5 mm diameter) and fall speeds (8–10 m/s). Since this fictitious tilting is significantly less than appears in the 2-D images there is an additional source of raindrop canting that must be explained. A plausible cause of intrinsic canting is the accelerated airflow ahead of the probe which tilts the raindrops but not the graupel because of differing response characteristics.

2. Theory and estimates

A semi-quantitative picture of canting can be gained from a comparison of the significant time scales. First, the interaction time of precipitation with the airflow disturbance is about $\tau \sim 6$ ms (based on assumptions discussed later). The response time of a large hydrometeor to a horizontal velocity change is $\tau_v = V_\infty/g$ (Beard and Jameson, 1983) or about 1 second for a raindrop or graupel. Since $\tau \ll \tau_v$, large hydrometeors cannot adjust their velocity appreciably to the airflow disturbance, and therefore, the *change* in motion can be ignored.

The drag force can be calculated from the total velocity as $F = BV^2 = B(u^2 + V_\infty^2)$ where u is the horizontally accelerated flow (see vector diagram on figure). The coefficient B is obtained from the size, shape and drag coefficient of the hydrometeor. If shape changes due to acceleration are negligible then B is essentially constant so that $F \approx mg(1 + u^2/V_\infty^2)$. The angle of the drag force (F) from the vertical is the same as the velocity (V), i.e., $\phi = \tan^{-1}(u/V_\infty)$, which result in a canting angle ϕ for a steady force (Brussard, 1974). The canting angle is also ϕ for a quasi-steady situation where the drop is able to respond to the changing force (Beard and Jameson, 1983).

If a steady or quasi-steady force is assumed the raindrop distortion is increased, since $F = mg(1 + \tan^2\phi)$. For a 3 mm diameter drop with $\phi = 30°$, the force is increased by 33%, but the calculated decrease in the axis ratio, using the formula of Green (1975), is only 4%. Hence in the steady or quasi-steady situation the change in drop shape for ϕ as large as 30° can be neglected so that the coefficient B is essentially the value obtained from the terminal velocity relation, $mg = BV_\infty^2$.

With the assumption that canting is a capillary wave response (not a rotation) a characteristic response time (τ_c) can be estimated from the time for a simple distortion which is about ¼ the oscillation period of the fundamental harmonic. For a 3 mm diameter drop $\tau_c = 4$ ms. A value of $\tau_c \sim 2$ ms is obtained using a capillary wave speed of 180 degrees per oscillation period for a canting angle of 15°. By either estimate canting should be significant for the stated interaction

---

[1] Also affiliated with the Department of Atmospheric Sciences.

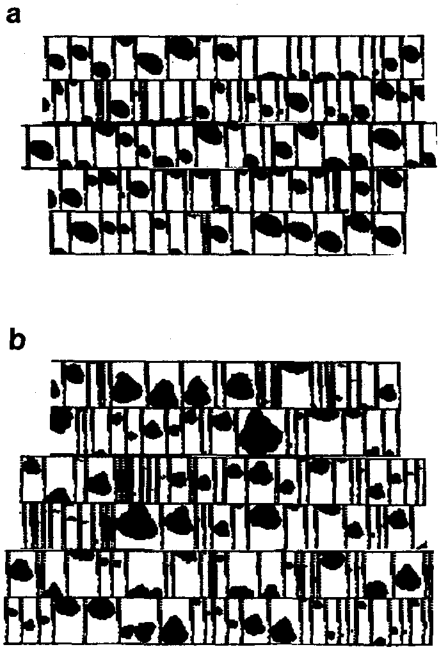

FIG. 1. Hydrometeor images from diode array probe (from Bringi *et al.*, 1984) for (a) raindrop images with apparent canting angles of about 30° and (b) graupel. Vertical frame dimension is 6.2 mm.

time of $\tau \sim 6$ ms. In contrast, the canting response time for graupel is much longer, in part, because more mass is involved in a solid body rotation than in the capillary wave adjustment. A rough calculation of the graupel response time can be made by assuming an approximate moment of inertia and torque. For simplicity the moment of inertia for a sphere of radius r is used ($0.4\ m\ r^2$) with a torque of $F_x r/2$ where $F_x = mg \sin\phi(1 + \tan^2\phi)$. For the response time from 0° to 30° it is assumed that the force is a constant given at $\phi = 15°$ so that $\omega = d\phi/dt$ and ϕ can be readily obtained from the angular acceleration equation, $d\omega/dt = F_x(r/2)/(0.4\ m\ r^2)$, resulting in $\phi \simeq 0.17\ gt^2/r$. With these approximations the orientation time for 0 to 30° is $\tau_0 \sim 22$ ms for a 3 mm diameter graupel, a time which is significantly longer than the raindrop response time. A completely unrealistic torque or moment of inertia is needed to produce a graupel orientation time comparable to the raindrop.

Quantitative estimates of canting angle can be made using an assumed velocity disturbance (u) for the PMS precipitation probe. The major disturbance should originate from the electronics pod which is 13.3 cm downstream from the sampling point. (The probe geometry is based on dimensions supplied by PMS Inc.) The airflow ahead of the hemispherical cap of the pod, on the flow axis, may be estimated from incompressible flow ahead of a sphere (Batchelor, 1967) of the same radius (Fig. 2) which is $u = UR^2/(R + x)^3$ where U is the airspeed. The computed value of u is shown in Fig. 2 for the average aircraft speed of $U = 85$ m s^{-1} given in Bringi *et al.* and cap radius of $R = 8.89$ cm. At the sampling point, where $x = 13.3$ cm, the disturbance velocity is about 5 m s.$^{-1}$. Calculations of the instantaneous canting angle from $\phi = \tan^{-1}u/V_\infty$ are also shown for 2 values of terminal velocity (8 and 10 m s^{-1}). This range in V_∞ includes raindrops that are ≥3 mm diameter for sea level conditions and raindrops from 2.5 to 4.5 mm diameter for 15 C and 800 mb.

Instantaneous canting angles are 35° ($V_\infty = 8$ m s^{-1}) and 29° ($V_\infty = 10$ m s^{-1}) at the sampling point ($x = 13.3$ cm). A comparison with the estimated intrinsic canting angle for raindrops of ~15° indicates that the canting response is appreciable but incomplete. This result is not surprising since the interaction time, as seen in Fig. 2 ($\tau \sim 6$ ms), is similar to the response time ($\tau \sim \tau_c$). In the case of the graupel the response can be calculated for the velocity disturbance shown in Fig. 2 by integration of the angular acceleration equation. The result for the torque and moment of inertia used in the previous estimate yields a canting angle of <1° at the sampling point. Again the outcome is not surprising since the interaction time is much shorter than the estimated orientation time ($\tau \ll \tau_0$).

At higher air speeds the artificial canting produced

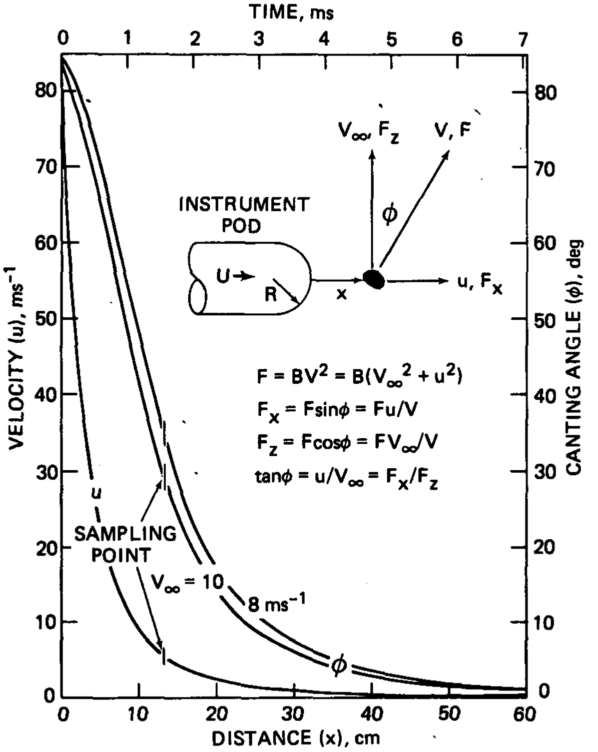

FIG. 2. The vector diagram and equations show the relation between the velocities and drag force. Curves give the disturbance velocity (u) and instantaneous canting angle (ϕ) as a function of distance (x) and time ahead of the instrument pod.

by the angle of motion would decrease but the intrinsic canting might be relatively unaffected since an increase in forcing is compensated by a decrease in response time. A doubling of the airspeed would alter the instantaneous canting angle at the sampling point from $\phi \simeq 30°$ to $50°$ but decrease the interaction time from $\tau_c \simeq 6$ ms to 3 ms. The artificial canting would be reduced from a range of 14–22° down to 7–12°.

In summary the differing response characteristics to the computed airflow has indicated intrinsic canting for raindrops but none for graupel. The graupel response to forcing ($\phi = 30°$) is negligible since $\tau \ll \tau_0$. The raindrop response of the fundamental distortion is appreciable since $\tau \sim \tau_c$, a conclusion consistent with the intrinsic canting deduced from the data on raindrops of about 15°. This analysis does not address distortions of shorter wavelength nor incorporate any increased flattening from acceleration. However, as the raindrop images testify, the dominant response to the accelerated airflow is a simple reorientation.

Acknowledgments. This work was supported in part by the National Science Foundation under Grants ATM81-08455 and ATM82-16847. The author appreciates the helpful comments of Bob Czys, Dave Johnson and Bernice Ackerman of the Illinois State Water Survey.

REFERENCES

Beard, K. V., and A. R. Jameson, 1983: Raindrop canting. *J. Atmos. Sci.*, **40**, 448–454.

Batchelor, G. K., 1967: *An Introduction to Fluid Dynamics.* Cambridge Press, 615 pp.

Bringi, V. N., T. A. Seliga and W. A. Cooper, 1984: Analysis of hydrometeor spectra and differential reflectivity measurements during the Cooperative Convective Experiment. *Radio Sci.*, (in press).

Brussard, G., 1974: Rain-induced cross polarization and raindrop canting. *Electron. Lett.*, **10**, 411–412.

Green, A. W., 1975: An approximation for the shapes of large raindrops. *J. Appl. Meteor.*, **14**, 1578–1583.