Extrapolation of Solar Radiation Measurements: Mesoscale Analyses from Arizona and Tennessee Valley Authority Regions

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ABSTRACT

Relationships for determining the maximum permissible distance for extrapolating daily totals of solar radiation from measurement sites are established for mesoscale monitoring networks in southern Arizona and the Tennessee Valley Authority (TVA) region. The method involves calculation of the standard deviation of the daily differences in solar radiation receipt for pairs of measurement stations in order to determine a coefficient of variability. This is then plotted as a function of distance between station pairs to establish an extrapolation distance relationship.

Results indicate that the solar climate of southern Arizona has much greater spatial coherence than that for the TVA region, thus permitting extrapolation of data over longer distances. However, extrapolation distances for daily totals of solar radiation are very small in either study area. Applied to monthly totals for an error tolerance of ±10% at a 90% confidence level, permissible extrapolation distances of more than 400 km for southern Arizona and ~200 km for the Tennessee Valley were found. However, the extrapolation distances may vary with season.

1. Introduction

Routine solar radiation measurements are taken at a very limited number of sites. Such data are being applied to numerous problems ranging from agricultural uses to solar energy studies. The applicability of these data to locations other than the original monitoring site is an important subject. It is well known that solar radiation receipt is quite variable on a wide range of spatial and temporal scales including the mesoscale (Kerr *et al.*, 1968; Atwater and Ball, 1978). One alternative method for determining the spatial distribution of solar radiation is through the use of satellite data (e.g., Tarpley, 1979; Hay, 1981; Gautier, 1982). Another method of supplementing solar radiation measurements is through the application of numerical models that utilize other meteorological data (e.g., Atwater and Brown, 1974; Davies *et al.*, 1975; Suckling and Hay, 1977). However, both the use of satellite information and numerical models are data intensive and represent approaches not readily available to many solar radiation data users. Therefore, data from our limited monitoring networks will continue to be widely used.

Recently, attempts have been made to assess the spatial variability of solar radiation measurements and determine the distance to which values can be reliably extrapolated from the monitoring site (Wilson and Petzold, 1972, 1976; Suckling and Hay, 1976; Hay and Suckling, 1979; WMO, 1981; Suckling, 1982). The basic approach involves calculation of the mean and standard deviation of the daily differences in incoming solar radiation for pairs of measurement stations. Hay and Suckling (1979) defined a coefficient of variability (c_v) which is calculated as a percentage from

$$c_v = \frac{\sigma}{0.5(K\downarrow_1 + K\downarrow_2)} \times 100\%, \qquad (1)$$

where σ is the standard deviation of daily differences in solar radiation for two given stations, and $K\downarrow_1$ and $K\downarrow_2$ are the mean daily values of solar radiation for the same two stations. Values of c_v are then plotted as a function of the distance between stations in order to establish a solar radiation extrapolation distance relationship. The approach requires data from a number of monitoring sites preferably in a mesoscale network.

Suckling (1982) has utilized this approach applying data from several previous studies in southern Canada. Results were then applied in a preliminary assessment of the spatial coverage provided by the United States solar radiation monitoring network. It was noted, however, that separate solar radiation extrapolation distance relationships representing the solar climates of the different regions of the United States would be preferable. Willmott and Vernon (1980) have suggested that there are ten distinct solar climates in the conterminous United States. It is the purpose of this paper to establish solar radiation extrapolation distance relationships for two of these re-

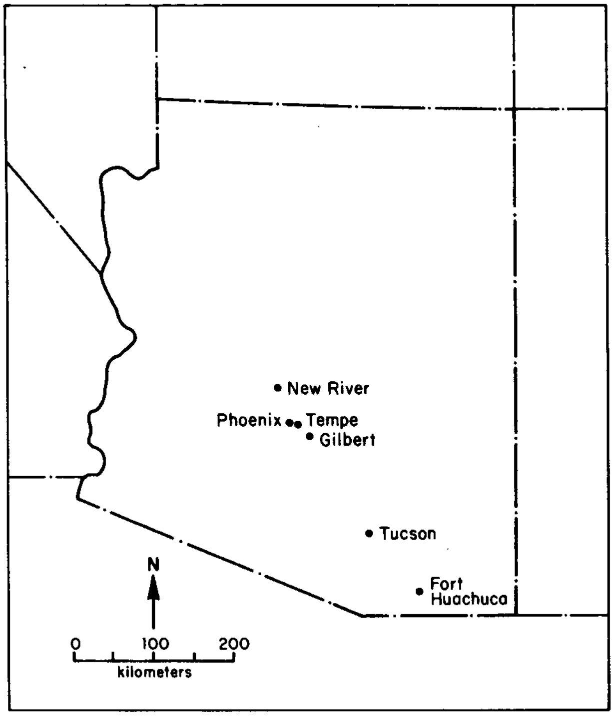

FIG. 1. Solar radiation measurement sites in southern Arizona.

gions utilizing data from mesoscale solar radiation networks in southern Arizona (Willmott and Vernon's solar climate 4) and the Tennessee Valley Authority (TVA) region (Willmott and Vernon's solar climate 9).

2. Data

Daily totals of incoming solar radiation for the year 1977 were obtained for a mesoscale monitoring network in southern Arizona from the Laboratory of Climatology at Arizona State University (State Climatologist for Arizona, 1978). Data from six sites (Fig. 1) were used. Distances between the 15 station pairs were calculated using the approach outlined in Robinson *et al.* (1978, p. 44–45) and are given in Table 1. Similar data for the year 1979 were obtained for a solar radiation network administered by the TVA. Twelve sites (Fig. 2) were used and the distances between the 66 station pairs are given in Table 2.

The original data sets included more than those sites given in Figs. 1 and 2. However, some sites were eliminated from the analyses since their data were deemed significantly different than most of the other sites. Wilson and Petzold (1972) suggest that such a determination can be made by considering the difference between the mean daily solar radiation for two stations. Since measurement error is often considered ±5.0%, then the difference between two stations would not be significant unless it exceeded $(5^2 + 5^2)^{1/2} = \pm 7.1\%$ of the average solar radiation received at the two stations. For Arizona, the two high altitude sites of Castle Creek and Seven Springs located in the eastern part of the state were found to have much lower values of solar radiation on average likely due to topographic/orographic cloud effects. On the other hand, the site of Yuma located in the southwestern corner of the state had a significantly higher mean daily solar radiation for the year. Also, Yuma would not assist in mesoscale analysis since it is located more than 230 km from all other sites.

In the TVA analysis, the sites of Cumberland, TN and Bull Run, TN were found to have significantly lower solar radiation values than most of the other sites while Browns Ferry, AL had much higher readings. In the absence of obvious climatic differences, it is probable that instrumentation errors such as calibration or siting were responsible for the observed differences at these three sites. It should be noted that tests including these sites in the following analysis did not affect the results since the standard deviation of differences in $K\downarrow$ was of central concern rather than absolute values. This fact further suggests that instrumentation errors rather than microclimatic factors are responsible for the significantly different values at these three sites. Since 12 sites constituting 66 station pairs were available without their inclusion, Cumberland, Bull Run and Browns Ferry were eliminated from further study.

3. Establishment of extrapolation distance relationships

Values of c_v were calculated for each pair of solar radiation stations in each of the study areas and are plotted as a function of the distance between stations (D) in Fig. 3. For the southern Arizona analysis, the relationship between c_v and D can be expressed as

$$c_v = 6.36 + 0.0626D, \tag{2}$$

with a correlation coefficient of 0.988 and a standard error of 0.97. In the study by Suckling (1982), the best relationship was one with D expressed as the natural logarithm. The Arizona data expressed in this fashion is

$$c_v = -10.6 + 5.60 \ln D, \tag{3}$$

TABLE 1. Distances (km) between stations for southern Arizona study area.

	Fort Huachuca	Gilbert	New River	Phoenix	Tempe	Tucson
Fort Huachuca	—	229	303	259	254	94
Gilbert		—	75	38	30	135
New River			—	47	49	209
Phoenix				—	8	160
Tempe					—	135
Tucson						—

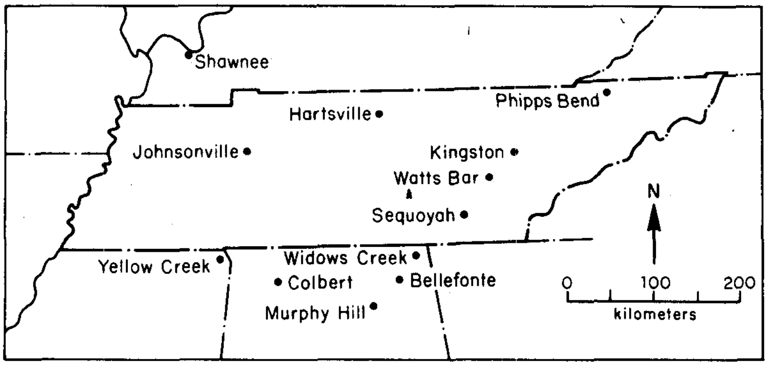

FIG. 2. Solar radiation measurement sites in the Tennessee Valley Authority region.

with a correlation coefficient of 0.938 and a standard error of 2.20. Thus, the linear relationship expressed by Eq. (2) proved to be slightly better.

The results for the TVA study area yield

$$c_v = 19.4 + 0.0630D, \qquad (4)$$

with a correlation coefficient of 0.925 and a standard error of 3.12. The logarithmic relation is

$$c_v = -25.5 + 11.3 \ln D, \qquad (5)$$

with a correlation coefficient of 0.923 and a standard error of 3.17. Again, no benefit is gained by the logarithmic transformation.

Although the slope terms in Eqs. (2) and (4) are similar, the much lower intercept value for the southern Arizona relationship implies a much more spatially coherent solar climate compared to the TVA study area.

The use of the standard deviation in Eq. (1) requires that the daily differences in solar radiation for a station pair be normally distributed. Chi-square tests were conducted with data for each station pair divided into nine groups; seven of which were 0.5σ in size, with the last two categories covering data 1.75σ above or below the mean. Examples of the frequency distributions are given in Fig. 4. Chi-square values were very high in all cases (above the 20.1 critical value for the 1% decision level) indicating that the distributions were not normal. Some of the station pairs studied by Wilson and Petzold (1972) exhibited similar characteristics with large frequencies near the mean of the distribution. It should also be noted that the use of the standard deviation in Eq. (1) implies only a 67% confidence in the relationship (Wilson, 1980). Higher levels of confidence can be achieved by multiplying the resulting coefficients in Eqs. (2)–(5) by appropriate Z-values such as 1.64 for a 90% confidence level. Such a confidence level would be much more meaningful. With this in mind, the frequency distributions (like those in Fig. 4) were examined to determine the percentage of the observations that fell within 1.64σ of the mean. For the 15 Arizona stations pairs, 89.1–93.0% (average 90.8%) of observations were within the 1.64σ boundaries while for the 66 TVA station pairs, 88.0–96.1% (average 90.7%) of observations were within the 1.64σ boundaries. These results suggest that although not normally distributed, the approach used in Eq. (1) may be applicable when the relationships are converted to the 90% confidence level.

Converting Eqs. (2)–(5) to the 90% confidence level gives southern Arizona relationships of

TABLE 2. Distances (km) between stations for Tennessee Valley Authority study area.

	Bellefonte	Colbert	Hartsville	Johnsonville	Kingston	Murphy Hill	Phipps Bend	Sequoyah	Shawnee	Watts Bar	Widows Creek	Yellow Creek
Bellefonte	—	174	182	236	182	34	341	93	372	141	22	212
Colbert		—	238	144	327	156	492	263	282	292	188	44
Hartsville			—	172	150	208	293	154	254	146	167	247
Johnsonville				—	311	238	465	275	144	292	237	121
Kingston					—	216	165	92	404	43	161	352
Murphy Hill						—	374	126	379	174	55	197
Phipps Bend							—	248	534	202	320	516
Sequoyah								—	392	49	72	286
Shawnee									—	397	370	249
Watts Bar										—	119	320
Widows Creek											—	224
Yellow Creek												—

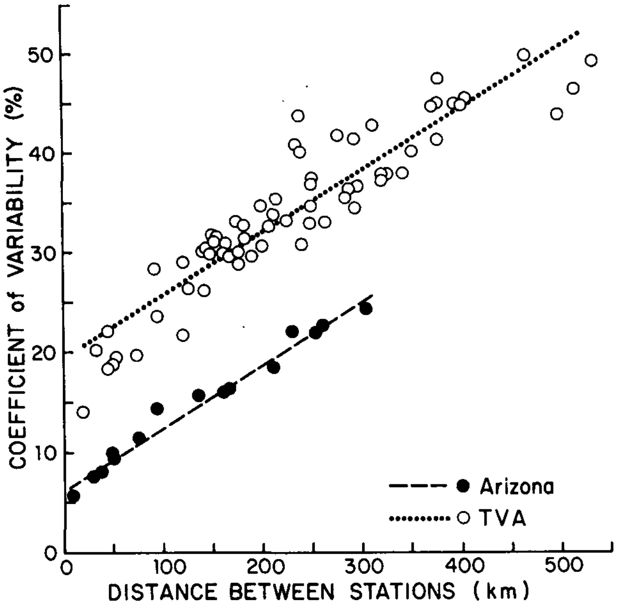

FIG. 3. Relationship between the coefficient of the variability of differences in daily values of solar radiation and distance between stations.

$$c_v = 10.4 + 0.103D, \tag{6}$$

or

$$c_v = -17.4 + 9.18 \ln D, \tag{7}$$

while yielding TVA relationships of

$$c_v = 31.8 + 0.103D \tag{8}$$

or

$$c_v = -41.8 + 18.5 \ln D. \tag{9}$$

These equations can then be used to estimate the distance to which one can extrapolate daily solar radiation totals at a 90% confidence level for specified error tolerances (as represented by values of c_v). For example, Table 3 lists extrapolation distances for error tolerances of ±15% [similar to values obtained by some numerical models (e.g., Davies *et al.*, 1975; Suckling and Hay, 1977)] and a rather large ±25%. There is a noticeable difference between the maximum extrapolation distances for the two study areas. Extrapolation from the TVA sites is very limited, if not nil, whereas southern Arizona data can be applied to a greater distance implying a higher spatial coherence in the solar climate. Nevertheless, even in Arizona, the data can only be applied a few tens of kilometers for an error tolerance of ±15%.

These results can be compared to the study by Suckling (1982) which utilized results primarily from the Canadian studies of Wilson and Petzold (1972) in parts of southern Canada, Wilson and Petzold (1976) for locations in Labrador, Hay and Suckling (1979) for British Columbia and Hay (1981) for a mesoscale network in southern British Columbia. The resulting relationship was

$$c_v = -17.5 + 8.73 \ln D, \tag{10}$$

with a correlation coefficient of 0.948 and standard error of 4.24. Translated into a 90% confidence relationship, Eq. (10) becomes

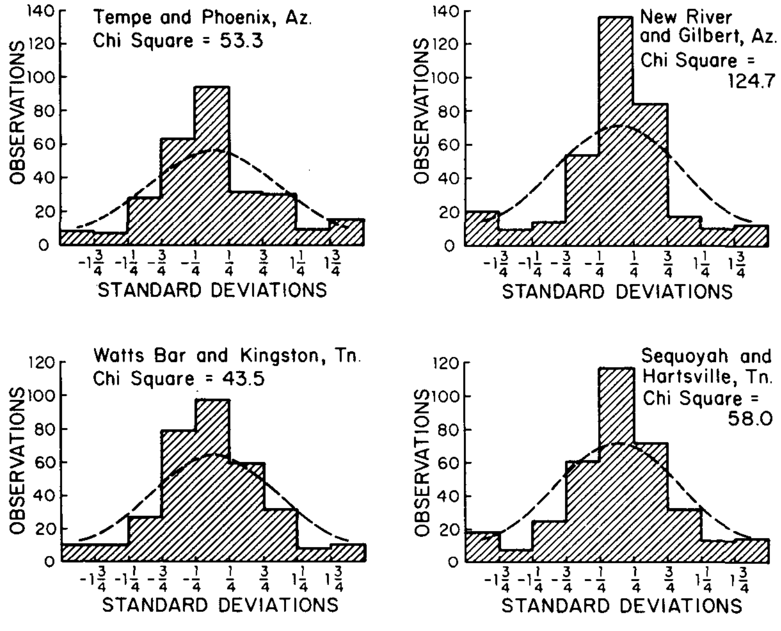

FIG. 4. Frequency distributions of differences in daily values of solar radiation for selected station pairs.

TABLE 3. Maximum extrapolation distance (km) from measurement sites for daily totals of solar radiation at a 90% confidence level.

	Arizona		TVA	
Error due to extrapolation (%)	Linear relation [Eq. (6)]	Logarithmic relation [Eq. (7)]	Linear relation [Eq. (8)]	Logarithmic relation [Eq. (9)]
15	45	34	0	22
25	142	101	0	37

$$c_v = -28.7 + 14.3 \ln D, \tag{11}$$

and yields maximum extrapolation distances of 21 and 43 km for error tolerances of ±15% and ±25% respectively. The results in the present analysis compared to these Canadian values suggest that much larger extrapolations are permissible in the southern Arizona solar climate but shorter extrapolations are in order for the TVA region.

4. Seasonal variability in the extrapolation distance relationships

Considerable seasonal variation can exist in the spatial coherence of solar radiation (Wilson and Petzold, 1976; WMO, 1981). The analysis in the preceeding section used all data for the years 1977 and 1979 for southern Arizona and the TVA region, respectively. Relationships for individual seasons can also be established and are summarized in Table 4. In most cases, the relationship with the linear distance scale remains slightly better than the logarithmic distance scale although no definitive case can be made. For illustration, the seasonal and annual relationships with the linear distance scale have been plotted in Fig. 5 at a 90% confidence level. Resulting extrapolation distances do show considerable variation especially between summer and winter for the TVA region. Seasonal variation in the relationships for southern Arizona appears not to be as critical.

TABLE 4. Coefficient of variability versus distance equations based on separate season analyses. (90% confidence level)

Study area	Season	Equation	Correlation coefficient	Standard error of estimate
Arizona	Winter	$c_v = 17.6 + 0.0476D$	0.72	2.83
		$c_v = 2.62 + 4.72 \ln D$	0.75	2.67
	Spring	$c_v = 7.32 + 0.0873D$	0.95	1.68
		$c_v = -16.0 + 4.71 \ln D$	0.90	2.48
	Summer	$c_v = 7.84 + 0.0932D$	0.98	1.03
		$c_v = -17.2 + 8.30 \ln D$	0.93	2.13
	Fall	$c_v = 14.1 + 0.0884D$	0.94	2.05
		$c_v = -12.7 + 8.53 \ln D$	0.96	1.67
TVA	Winter	$c_v = 37.7 + 0.122D$	0.87	4.99
		$c_v = -45.5 + 21.2 \ln D$	0.84	5.52
	Spring	$c_v = 27.1 + 0.128D$	0.87	5.46
		$c_v = -63.1 + 22.7 \ln D$	0.86	5.59
	Summer	$c_v = 28.9 + 0.0798D$	0.80	3.99
		$c_v = -23.9 + 13.1 \ln D$	0.82	3.78
	Fall	$c_v = 32.0 + 0.0979D$	0.86	4.30
		$c_v = -36.8 + 17.4 \ln D$	0.85	4.44

The seasonal results presented here constitute approximately 90 days of data per season for a station pair. In order to establish meaningful relationships in a climatological sense, more data is needed. Nevertheless, these results indicate that seasonal variation does exist for the solar climates under study.

5. Application to monthly solar radiation totals

The previous sections have considered daily solar radiation totals and indicated that the permissible extrapolation of solar radiation values to distances away from the measurement site is very limited. Wilson (1980) noted that spatial extrapolation of solar radiation data can be improved if one is interested in values over long time periods. The Central Limit

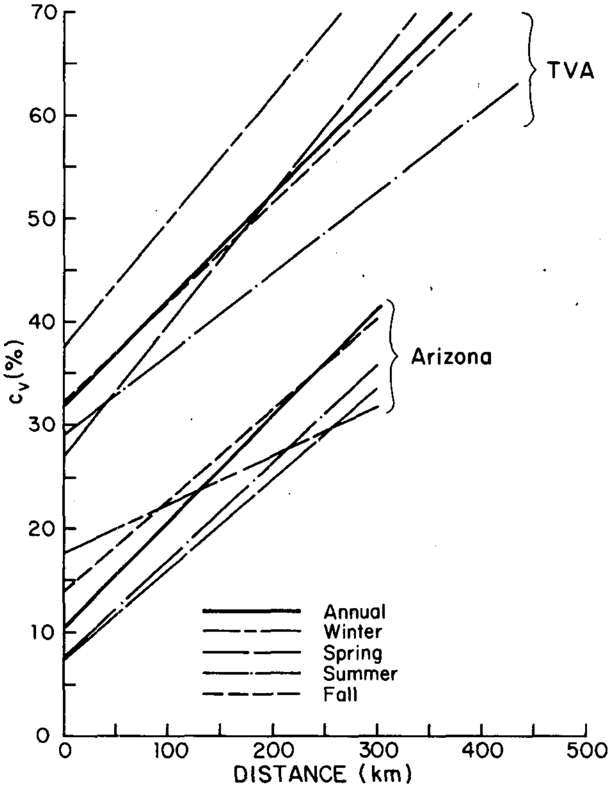

FIG. 5. Extrapolation distance–coefficient of variability relationships for daily solar radiation totals at a 90% confidence level based on data for individual seasons and all seasons combined (labelled annual).

TABLE 5. Maximum extrapolation distance (km) from measurement sites for monthly totals of solar radiation at a 90% confidence level.

Error due to extrapolation (%)	Arizona: Linear relation [Eq. (13)]	Arizona: Logarithmic relation [Eq. (14)]	TVA: Linear relation [Eq. (15)]	TVA: Logarithmic relation [Eq. (16)]	Canadian studies (Suckling, 1982) [Eq. (17)]
5	164	130	0	42	50
10	430	2554	222	182	337

Theorem can be used to estimate the standard deviation for n-day periods (σ_n) as

$$\sigma_n = \sigma n^{-1/2}. \tag{12}$$

Thus, for 30-day periods (i.e., monthly totals of solar radiation), coefficients in daily relationships should be multiplied by 0.183. At a 90% confidence level for monthly values, the relationships for southern Arizona become

$$c_v = 1.91 + 0.0188D \tag{13}$$

or

$$c_v = -3.18 + 1.68 \ln D. \tag{14}$$

For the TVA region, the monthly relationships are

$$c_v = 5.82 + 0.0188D \tag{15}$$

or

$$c_v = -7.65 + 3.39 \ln D. \tag{16}$$

In both cases, the relationships with D expressed as a linear scale had slightly higher correlation coefficients. The relationship of Suckling (1982) based on Canadian data expressed for monthly values is

$$c_v = -5.25 + 2.62 \ln D. \tag{17}$$

The differences between these three study regions are illustrated in Fig. 6 where the linear distance relationships are shown for Arizona and the TVA region. Again the Arizona region exhibits the most spatial coherence in solar climate while the TVA region shows the least. Maximum extrapolation distances from measurement sites in the monthly case are given in Table 5 for error tolerances of ±5 and ±10%. For ±5%, very little extrapolation is permissible for the TVA region whereas monthly solar radiation totals can be applied to distances of more than 100 km from the Arizona measurement sites. A ±5% error tolerance may be better than necessary for many applications of solar radiation data. In the study of the adequacy of the United States solar radiation monitoring network, Suckling (1982) used a ±10% error tolerance with a 337 km (see Table 5) permissible extrapolation distance. The results in this study indicate that greater extrapolation distances of more than 400 km are permissible in the southern Arizona solar climate. It should be noted that the values calculated for Arizona, however, extend beyond the distances between stations in the original data set. A considerably shorter distance (~200 km) is appropriate for the TVA region. This indicates that a uniform network density across the United States is not appropriate.

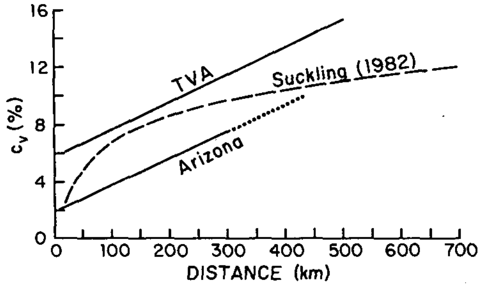

FIG. 6. Extrapolation distance–coefficient of variability relationships for monthly solar radiation totals at a 90% confidence level based on data for all seasons combined.

6. Conclusion

This study suggests that the permissible spatial extrapolation of solar radiation values from measurement sites is dependent upon the characteristics of the region's solar climate and therefore will vary in different parts of the country. In this analysis, the solar climate of southern Arizona was shown to have much greater spatial coherence than that for the Tennessee Valley Authority region.

For daily totals of solar radiation, extrapolation of data from measurement sites is very limited and therefore of limited use. For monthly totals, at a 90% confidence level for an error tolerance of ±10%, extrapolation distance of ~200 km was found for the TVA region while a distance of more than 400 km was applicable in southern Arizona. However, the permissible extrapolation distances may vary with season. Since a considerable difference was found for the two areas investigated in this study, further analyses of the mesoscale variability of solar radiation in additional regions is warranted.

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