A Model for Estimating One-Minute Rainfall Rates

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ABSTRACT

A model for estimating one-minute rainfall rates has been developed using stepwise multiple regression analysis. The model is made up of six regression equations to estimate rates that are equalled or exceeded 0.01, 0.05, 0.10, 0.50, 1.0, and 2.0 percent of the time during a month at a given location. Information required to make the estimates consists of monthly mean temperature, monthly mean precipitation, number of days in the month with precipitation (based on any of three threshold values that define a rainy day), and latitude. The model is not valid when the mean monthly temperature is ≤0°C (32°F), when there is less than one rainy day in the month, or when a precipitation index (the ratio of monthly precipitation to the number of rainy days) is less than 2 mm day^{-1}.

1. Introduction

Rain is a major cause of attenuation of radio waves (frequencies >10 GHz) traversing the troposphere. Knowledge of the frequency distribution of short-duration rainfall rates is needed to estimate signal loss for the design and operation of systems that utilize these frequencies (e.g., communications, surveillance). One-minute rainfall rates, commonly referred to as "instantaneous" rates, have been recognized as most practical for these calculations. Instantaneous rainfall-rate statistics also have applications in other areas, such as the design and operation of aerospace vehicles and radar systems.

Records of rainfall amounts are available for thousands of locations worldwide over more than 100 years in many instances. However, data collection was oriented toward agricultural and hydrological purposes for which monthly, daily, and less commonly, 3 and 6 h totals were collected. Precipitation amounts for intervals of 3 h down to 5 min are available for many locations in the United States, but for few locations in other parts of the world. Much of the meager amount of data on 1 min rates was collected during special field programs conducted for limited time periods (e.g., 1–3 years). This has prompted the development of numerous models to estimate instantaneous rates.

Tattelman and Grantham (1982) discuss sources of one minute data and present a survey and comparison of models for estimating one minute rates. They found six models that estimate annual rainfall-rate distributions, but only one that estimates rates on a monthly basis (Lenhard, 1974). Due to the importance of worst-month considerations for design and operation problems, an improved monthly model that overcomes some of the shortcomings of the Lenhard model has been developed and is presented in this report.

2. Data

Frequencies of occurrence of short-duration precipitation rates for a wide variety of the world's rain climates were published by Jones and Sims (1971) and Sims and Jones (1973). These reports include the monthly frequency distribution of instantaneous rates for the 12 locations in Table 1. The type of rain gauge used to collect the data varied with location, but all were modified to enable the determination of one minute rates. Rates were determined from either the original recorder charts, paper copies, or microfilm copies. Details on the sites, the rain gauges, and analysis of the chart records are given in the references.

Three years of data were available for Urbana and two years for Paris, whereas one year or less was available for the other stations. Therefore, data for the same month in different years were averaged to avoid bias by the stations with longer records. This resulted in 122 monthly instantaneous precipitation rate distributions for the 12 locations. Lenhard smoothed the monthly cumulative frequency distributions by fitting the data with a polynomial. The degree of the polynomial with the best fit was determined by analysis of variance. This was done to the highest degree that additional explained variance was significant, but no higher than degree six.

TABLE 1. Locations and number of months of one-minute data available for analysis. Data for the same month in different years were averaged. The resulting calendar months of data are indicated.

Station	Coordinates	Months of data	Calendar months of data
1. Flagstaff, AZ	35°14′N, 111°45′W	4	2
2. Franklin, NC	35°02′N, 83°28′W	17	12
3. IL Gauge 97 (20 mi NW of Urbana)	40°27′N, 88°15′W	10	10
4. Island Beach, NJ	39°52′N, 74°05′W	13	12
5. Majuro, Marshall Islands	07°05′N, 171°23′E	13	12
6. Miami, FL	25°45′N, 80°19′W	13	12
7. Panama, Canal Zone	09°21′N, 79°59′W	4	4
8. Paris, France	48°52′N, 02°20′E	24	12
9. Preston, England	53°46′N, 02°42′W	12	12
10. Reading, England	51°28′N, 00°59′W	13	12
11. Urbana, IL	40°07′N, 88°12′W	36	12
12. Woody Island, AK	57°47′N, 150°20′W	11	10

Cole and Donaldson (1965) found a high positive correlation between precipitation rates and the average amount of rain falling on a rainy day. A precipitation index that expresses this is the total monthly precipitation divided by the number of days with precipitation. The minimum threshold amount of precipitation to define a rainy day varies with country. Three of the most common threshold values used worldwide to define a rainy day are 0.25 mm, 1 mm, and 2.54 mm. The number of rainy days during the month based on each of these threshold amounts, as well as monthly precipitation and monthly mean temperature, were observed coincident with the rain-rate frequencies (dependent data). The number of days per month with another frequently used threshold called a "trace" differed only slightly with the number of days equal to or greater than 0.25 mm, and was not used. Long-term means of these monthly parameters are available for a large number of locations throughout the world. Therefore, the primary independent variables used to develop the model are

T monthly mean temperature (°F or 1.8 × °C + 32),

I precipitation index (mm day^{-1}).

3. Model development

a. The Lenhard model

The model developed by Lenhard (1974) was based on data from the same sources described in the previous section. He also used an additional 12 months of data for Bogor, Indonesia. We chose not to use the Bogor data because Jones and Sims (1971) state that the gauge there was not operational at all times and that little confidence can be placed on the frequency distribution of the rates. The independent variables in Lenhard's model are the monthly mean temperature (T) and the precipitation index (I). He found that T had no appreciable effect on rates estimated for his three tropical locations, but it did make an important contribution to explaining the observed variance for rates estimated at the other 10 locations. He therefore developed two basic models, one for the tropics and one for extra-tropical stations. The two models estimate one-minute rates (mm min^{-1}) that are equalled or exceeded 2, 1, 0.5, 0.1, 0.05, and 0.01% of the month. Regression coefficients for each model at each of these six exceedance levels (p) are determined by a quadratic equation of log p.

A drawback of the Lenhard model is the need to classify a station as tropical or extra-tropical. Such a categorization is frequently difficult in the transition zone between the two, and differences in estimates from the two models can be large. Another drawback is the constraint imposed by using a quadratic equation to determine the regression coefficients. Better accuracy would be attained if the coefficients were calculated using least squares estimation without restrictions. Also, use of the questionable Bogor data causes bias towards high estimates of rates at tropical locations.

b. The improved model

It has been established that the contribution of temperature to estimating rates in the tropics is minimal, but becomes substantial at middle latitudes. Since latitude is known for any location of interest, its use in weighting the importance of temperature is logical. Thus, the product of latitude (L) and monthly mean temperature was included as an additional independent variable to establish a transition from tropical to extra-tropical latitudes. Since the term should have a value of zero in the tropics, it was necessary to establish a cut-off latitude where the tropics ended. This was chosen to be 23.5°, since this is approximately the parallel of maximum solar declination. The latitude–temperature term $f(L, T)$, defined as

$$f(L, T) = \begin{cases} 0, & L \leqslant 23.5 \\ (L - 23.5)T, & L > 23.5, \end{cases}$$

and two other independent variables, monthly mean temperature (T) and the precipitation index (I) were used to determine a multiple regression equation to estimate rates for each of six exceedance levels (p = 0.01, 0.05, 0.10, 0.50, 1.0, and 2.0% of the time during the month). The equation is expressed by

$$R_p = A_p + B_pT + C_pI + D_pf(L, T), \quad (1)$$

where R_p is the estimated precipitation rate (mm min^{-1}) for exceedance level p, A_p is the constant for

TABLE 2. Results of stepwise multiple regression analysis for exceedance levels p = 0.01, 0.05, 0.10, 0.50, 1.0, and 2.0% based on a threshold value of 2.54 mm for I. The regression coefficients are given for each independent variable. The t-statistic corresponding to the variable is shown in parentheses.

p	Constant (A_p)	T (B_p)	$I_{(2.54)}$ (C_p)	$f(L, T)$ (D_p)	R	SEE (mm min^{-1})
0.01	−0.91	2.8×10^{-2} (10.7)	2.3×10^{-2} (5.3)	-3.4×10^{-4} (−3.1)	0.83	0.43
0.05	−0.50	1.6×10^{-2} (10.7)	1.9×10^{-3} (6.7)	-3.1×10^{-4} (−5.2)	0.86	0.24
0.10	−0.31	1.1×10^{-2} (9.5)	1.4×10^{-2} (6.7)	-3.0×10^{-4} (−6.2)	0.85	0.19
0.50	−0.01	2.5×10^{-3} (4.5)	5.4×10^{-3} (5.1)	-1.5×10^{-4} (−6.4)	0.76	0.09
1.0	0.03	7.4×10^{-4} (2.2)	2.9×10^{-3} (4.7)	-7.6×10^{-5} (−5.5)	0.67	0.06
2.0	0.04	-2.0×10^{-4} (−1.2)	1.5×10^{-3} (5.6)	-3.2×10^{-5} (−5.4)	0.64	0.02

exceedance level p, and B_p, C_p, and D_p are multiple regression coefficients for T, I and $f(L, T)$, respectively, for exceedance level p.

The resulting multiple correlation coefficients (R) and standard errors of estimate (SEE) were encouraging, but negative precipitation rates were commonly estimated for locations with a latitude greater than 40°. The term was then redefined as follows:

$$f(L, T) = \begin{cases} 0, & L \leq 23.5° \\ (L - 23.5)T, & 23.5° < L \leq 40° \\ (40 - 23.5)T, & L > 40°. \end{cases} \quad (2)$$

Results of the stepwise multiple regression analysis for each of the six exceedance levels are given in Table 2, 3 and 4 for indices based on rainy day threshold values of 2.54 mm, 1 mm and 0.25 mm, respectively. The t-statistic (Draper and Smith, 1981) is used to determine if there is a significant reduction in variation between the estimated and actual rates for each of the independent variables. The larger the absolute value of the t-statistic, the greater the reduction in variance. An absolute value greater than 1.96 indicates the variable is significant at the 5% level. All of the independent variables significantly reduce the variance of the estimated rates except for T at p = 2.0%. Since T significantly reduces the variance at five of the exceedance levels, its use in the model in addition to the $f(L, T)$ term is justified. The regression coefficients and constants to be used to estimate rates in Eq. (1) are provided in Tables 2, 3 and 4.

A comparison of R and the SEE between the two

TABLE 3. Results of stepwise multiple regression analysis for exceedance levels p = 0.01, 0.05, 0.10, 0.50, 1.0, and 2.0% based on a threshold value of 1 mm for I. The regression coefficients are given for each independent variable. The t-statistic corresponding to the variable is shown in parentheses.

p	Constant (A_p)	T (B_p)	$I_{(1.00)}$ (C_p)	$f(L, T)$ (D_p)	R	SEE (mm min^{-1})
0.01	−1.00	2.8×10^{-2} (11.2)	3.6×10^{-2} (6.5)	-2.2×10^{-4} (−2.1)	0.84	0.41
0.05	−0.56	1.6×10^{-2} (11.5)	2.5×10^{-2} (8.1)	-2.4×10^{-4} (−4.0)	0.88	0.23
0.10	−0.36	1.1×10^{-2} (10.2)	2.0×10^{-2} (8.2)	-2.4×10^{-4} (−5.1)	0.87	0.18
0.50	−0.03	2.4×10^{-3} (4.7)	7.8×10^{-3} (6.7)	-1.2×10^{-4} (−5.4)	0.79	0.09
1.0	0.02	6.9×10^{-4} (2.2)	4.2×10^{-3} (6.0)	-6.2×10^{-5} (−4.5)	0.71	0.05
2.0	0.04	-1.8×10^{-4} (−1.3)	2.0×10^{-3} (6.4)	-2.6×10^{-5} (−4.4)	0.67	0.02

TABLE 4. Results of stepwise multiple regression analysis for exceedance levels p = 0.01, 0.05, 0.10, 0.50, 1.0, and 2.0% based on a threshold value of 0.25 mm for I. The regression coefficients are given for each independent variable. The t-statistic corresponding to the variable is shown in parentheses.

p	Constant (A_p)	T (B_p)	$I_{(0.25)}$ (C_p)	$f(L, T)$ (D_p)	R	SEE (mm min^{-1})
0.01	−1.00	2.8×10^{-2} (11.4)	4.2×10^{-2} (6.8)	-2.2×10^{-4} (−2.2)	0.85	0.41
0.05	−0.56	1.6×10^{-2} (11.9)	3.0×10^{-2} (8.9)	-2.3×10^{-4} (−4.1)	0.88	0.22
0.10	−0.36	1.1×10^{-2} (10.6)	2.4×10^{-2} (9.2)	-2.3×10^{-4} (−5.2)	0.88	0.17
0.50	−0.03	2.3×10^{-3} (4.8)	1.0×10^{-2} (8.6)	-1.2×10^{-4} (−5.6)	0.82	0.08
1.0	0.01	5.6×10^{-4} (2.0)	6.0×10^{-3} (8.3)	-5.6×10^{-5} (−4.5)	0.77	0.05
2.0	0.03	-2.3×10^{-4} (−1.9)	2.8×10^{-3} (8.8)	-2.4×10^{-5} (−4.4)	0.74	0.02

basic Lenhard models and the improved model, based on a threshold of 2.54 mm for I, is given in Table 5. At each exceedance level the correlations, and hence the percent variation of the estimated one minute rates accounted for by the independent variables, is greater for the improved model. The SEE's for the improved model are much smaller than those for Lenhard's tropical model, and nearly the same as those for his extratropical model.

The improved model was derived from all of the available one-minute monthly rate distributions. However, a limited test of the model with independent data can be made using the maximum short-duration precipitation statistics published by NOAA (1980). They include the highest monthly precipitation intensities for intervals of 5 to 180 min at approximately 250 U.S. locations. They also contain monthly mean temperature, monthly precipitation, and the number of days each month with at least 0.25 mm precipitation. If we assume that the highest 5 min monthly intensity is equivalent to the highest five one-minute intensities, then the monthly 5 min maximum precipitation is the value equalled or exceeded 0.0116% of the time during a 30-day month, and 0.0112% of the time during a 31-day month. This is quite close to the 0.01% exceedance level (p). Therefore, model estimates of the 1-min rate for p = 0.01 can be compared to the observed 5-min maximum monthly intensity as an independent check of the model.

Comparison of model estimates with the NOAA data at less extreme exceedance levels would not be valid. For example, at p = 0.05, the maximum precipitation observed in a 20 min interval from the NOAA data would be the basis of comparison. However, the maximum precipitation observed in such a long interval could be much less than the sum of the precipitation amounts occurring in the 20 most intense one minute intervals, which would be comparable to the model estimate. The discrepancy would be even worse for $p > 0.05$. Therefore, a check of the model estimates with independent data has been limited to p = 0.01.

Table 6 lists the locations for which the independent data were used. The months and the number of years of data used for each month, are indicated. The stations and the months of January, April and July were chosen

TABLE 5. Comparison of multiple correlation coefficients (R) and the standard errors of estimate (SEE) between the two basic Lenhard models and the improved model based on a threshold of 2.54 mm for I.

	Exceedance level (%)											
	0.01		0.05		0.10		0.50		1.0		2.0	
	R	SEE	R	SEE	R	SEE	R	SEE	R	SEE	R	SEE
Lenhard's tropical	0.50	1.02	0.58	0.52	0.62	0.38	0.66	0.17	0.66	0.10	0.50	0.07
Lenhard's extratropical	0.77	0.42	0.80	0.22	0.78	0.16	0.63	0.06	0.52	0.03	0.36	0.02
Improved model	0.83	0.43	0.86	0.24	0.85	0.19	0.76	0.09	0.67	0.05	0.64	0.02

TABLE 6. Locations, months, and number of years of data used in an independent check of the improved model rate estimates at the 0.01% exceedance level.

Location	Month (years of data)
Billings, Montana	Apr (8), Jul (8)
Birmingham, Alabama	Jan (8), Apr (14), Jul (18)
Chicago, Illinois	Apr (9), Jul (10)
Key West, Florida	Jan (9), Apr (8), Jul (16)
Las Vegas, Nevada	Jan (8)
Memphis, Tennessee	Jan (8), Apr (12), Jul (20)
Newark, New Jersey	Apr (8), Jul (9)
Seattle, Washington	Jan (8), Apr (8), Jul (8)
Washington, D.C.	Jan (8), Apr (9), Jul (10)

such that they represent a variety of rainfall regimes. All three calendar months were not used for some locations because of missing data, monthly mean temperature less than freezing or years with no rainfall. The model does not give valid estimates of rates for the latter two conditions. This is discussed in Section 4. The number of years for each month at each location varies because the manner in which the NOAA data were presented changed in 1973. Prior to that year, the maximum 5 min precipitation for a month was published only if it exceeded 6.4 mm (0.25 inches). Therefore, data for each month were used only as far back as the 5 min intensity is available.

Model estimates for each month at each location were made using the mean temperature, mean precipitation, and mean number of days with precipitation $\geqslant$0.25 mm observed during the indicated period for each month (see Table 6). The maximum 5-min precipitation observed each year for the same period was averaged. The rms difference between the estimated rates for each month, and the average observed 5 min maximum precipitation is 0.39 mm min^{-1}. This compares favorably with the *SEE* of 0.41 mm min^{-1} for $p = 0.01$ in Table 4.

The following example is presented to illustrate the use of the model. To estimate the rainfall rate equalled or exceeded 0.01% of the time at Washington, D.C., during July, the regression coefficients in Table 4 are used because the climatic data on the number of rainy days for Washington is based on a threshold of 0.25 mm (0.01 inch). For $p = 0.01\%$, Eq. (1) becomes

$$R_{(0.01)} = -1.00 + 0.028T + 0.042I - 0.00022f(L, T). \quad (3)$$

Since L is 38.9°N and T is 79.4°F (26.3°C), $f(L, T)$ from Eq. (2) becomes $(38.9 - 23.5) \times 79.4 = 1222.76$. The mean precipitation for July is 112.3 mm, and the mean number of days with at least 0.25 mm of rain is 10.2. Therefore, $I = 112.3/10.2$ and Eq. (3) becomes

$$R_{(0.01)} = -1.00 + 0.028(79.4) + 0.042(112.3/10.2) - 0.00022(1222.76) = 1.42 \text{ mm min}^{-1}.$$

The climatic data to make the estimate were the mean values for the 10-year period for July used in the independent test of the model (see Table 6). The estimate can be compared to the mean of the highest observed 5 min intensities for July of each year for the same period, which is 1.62 mm min^{-1}.

c. *A comprehensive model*

As an alternative to having six equations, one for each of the exceedance levels, a one-equation model was developed. For this model, the exceedance level (p) was tried as an independent variable, and since p varies logarithmically with the rate, ln p was also included with the set of independent variables. Since p did not significantly reduce the variation between the actual and estimated rates, the resulting regression equation is

$$R_p = b_0 + b_1T + b_2I + b_3f(L, T) + b_4 \ln p, \quad (4)$$

where b_0 is a constant, and b_1, b_2, b_3, and b_4 are multiple regression coefficients, and p is the exceedance level.

Results of the stepwise multiple regression analysis, including the coefficients and constants for estimating rates in Eq. (4), are given in Table 7 based on a threshold value of 2.54 mm for I. The *SEE*'s for the comprehensive model calculated for each exceedance level based on a threshold of 2.54 mm for I are 0.61, 0.30, 0.21, 0.25, 0.32, and 0.39 for p = 0.01, 0.05, 0.10, 0.50, 1.0, and 2.0%, respectively. Comparison with the *SEE*'s for the improved model in Table 5 indicates a substantial loss in precision using the comprehensive model. When estimates of rates are required at other than the six exceedance levels used for the improved model, better results would be obtained by interpolating between exceedance levels rather than by using the comprehensive model.

TABLE 7. Results from stepwise multiple regression analysis using the data for all exceedance levels based on a threshold value of 2.54 mm for I.

Variable		Regression coefficient	t-statistic
		$b_0 = -0.558$	
T		$b_1 = 0.0098$	12.34
$I_{(2.54)}$		$b_2 = 0.0115$	7.87
$\ln p$		$b_3 = -0.159$	−24.67
$f(L, T)$		$b_4 = -0.00019$	−5.98
	$R = 0.78$		
	$SEE = 0.32$		

4. Model limitations

The improved model was subjectively evaluated by estimating rates at independent locations representing a wide variety of the earth's climates. Results indicated circumstances when the model is either invalid or should be used with discretion. This occurred for very dry or cold months for which there were little or no data among the dependent stations. The model was found to be generally invalid when any of the following conditions exist for a specific month at a location:

1) $T \leqslant 32°F$ (0°C),
2) $I < 2$ mm day^{-1},
3) Number of rainy days < 1.

Inconsistencies such as negative rates, or increasing rates with increasing exceedance level occasionally occur when T is between 32 and 40°F (0 to 4.4°C). When T is in this range, estimated rates are acceptable if they are positive and decrease with increasing exceedance level. If the estimated rates are consistent, but a negative rate is estimated for $p = 2.0\%$, this may indicate that it rains less than 2% of the time during that month.

When there are between one and three rainy days during the month, the model may estimate rates for each of the exceedance levels which, when integrated, indicate a total rainfall two or more times greater than the monthly mean precipitation. Under these circumstances heavy, but infrequent, convective precipitation accounts for just about all of the precipitation in the month. Therefore, the most accurate estimates are for the two or three lowest exceedance levels, and significant rainfall might not occur more often than 0.5% of the month. These possibilities should be considered before accepting rate estimates in arid locations.

5. Summary

A model for estimating one-minute precipitation rates at a given location was developed using stepwise multiple regression analysis. The model is made up of six regression equations to estimate rates that are equalled or exceeded for exceedance levels, $p = 0.01$, 0.05, 0.10, 0.50, 1.0 and 2.0% of the time during a month. Information required to make the estimates are monthly mean temperature, monthly mean precipitation, number of days in the month with precipitation (based on any of three threshold values that define a rainy day) and latitude. A one-equation model was also developed by using $\ln p$ as an additional independent variable. However, it is not recommended because its precision is substantially less than the six-equation model.

The six-equation model is not valid when the monthly mean temperature is equal to or less than 32°F (0°C), when there is less than one rainy day in the month or when a precipitation index (the ratio of monthly precipitation to the number of rainy days) is less than 2 mm day^{-1}. Also, the model is occasionally invalid when the monthly mean temperature is between 32°F to 40°F (0°C to 4.4°C), or when there are between one to three rainy days in the month. Precipitation rates are not usually of concern during these cold or dry months, since they are likely to be quite low.

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