NOTES

On the Thermodynamic Method for Estimating Maximum Tornado Windspeeds

John T. Snow and Randal L. Pauley

Department of Geosciences, Purdue University, West Lafayette, IN 47907

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1. Introduction

The maximum windspeed[1] that can occur in a tornado remains an unknown quantity of great interest. Current estimates of peak tornadic velocities are founded largely on damage surveys and photogrammetric analyses, neither of which address the physics of the question. One technique that seeks to estimate tornado windspeeds on the basis of what is physically realizable is the "thermodynamic" or "tephigram" method. While several different versions of what we call the thermodynamic method have been formulated, all yield an estimate of the maximum windspeed that can be produced beneath a column of buoyant air, given a standard environmental sounding and some assumptions about the flow field. Such an approach is appealing as it relates a quantity very difficult to determine (the maximum tornado windspeed) to ones much more easily measured (the profiles of temperature and moisture in the environment).

Vonnegut (1960) appears to have been the first to have estimated tornado windspeeds using thermodynamic principles, modeling the tornado as a perfect heat engine. The thermodynamic method expands on this idea, assuming adiabatic processes, including the release of latent heat, and also considering descending as well as ascending motion. Lilly (1969) and Kessler (1970) applied the thermodynamic method to tornado-proximity soundings to demonstrate that Joule (electrical) heating is unnecessary to account for strong tornadoes. Dergarabedian and Fendell (1970a,b) used the technique to help argue that maximum tornadic velocities should be much less than the sonic values sometimes quoted. Most recently Redmann *et al.* (1983) advocated using this approach to establish regional windspeed criteria for structural engineering purposes. Although all of these authors recognized some of the limitations of the thermodynamic method, we believe its application deserves further scrutiny. It is the objective of this note to critically review the thermodynamic method in light of current understanding of tornado dynamics. An earlier critique of the method can be found in Lewellen (1976).

2. An overview of the thermodynamic method

The details of the thermodynamic method are provided in the above references and only the essential features will be outlined here. The idealized tornado commonly considered is taken to be steady and axisymmetric, with a cylindrical or quasi-cylindrical core (delimited by the surface of maximum tangential winds) extending vertically from surface to near tropopause. Inviscid flow is usually assumed, at least outside of a frictional surface boundary layer. The meridional flow within the core is allowed to be either one-celled, with upflow along the axis, or two-celled, with downflow along the axis and upflow in a surrounding annular region. In either case the vertical motions in the core are hypothesized to be adiabatic and undiluted. Latent heat release in the one-celled flow and compressional heating in the two-celled case lighten the tornado core in comparison to the ambient. In the thermodynamic method, the difference in weight between the column of air constituting the core and an ambient column is vertically integrated over the full depth of the buoyant column to determine the surface pressure drop at the center of the vortex. This pressure drop is customarily related to the peak windspeed either via Bernoulli's equation for inviscid flow along a streamline or by radially integrating the cyclostrophic relationship for an assumed form of tangential velocity distribution. In the latter approach, the familiar Rankine vortex might be assumed for a one-celled tornado, while for the two-celled case a "hollow-core" vortex with a potential vortex in the upflow region and a nonrotating core

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[1] The windspeed "maximum" is a designation which is often used rather loosely. Since the greatest interest is in winds capable of effecting damage and destruction, it seems reasonable to consider a maximum averaged over no less than 1 s in time and 10 m^2 in cross-sectional area, restricted to the lowest 100 m or so above ground. An unrestricted instantaneous point value would of course be greater, a broader average less.

of downflow might be considered.[2] Implicit in this relation of the surface pressure drop to the maximum windspeed is the assumption that the surface pressure deficit is very close to that at the top of the ground boundary layer (or at whatever level the maximum velocity is located).

When applied to actual tornado-proximity soundings, the thermodynamic method yields peak windspeeds for one-celled vortex cores well below the current consensus estimate of $\sim$100 m s^{-1} for maximum violent-tornado windspeeds. For two-celled vortices, maximum velocities generally greater than the consensus are obtained. For example, from the highly unstable Watonga, Oklahoma sounding of 2300 GMT 5 June 1966, Lilly (1969) calculated a 4.5 kPa surface pressure deficit for a one-celled flow and a 12.0 kPa deficit for the two-celled case. If Rankine and "hollow-core" velocity distributions are assumed for one- and two-celled tornadoes respectively, the cyclostrophic integration yields a maximum windspeed of $\sim$65 m s^{-1} for a one-celled vortex and $\sim$145 m s^{-1} for a two-celled. This illustrates that for the most unstable soundings, the two calculations bracket the consensus estimates for maximum violent-tornado windspeeds.

Since the windspeeds calculated for the one-celled vortices fall well short of 100 m s^{-1}, it is inferred that violent tornadoes must be two-celled. Two hypotheses have been offered to explain why calculated windspeeds for the two-celled cases usually exceed the consensus maximum: the warm downflow may be diluted during its descent, and/or the downdraft does not extend all the way to the surface so the flow in the lower reaches of the vortex remains one-celled. In this vein Redmann *et al.* (1983) proposed an empirical combination of the one-celled and fully two-celled velocity calculations as a means of estimating the actual maximum likely to occur. Good agreement was claimed between their empirical formula and independent estimates of windspeeds for a diverse sampling of tornadoes.

3. Shortcomings of the thermodynamic method

A serious question about the thermodynamic method concerns the calculation of the pressure drop supposed to be related to the maximum velocity. The several variations of the thermodynamic method all effectively calculate a pressure drop at the surface. To illustrate, first consider the vertical momentum equation on the axis of a steady, axisymmetric, inviscid tornado

$$\frac{\partial}{\partial z}\left(\frac{w^2}{2}\right) = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - \frac{\rho'}{\rho} - g, \qquad (1)$$

where the primes indicate core departures from a hydrostatic ambient. When this equation is integrated from the top of the vortex (where $w = 0$) to the surface (where again $w = 0$), the contribution from the vertical acceleration term on the left-hand side vanishes. The resulting surface pressure drop is determined solely by the core hydrostatic contribution on the right-hand side. If this surface pressure drop is to be related to the maximum velocity which must occur at some point aloft, then it must be assumed that the vertical pressure gradient across the surface boundary layer of the tornado is weak (i.e., nearly hydrostatic).

Is the above assumption a safe one in all cases? Certainly at large radius the Prandtl boundary layer hypotheses hold, and $\partial p/\partial z$ is small near the surface. However as the axis of the vortex is approached, the inflow boundary layer must erupt and turn into the vertical upflow of the core. In this "turnaround" region the traditional boundary layer assumptions fail, and strong vertical accelerations may occur. Photogrammetric analyses of tornadoes by Hoecker (1960) and Golden and Purcell (1977) indicate upward accelerations of 3 g or more in the lowest 100 m of the vortex core with vertical velocities reaching 60 m s^{-1}. Such vertical accelerations necessarily indicate concomitant large nonhydrostatic pressure gradients in the turnaround region. A 60 m s^{-1} upflow corresponds to a nonhydrostatic pressure drop of $\sim$2 kPa, but due to time-averaging and the inability to observe windspeeds inside the tornado funnel, these photogrammetric results should be *under*estimates of what may actually occur in violent tornadoes. Since the pressure fall increases as the square of the velocity, a 75 m s^{-1} upflow, for example, would be accompanied by a pressure drop of $\sim$3 kPa beyond the hydrostatic surface value. So for tornadoes that are one-celled near surface, we expect that the greatest pressure drop occurs at some point aloft and cannot be calculated from the thermodynamic method.

Similar flow structure appears in one-celled vortices investigated in the Purdue Tornado Vortex Chamber. Measurements by Baker (1981) show that a cyclostrophic integration of the outer-flow tangential velocity profile agrees well with the axial pressure drop *aloft*, but not with the (much smaller) surface pressure deficit. The axial pressures measured in the one-celled laboratory vortex also correlate well with the vertical velocities as per Eq. (1). Of course, there are no thermodynamic effects in the homogeneous laboratory flow, the convergence and convection being

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[2] The Rankine and "hollow-core" vortex models are perhaps not unreasonable idealizations of tornado windfields outside the boundary layer. However, when integrated cyclostrophically they always yield peak swirl velocities *greater* than do continuous and less-sharply-peaked radial distributions of tangential velocity for the same pressure drop. Likewise, a calculation of the maximum windspeed using the Bernoulli-type relationship will tend to overestimate that windspeed, partly due to frictional losses and also because no streamline joins the region of maximum horizontal velocity to the region of maximum pressure drop.

mechanically forced. In the atmosphere, both dynamic and hydrostatic pressure contributions may be present. As suggested by Ward (1972), an example of mechanical forcing in the atmosphere is the thunderstorm outflow, which may provide the strong local convergence to produce a transient tornado even without local buoyancy. The relative importance of hydrostatic and nonhydrostatic effects in determining the intensity of long-lived, violent tornadoes spawned by supercell thunderstorms remains an open question. The thermodynamic method, however, effectively neglects the nonhydrostatic contribution altogether.

Two additional points merit consideration in judging the validity and range of application of the thermodynamic method:

1) Cyclostrophic balance is a good approximation outside the tornado boundary layer, but does not within this layer. If the maximum velocity should occur in the boundary layer or in the turnaround, then a cyclostrophic calculation may err. However, it is reasonable to expect that the magnitude of the vector sum of the radial and tangential velocities is roughly constant with height across a tornado boundary layer, except very near the surface where the no-slip condition must be met (see Burggraf *et al.*, 1971). So the validity of the cyclostrophic calculation is called into question only if the the inflow boundary layer penetrates to smaller radius than the outer (approximately cyclostrophic) flow. Then, despite frictional losses, approximate conservation of angular momentum may yield windspeeds in the turnaround greater than the maximum in the outer flow. Just such a vortex configuration may occur in some two-celled flows, or when the interface between the one-celled and two-celled meridional flows is near the surface (for a fuller description of this "vortex jump", see Maxworthy, 1973). The occurrence of supercyclostrophic velocities in the turnaround must depend critically upon the turbulence of the boundary layer, but it has been observed in both numerical (Rotunno, 1979) and laboratory (Baker and Church, 1979) vortex models.

2) Since axisymmetry is assumed in the thermodynamic method, the possibility of secondary vortices within a tornado core is not considered. The secondary vortices (when present) provide local centers of convergence and rotation embedded within the parent vortex. In the laboratory, secondary vortices form in the strongly sheared upflow region of large two-celled parent flows, but the secondary vortices themselves appear to be one-celled in character. Evidence suggests that secondary vortices may contain greater pressure falls than exist near the tornado center and may also produce significantly higher windspeeds than a calculation based on an axisymmetrical model of the parent flow would indicate (Rotunno, 1984). Although this "multiple-vortex" phenomenon is not fully understood, it represents an important dynamic complication in the tornado flow which the thermodynamic method is unable to address.

4. Conclusions

We conclude that the thermodynamic method is not a physically sound basis for determining maximum tornadic windspeeds. The method is based on a simplified model of the tornado which cannot account for complications in the flow structure of the turnaround region. In particular, the method implicitly assumes that the surface pressure deficit characterizes the maximum velocity (or equivalently, that hydrostatic balance holds in the turnaround region) in all tornadoes. Thermodynamics do play a role in determining the intensity of a tornado, but at present the internal dynamics are not known well enough to reliably bound maximum tornadic velocities from physical principles.

We believe that the results of the thermodynamic method sometimes *appear* successful for the following reasons. The undiluted adiabatic ascent/descent assumed in the core is an idealization which errs toward a too large hydrostatic pressure drop at the surface. Neglect of entrainment consequently contributes toward an overestimate of the maximum windspeed. The commonly used Rankine vortex and the "hollow-core" vortex profiles both err toward too high maximum velocities for a given central pressure drop, so the use of these profiles with the cyclostrophic balance assumption also tends to overstate the windspeed. On the other hand, the assumption that the maximum pressure drop occurs at the surface (i.e., that hydrostatic balance holds in the turnaround) errs toward too small pressure drops and so ultimately toward too low velocities. These opposing errors cancel one another to varying degrees in the one-celled and two-celled calculations so that the 100 m s^{-1} consensus value for maximum windspeeds is bracketed and a combination formula seems to give good results. The agreement, however, is more fortuitous than physical. Moreover, there is no guarantee that the 100 m s^{-1} censensus figure will finally prove to be the upper bound.

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