Comments on "Extreme High Temperature Events: Changes in Their Probabilities with Changes in Mean Temperature"

DENNIS M. DRISCOLL

Dept. of Meteorology, Texas A & M University, College Station, TX 77843

11 April 1985

Mearns, Katz, and Schneider (1984; hereafter MKS) develop and apply a mathematical–statistical model for generating daily maximum temperatures by an autoregressive process which utilizes climatic values of mean temperature, its variability, and the serial correlation of successive daily maximum temperatures. They then show how changing these input parameters modifies various temperature event probabilities of importance to agriculture and human health. I would like to offer some suggestions for improving this model, and attempt to answer some of the questions they raise.

Their model is

$$X_t - \mu = \phi(X_{t-1} - \mu) + \epsilon_t, \qquad (1)$$

where X_t and X_{t-1} are the daily maximum temperature generated and that of the previous day, respectively, μ is the mean monthly daily maximum temperature, ϕ the serial correlation coefficient, lag 1 (one day), and ϵ_t a random variable with mean zero. This model does not describe a stationary process, but one in which X_t decreases monotonically to the mean μ; i.e., X_t approaches μ as a limit for increasing repetitions. This is easily seen if (1) is rewritten as

$$X_t = \phi X_{t-1} + \mu(1 - \phi) + \epsilon_t. \qquad (2)$$

Over the long run the ϵ_t are zero and can be ignored. For constant μ and ϕ the middle term on the right side is thus also constant, and, since $0 < \phi < 1$, successive values of X_t approach μ at a rate that is inversely proportional to ϕ. I suspect that this slippage was not apparent to MKS because 1) the process was run over only 30 repetitions (and then, presumably, a new random "seed" temperature was chosen and a new "month" of temperatures generated); 2) ϕ is fairly high (0.55 to 0.63); and 3) the ϵ_t are large enough, relative to the X_t, to mask the slippage.

The development and application of a stationary model has been previously shown (Gringorten, 1966; Hansen and Driscoll, 1977). Using MKS symbols, this is

$$X_t = \phi X_{t-1} + (1 - \phi^2)^{1/2}\epsilon_t. \qquad (3)$$

This model requires that the X_t and ϵ_t be standardized (normally distributed with mean zero and variance unity). After the model is run with standardized variables the conversion to temperature (in °F) is accomplished by substituting $\bar{X}_t$ (or, in MKS symbols, μ) for z, the standardized variate, and the actual variance—determined, as MKS did, from the climatic record—for unit variance. Our application involved generating hourly temperatures for a year, and thus (3) was modified to account for annual and diurnal cycles. But (3) is recommended for any application in which one desires to simulate a stationary, autoregressive process in which the X_t are linear functions of the X_{t-1} and random numbers.

The frequency distributions of daily maximum temperatures in July, which MKS modeled for four United States locations, are essentially normally distributed, and thus no appreciable error is introduced by a model which generates temperatures in this way. However, we have shown (Hansen and Driscoll, 1977) that for more southerly locations in the United States maximum daily temperatures in summer are appreciably negatively skewed, and our model incorporates a correction for this skewness. It is likely that such temperatures for other areas and times, and daily minimum temperatures as well, also are not normally distributed.

Mearns, Katz and Schneider correctly indicate that the standard deviations of daily maximum temperatures in July are, for their stations, inversely related to the mean monthly temperatures (their Fig. 1). More generally, the variability of mean monthly temperatures, and undoubtedly therefore that of daily maximum temperatures, is related to the annual range of temperature, and thus to both continentality and latitude (Bailey, 1968; Summer, 1953).

The authors speculate about relationships among the parameters controlling their model, *viz.*, mean, standard deviation, and serial correlation. My own studies, which apply to mean monthly temperatures in the United States, and which are in Hansen and Driscoll (1977) in part, show that as the mean increases both the variability and serial correlation decrease. Thus, Miami in July has relatively small variation among days and low serial correlation. Minneapolis in January has the reverse. It seems reasonable to suggest, therefore, that similar relationships would be observed if there were changes in the mean over time at one location.

REFERENCES

Bailey, H. P., 1968: Hourly temperatures and annual range. *Eclectic Climatology,* Oregon State University, 25–40.

Gringorten, I. I., 1966: A stochastic model of the frequency and duration of weather events. *J. Appl. Meteor.,* **5,** 606–624.

Hansen, James E., and Dennis M. Driscoll, 1977: A mathematical model for the generation of hourly temperatures. *J. Appl. Meteor.,* **16,** 935–948.

Mearns, Linda O., Richard W. Katz and Stephen H. Schneider, 1984: Extreme high-temperature events: Changes in their probabilities with changes in mean temperature. *J. Climate Appl. Meteor.,* **23,** 1601–1613.

Sumner, Alfred R., 1953: Standard deviation of mean monthly temperatures in Anglo-America. *Geogr. Rev.,* **43,** 50–59.