Empirical Estimation of Daily Clear Sky Solar Radiation

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ABSTRACT

The suitability of two simple empirical equations for the estimation of clear sky radiation was investigated. Results indicated that latitude and altitude were sufficient to estimate the empirical equation coefficients and that the estimates of evapotranspiration using the derived values of clear sky radiation were sufficiently accurate for irrigation scheduling or hydrologic modeling purposes. It was also found that there was a significant change of approximately 10% in the calibration of NOAA instruments between 1972 and 1977.

1. Introduction

Modern methods of estimating evapotranspiration for hydrological studies and irrigation scheduling often incorporate the use of the combination equation as presented by Jensen *et al.* (1971). Saxton (1975) has shown that net radiation is the most sensitive parameter in this equation. Unfortunately, measurements of net radiation are not available for many locations. However, measurements of global solar radiation are more common; thus, methods have been developed (Jensen *et al.*, 1971) to estimate net radiation from global radiation and estimated back radiation. The method uses a daily clear sky radiation as input to the estimation procedure. The objective is to develop a simple technique to estimate daily clear sky solar radiation.

Net radiation (R_n) may be estimated as follows:

$$R_n = (1 - \alpha)R_s - R_b, \qquad (1)$$

where $(1 - \alpha)R_s$ represents the net shortwave radiation received by a green crop with full cover, α is the mean daily shortwave reflectance or albedo, R_s the measured solar radiation and R_b the net outgoing long-wave radiation,

$$R_b = (aR_s/R_{s0} + b)R_{b0}, \qquad (2)$$

where R_{b0} is the net outgoing longwave radiation in MJ m^{-2} on a clear day, R_{s0} the estimated clear sky solar radiation on that day, and a and b are empirical constants; R_{b0} is estimated as

$$R_{b0} = (a_1 - b_1 e_d^{1/2})(4.895 \times 10^{-9})(T_2^4 + T_1^4)/2, \qquad (3)$$

where e_d is the saturation vapor pressure (mb) at mean dew-point temperature, 4.895×10^{-9} is the Stefan-Boltzmann constant (MJ m^{-2} d^{-1} deg K^{-4}) and T_1 and T_2 are maximum and minimum daily temperatures (K), respectively. The coefficients a_1 and b_1 with values of 0.37 and 0.044, respectively, were calibrated by Kincaid and Heermann (1974) for Mitchell, Nebraska.

In order to assess the effect of errors in estimation of clear sky radiation on evapotranspiration estimates, a sensitivity analysis was performed using the techniques described by Saxton (1975). In this technique the sensitivity coefficients were derived as follows:

$$S = E_{te}/R_{s0e} = (\partial E_t/\partial R_{s0})(R_{s0}/E_t), \qquad (4)$$

where S is the sensitivity coefficient, E_{te} the percentage error in estimated evapotranspiration and R_{s0e} the percentage error in clear sky solar radiation and E_t is the evapotranspiration estimate.

Using the form of the Penman combination equation presented by Kincaid and Heermann (1974), we note:

$$E_t = 0.408556C_1(R_n - G) + 6.426C_2(1.1 + 0.01063W)(e_s - e_d), \qquad (5)$$

where E_t is in mm of water; and

C_1, C_2 mean air temperature weighting factors whose sum is 1,

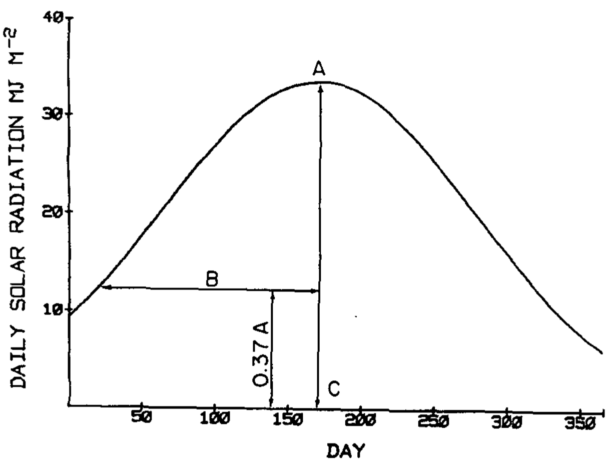

FIG. 1. Determination of coefficients for Eq. (7).

e_s mean saturation vapor pressure (kPa) as mean of the saturation vapor pressures at maximum and minimum daily air temperatures,
e_d saturation vapor pressure (kPa) at mean dew-point temperature (mean of six 4 hourly readings),
W total daily wind movement (km),
R_n daily net radiation in MJ m^{-2} and
G daily soil heat flux (MJ m^{-2}).

Using Eqs. 1–5, we observe that the sensitivity coefficient reduces to

$$(\partial E_t/\partial R_{s0})(R_{s0}/E_t) = a0.408556C_1(R_sR_{b0}/R_{s0}E_t), \quad (6)$$

where C_1 is a mean air temperature weighting factor.

The sensitivity coefficient, evaluated over a 120-day irrigation season at Greeley, Colorado, ranged in daily values from 0.14 to 0.41. The larger values tended to

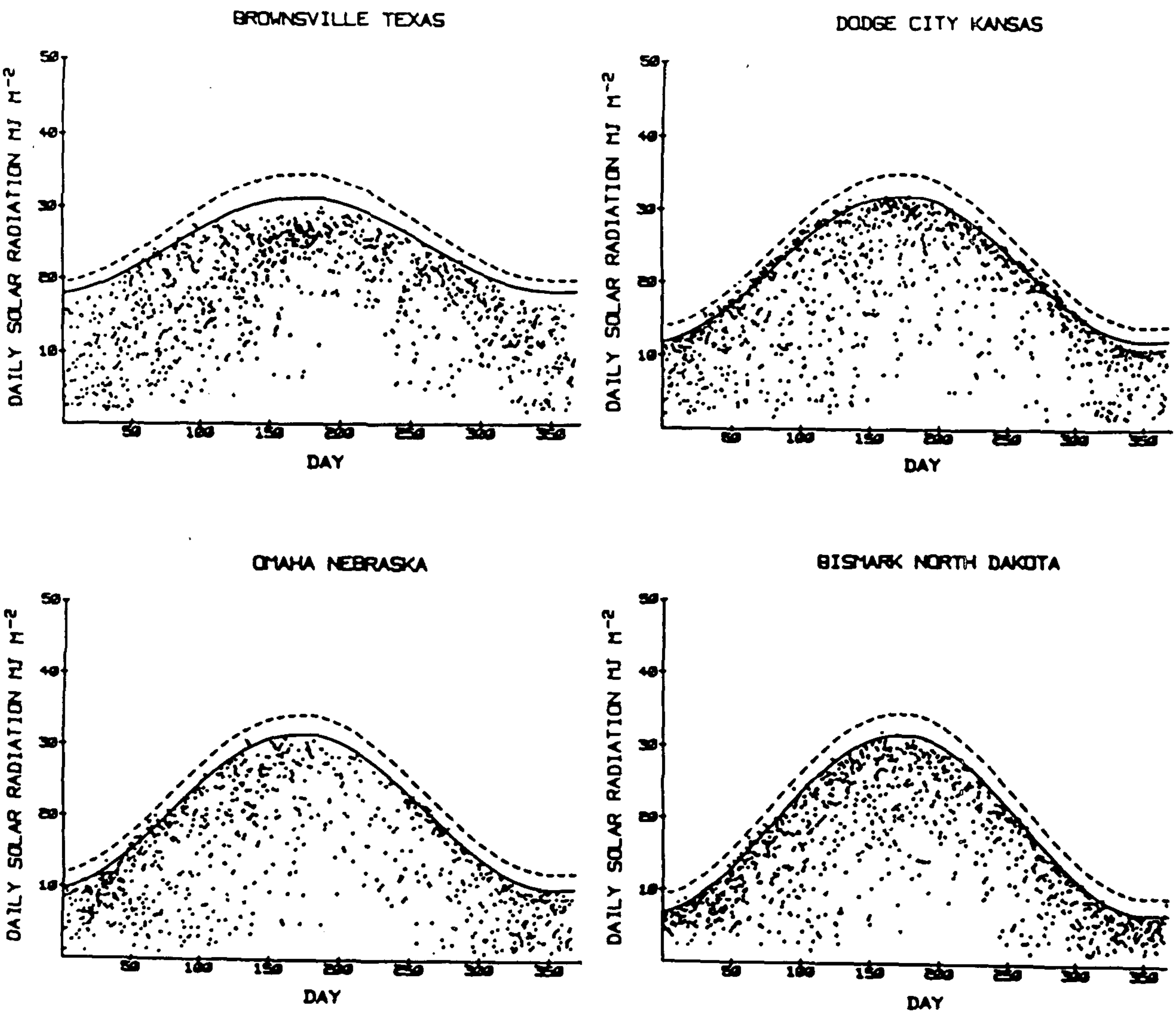

FIG. 2. Clear sky daily solar radiation fitted to NOAA 1977–80 data (solid curve) and similar fit to NOAA 1968–72 data (dashed curve).

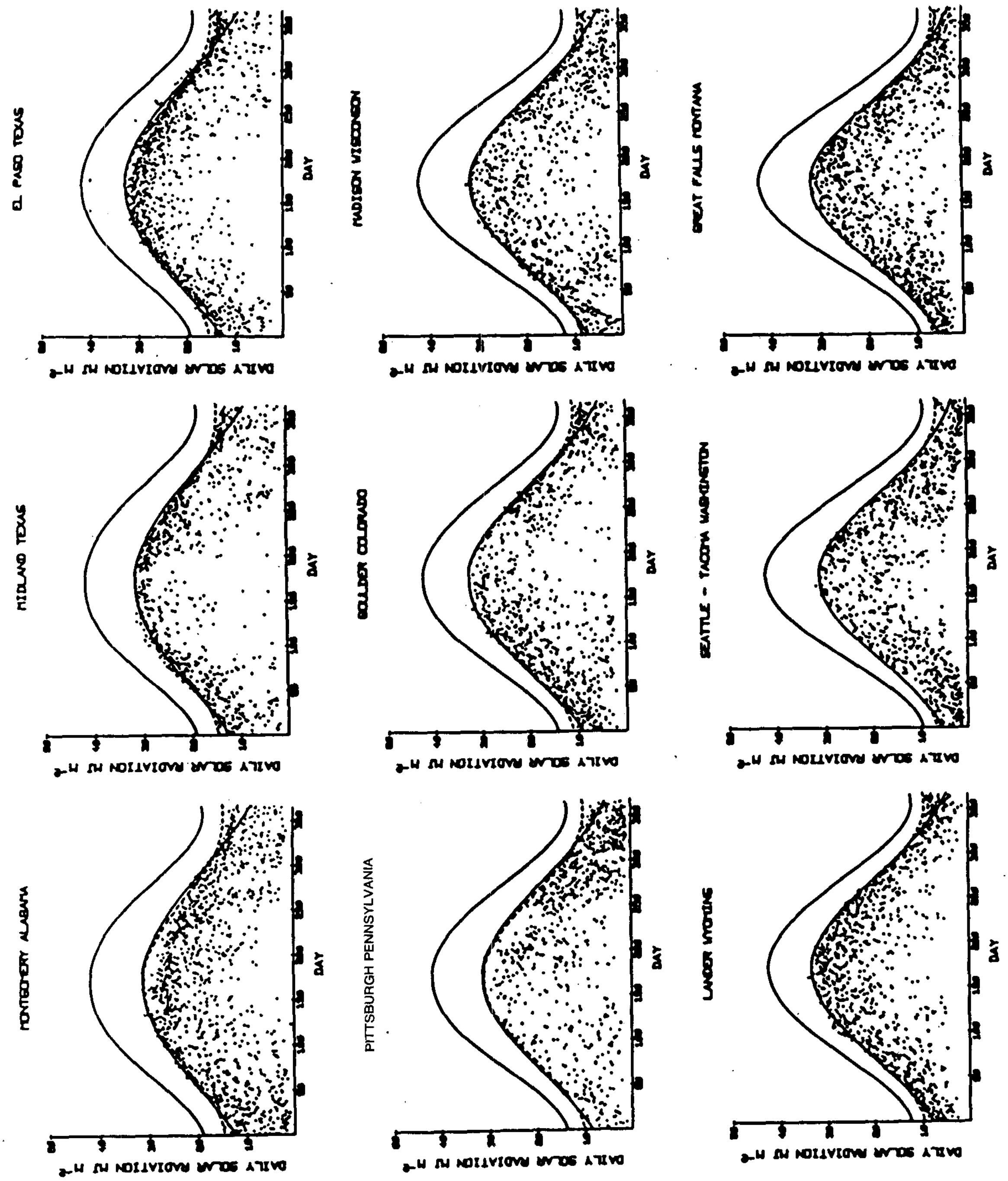

FIG. 3. National Oceanic and Atmospheric Administration 1977–80 data used to fit the empirical equations: upper solid line is extraterrestrial radiation; lower solid line and dashed line are best fits for Eqs. (7) and (8), respectively.

TABLE 1. The coefficients as curve fit to Eqs. (7) and (8) for the 1977–80 NOAA data.

Site	Latitude (°N)	Altitude (m)	Exponential (7) A	B	C	Cosine (8) A′	B′	C′
Montgomery, AL	32.3	68	31.4	174	170	22.8	8.4	2.92
Midland, TX	32.0	872	31.9	174	170	23.2	8.5	2.92
El Paso, TX	31.8	1206	33.1	174	170	24.1	8.8	2.92
Pittsburgh, PA	40.5	371	31.5	148	170	20.6	10.6	2.92
Boulder, CO	40.0	1634	32.9	148	170	21.8	11.0	2.92
Madison, WI	43.1	271	31.8	140	170	20.0	11.5	2.92
Lander, WY	42.8	1699	33.5	140	170	21.4	12.0	2.92
Seattle-Tacoma, WA	47.4	143	31.5	128	170	18.9	12.5	2.92
Great Falls, MT	47.5	1118	31.9	128	170	19.2	12.8	2.92

occur in spring and autumn. The average value for the season was 0.28. Thus a systematic 10% overestimation of daily clear sky solar radiation would result in a 2.8% change in total evapotranspiration throughout the irrigation season at Greeley.

Cloudless day solar radiation values can be obtained from estimates by Fritz (1949) or by plotting observed daily values to obtain an envelope curve through the high points. The latter method can be conveniently performed on a computer driven plotter which enables the derivation of a simple empirical equation. This technique was used to derive coefficients for equations to estimate clear sky radiation. Estimates were initially derived for 13 sites of the U.S. National Oceanic and Atmospheric Administration data from 1968 to 1972. Through regression analysis, simple equations were derived to estimate daily clear sky radiation using latitude as the principal variable.

When these original equations were tested against an independent data set, the National Oceanic and Atmospheric Administration data appeared to be suspect. A new data set was chosen from more recent (1977–80) National Oceanic and Atmospheric Administration data and a high and low altitude were paired at each latitude.

2. Derivation of clear sky radiation equations

Daily clear sky solar radiation measured at any location on earth is affected by solar elevation and hence also calendar date (Neuwirth, 1980). The radiation is attenuated as it passes through the atmosphere by water vapor, carbon dioxide, ozone, molecular oxygen and dust.

Since the pathlength of the sun's rays through the atmosphere is reduced at higher altitudes, it could be expected that there would be an increase in measured solar radiation. Neuwirth (1980) showed this dependence on altitude. Altitude alone is not the cause of increases in solar radiation at higher elevations. An increase in solar radiation can also be caused by a reduction in atmospheric turbidity and water vapor which can have a more significant effect than altitude (Threlkeld, 1958). Data on turbidity, however, is not readily available at most sites. Therefore, in this study, latitude and altitude were used with solar radiation data to determine the coefficients for Eqs. (7) and (8). This allows the technique to be easily calibrated for use at any site.

TABLE 2. The clearness index as calculated with Eq. (8) by season.

Site	Spring equinox	Summer solstice	Fall equinox	Winter solstice
Montgomery, AL	0.71	0.75	0.70	0.78
Midland, TX	0.72	0.76	0.70	0.79
El Paso, TX	0.75	0.79	0.73	0.81
Pittsburgh, PA	0.72	0.75	0.69	0.75
Boulder, CO	0.75	0.78	0.73	0.79
Madison, WI	0.73	0.75	0.70	0.73
Lander, WY	0.77	0.79	0.74	0.79
Seattle-Tacoma, WA	0.74	0.74	0.70	0.71
Great Falls, MT	0.75	0.76	0.71	0.71

TABLE 3. Regression coefficients for clear sky radiation prediction as a function of latitude and altitude.

	F	R^2 (%)
Exponential Eq. (7)		
$A = 31.25 + 0.001113$ A	28.54*	80.3
$B = 270 - 3.008$ L	2995.406**	99.8
Cosine Eq. (8)		
$A' = 31.54 - 0.2734$ L $+ 0.0007813$ A		
Overall	210.7**	98.6
Latitude (L)	72.1***	
Altitude (A)	31.8***	
$B' = -0.2986 + 0.2678$ L $+ 0.0004102\ A$		
Overall	733.6**	99.6
Latitude (L)	252.9***	
Altitude (A)	33.7***	

Significant F Values at 99.5% probability level

* $F_{1,7} = 16.2$
** $F_{2,6} = 14.5$
*** $F_{1,6} = 18.6$

Two types of equations were used in this study. The first is an exponential equation of the form:

$$R_{s0} = Ae^{-[(d-C)/B]^2}, \quad (7)$$

where A is the peak value of radiation, C the calendar day number at which the peak occurs, d the calendar day number and B determines the width of the curve (Fig. 1).

The second equation studied uses a cosine function to describe the daily solar radiation throughout the year. Although the equation is empirical, it has some theoretical justification. Since the earth rotates about the sun in a nearly circular orbit, the maximum height of the sun will vary according to a cosine pattern throughout the year. This relationship has the greatest validity in the midlatitudes, because in high latitudes, the radiation may be zero for extended periods and in the tropics the sun may appear directly overhead more than once a year. The form of the equation used is as follows:

$$R_{s0} = A' + B' \cos(2\pi d/365 - C'), \quad (8)$$

where A' is mean daily solar radiation, B' the amplitude, d the calendar day number, and C' the phase constant which should theoretically be set at a value corresponding to the longest day of the year (i.e., day 172).

3. Determination of equation coefficients

Envelope curves were fit to observed data by trial and error methods. A visual assessment of the curves fit to the data points was used to select suitable equation coefficients. An effort was made to standardize the time of the peak of Eq. (7) (i.e., coefficient C) and the phase shift parameter of Eq. 8 (i.e., coefficient C'). Theoretically the peak solar radiation should occur at calendar day 172. It occurred slightly earlier, however. For this reason, calendar day 170 was chosen for both, and other parameters were varied to subjectively fit the data.

When the predicted parameters from the 1968–72 National Oceanic and Atmospheric Administration data set were plotted against U.S. Department of Agriculture, Agricultural Research Service collected data, the predicted curves were consistently 10% higher than the observed data, and it was discovered that the original National Oceanic and Atmospheric Administration data were suspect. Figure 2 compares the 1977–80 National Oceanic and Atmospheric Administration data and the fitted curves for Eq. (8). The dashed line is the curve for the 1968–72 data and the solid line the curve for the 1977–80 data. This demonstrates the use of the curve fitting technique for clear sky radiation to check the validity of observed solar radiation.

The curves fit to the 1977–80 National Oceanic and Atmospheric Administration data are shown in Fig. 3. The solid envelope curve is Eq. (7), and the dashed curve Eq. (8). The coefficients are given in Table 1. In addition, the plot of extraterrestrial radiation (ETR) is also shown as the solid curve about 25% higher than the data. The ETR was calculated using an algorithm developed by Conley and McKee (1983).

The clearness index is defined as the ratio of hemispheric radiation to ETR. (The radiation data given here is hemispheric radiation.) Doesken *et al.* (1982) state that the clearness index for clear or nearly clear days ranges from a minimum of 0.65 to a maximum of 0.80. The clearness indices of our clear sky equations range from 0.69 to 0.81. They are both season and altitude dependent. The clearness index for each site by season is given in Table 2.

A number of the sites show a few data points above the envelope curves in the spring. According to Hoyt (1979), a maximum in transmission occurs in the spring because depth of air mass decreases rapidly, while total precipitable water increases very slowly. The sites near 30°N latitude were difficult to fit because of the pronounced bulges in both the spring

TABLE 4. Predicted coefficients for the test sites.

			Exponential			Cosine		
Site	Latitude	Altitude	A	B	C	A'	B'	C'
Guam, GU	13.3	112	31.4	230	170	28.0	3.3	2.92
Brownsville, TX	25.9	12	31.3	192	170	24.5	6.6	2.92
Medford, OK	36.6	376	31.7	160	170	21.8	9.6	2.92
Dodge City, KS	37.7	795	32.1	156	170	21.8	10.1	2.92
Garden City, KS	37.9	892	32.2	156	170	21.9	10.2	2.92
Akron, CO	40.2	1417	32.8	149	170	21.6	11.0	2.92
Greeley, CO	40.4	1448	32.9	148	170	21.6	11.1	2.92
Crook, CO	40.8	1143	32.5	147	170	21.3	11.1	2.92
Albin, WY	41.3	1463	32.9	146	170	21.4	11.4	2.92
Omaha, NE	41.3	404	31.7	146	170	20.6	10.9	2.92
Bismark, ND	46.8	511	31.8	129	170	19.1	12.4	2.92
Fairbanks, AK	64.8	143	31.4	75	170	13.9	17.1	2.92

and the fall. These are probably caused by a combination of the increased transmission described earlier and the broad peak that ETR exhibits at this latitude. The scattered points above the envelope curves fall into two categories: those with a clearness index below 0.9 are most likely exceptionally clear days; those above 0.9 are most likely data errors; Doesken *et al.* (1982).

4. Regression analysis

One of the objectives of this study was to determine the coefficients for Eq. (7) and/or Eq. (8) to predict

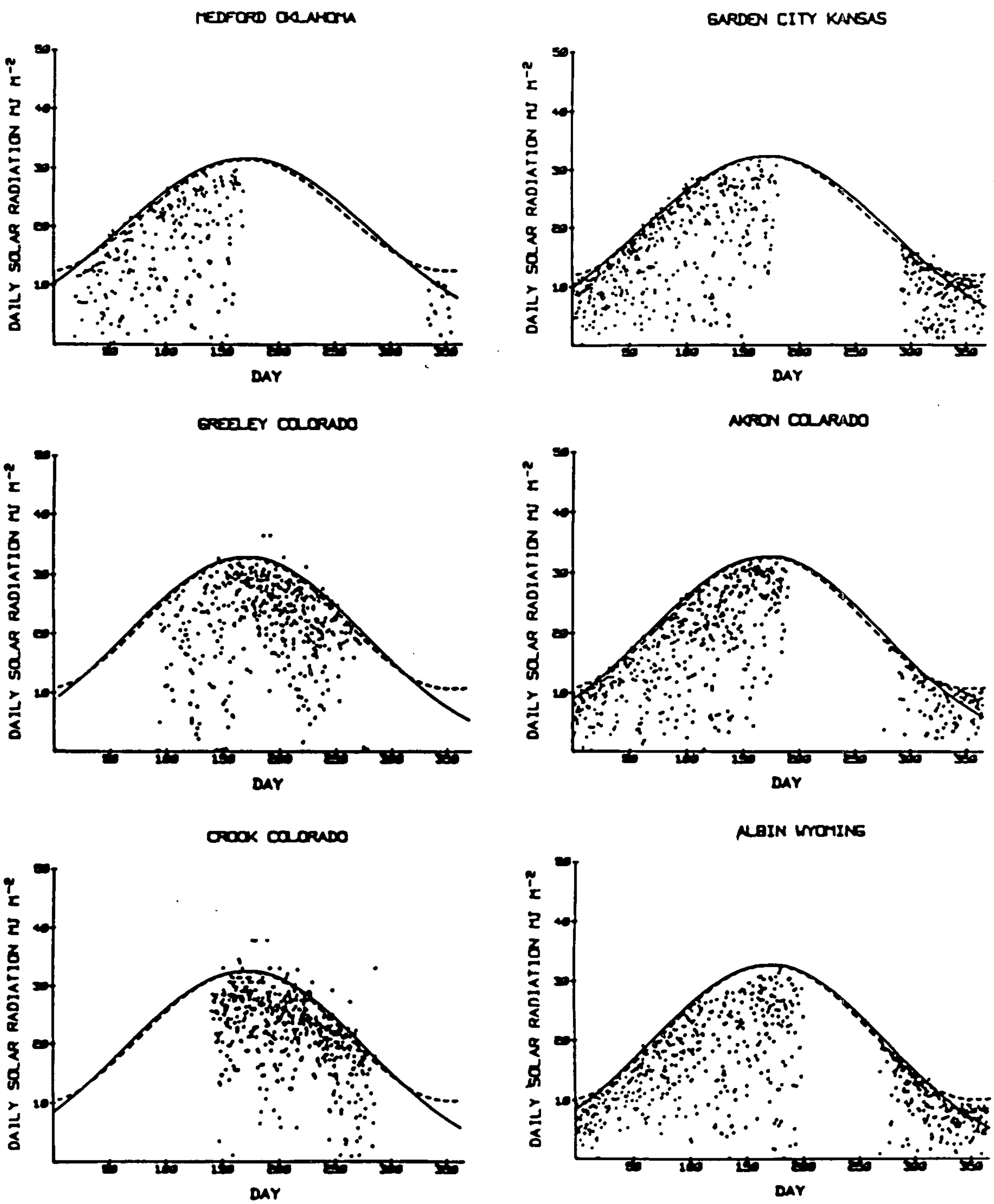

FIG. 4. Predicted equations based on latitude and altitude plotted against independent U.S. Department of Agriculture, Agricultural Research Service data sets: solid line is Eq. (7); dashed line is Eq. (8).

clear sky radiation at sites with no previous records. It is assumed that latitude and altitude would be the two variables that would most influence the empirical coefficients. Daily extraterrestrial radiation for the peak day (170) has less than a 2% variation between 30 and 50° latitude; therefore, the coefficient A in Eq. (7) is not a function of latitude. The results of these regressions for the exponential function (7) and the cosine function (8) are shown in Table 3. The coefficients for the cosine function (8) with the single degree of freedom analysis of variance showed that latitude and altitude were significant predictors for both A' (mean daily radiation) and B' (amplitude).

5. Verification of equations using different data

The empirical equations were tested by using the regression equations to predict clear sky solar radiation based on latitude and altitude at 12 sites. The predicted coefficients are given in Table 4. The test data were composed of six U.S. Department of Agriculture, Agricultural Research Service sites and six National Oceanic and Atmospheric Administration (1977–80) sites. The U.S. Department of Agriculture, Agricultural Research Service solar radiation was collected during either the winter wheat season or the irrigation season in various years from 1971–80 (Fig. 4). Of the six National Oceanic and Atmospheric Administration sites, one was within the United States but outside the latitude range (32–47°N) of the regression data (Fig. 5), and two were outside the continental United States (Fig. 6).

Of the nine sites within the latitude range of the original regressions, seven agreed very well. At two sites, Greeley and Crook, Colorado, a number of points were above the predicted line. This raised the

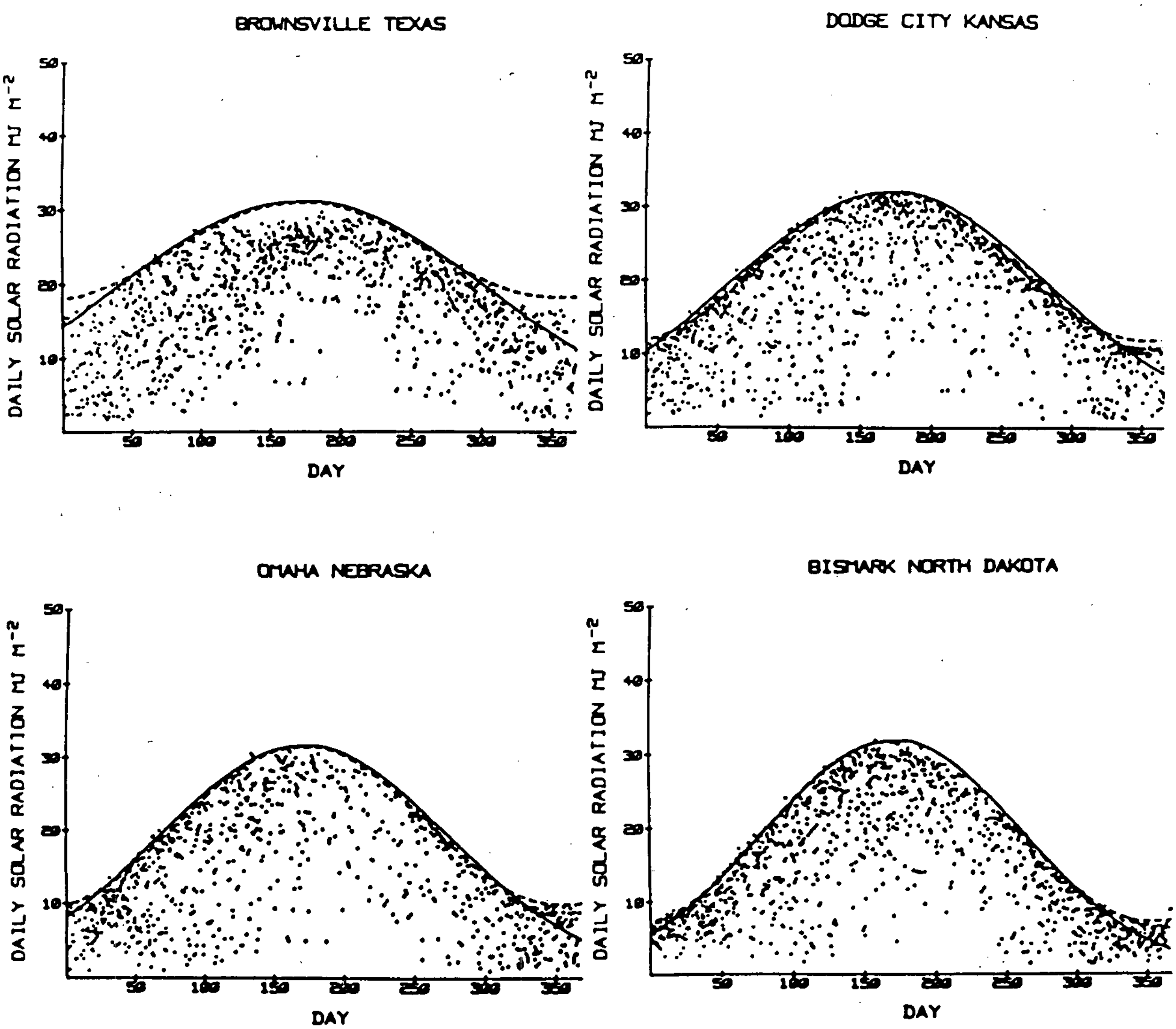

FIG. 5. Predicted equations based on latitude and altitude plotted against NOAA 1977–80 data that were not used to fit the coefficients: solid line is Eq. (7); dashed line is Eq. (8).

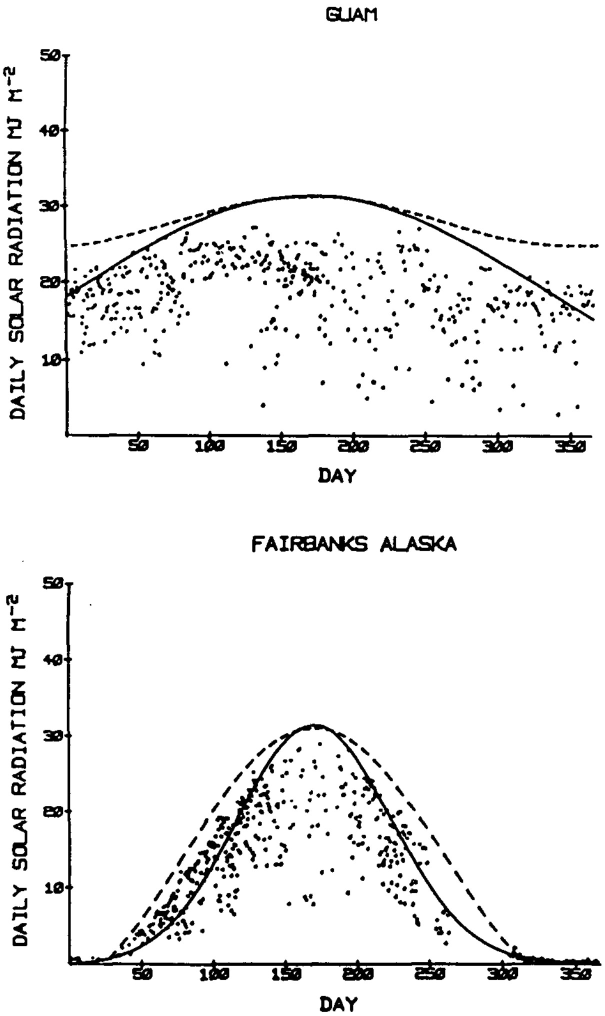

FIG. 6. Predicted equations based on latitude and altitude vs. NOAA 1977–80 data. Latitude has been extrapolated beyond the range of the regression.

possibility of an instrument calibration error. To check this further, the clearness index for each point was calculated and a frequency distribution of the clearness indices at each site was constructed. This was compared to a similar distribution constructed at Akron, Colorado, by Doesken *et al.* (1982) and also to the actual data at Akron. The frequency diagrams for both Greeley and Crook compared well with the Akron diagram. In examining the high points, there were no points at Greeley that had an absolute value over 36 MJ m^{-2} or a clearness index greater than 0.9. This was also in agreement with the Akron data. Crook, on the other hand, showed 4 points above these limits, which are probably in error. The 1980 Greeley data was higher than the other three years and this could have been an indication of a sensor calibration error during that year. However, Doesken's data at both Akron and Fort Collins were also higher than normal during the summer of 1980, implying that the summer of 1980 was exceptionally clear. The general conclusion reached was that there was no evidence of sensor miscalibration at Greeley and Crook, although there were 4 bad individual points at Crook.

Brownsville, Texas at approximately 26°N latitude was similar to Montgomery, Alabama in the fit data set. Brownsville also exhibits pronounced bulges in the spring and fall and a broad peak. These sites are close enough to the Tropic of Cancer that the peak ETR values are nearly constant for two months. Given the broad peak, both models overestimate solar radiation in the midsummer by 2–3 MJ m^{-2}. In addition the mid-winter low is overestimated by the cosine equation and underestimated by the exponential equation by approximately the same amount.

Of the test sites, the predicted equations had the worst fits at Brownsville, Texas and Albin, Wyoming. An estimate of the envelope curve for the data at each site yields clear sky values approximately 5% lower than the predicted curves during the summer. Based on the sensitivity analysis, this would result in an error in evapotranspiration of less than 1.5%.

To test the ability of the equations to extrapolate beyond the latitude range of the fit data, Guam (13°N) and Fairbanks, Alaska, (65°N) were used. At Guam neither equation was adequate; the cosine (8) performed much worse than the exponential (7). At Fairbanks Eq. (7) underestimated and Eq. (8) overestimated the data during the spring and fall. Both were high in the summer and neither was acceptable. These equations then do not work well when extrapolated beyond midlatitudes.

6. Conclusions

1) The estimation of clear sky radiation using the empirical formulas discussed in this study are well suited to estimating evapotranspiration for irrigation scheduling and for hydraulic modeling purposes. The maximum error of 5% in estimating clear sky radiation results in an error of less than 1.5%.

2) The exponential form, Eq. (7), fits the spring and fall data better than the cosine, Eq. (8), but is not acceptable for estimating winter data. Both equations provide the same summer estimates, which are acceptable.

3) Both equations perform well between 35 and 47°N latitude. They yield clearness indices within the range of known clear days. They are acceptable but not as accurate between 25–35°N latitude. Although not tested, these equations probably will work as far north as 60° where there is measurable solar radiation through the winter. They fail at 65°N, which is near the arctic circle which has a winter period of near zero radiation. They also fail at 13°N latitude, and will most likely fail anywhere south of the Tropic of Cancer.

4) There is approximately a 10% difference in calibration of the National Oceanic and Atmospheric

Administration instruments between the 1968–72 and the 1977–80 time periods.

5) The data collected by the U.S. Department of Agriculture, Agricultural Research Service shown in Fig. 4 is in reasonable agreement with the most recent calibration of the National Oceanic and Atmospheric Administration instruments.

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