Evaluation of Several "Single-Pass" Estimators of the Mean and the Standard Deviation of Wind Direction

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ABSTRACT

Proposed single-pass methods for estimating the mean ($\bar{D}$) and the standard deviation (σ_d) of wind direction and other problems in wind statistics have been evaluated using extensive field data. It can be concluded that Mardia's methods for estimating $\bar{D}$ and σ_d provide good estimators in practical application. Yamartino's empirical method also provides a good estimator of σ_d.

The relationship between the vector mean wind speed **Vv** and the scalar mean wind speed **Va** has been investigated in connection with σ_d and the standard deviation of the crosswind velocity σ_v. It is found that Kampé de Fériet–Frenkiel's equation, which represents the relation for **Vv** and **Va** in terms of σ_v, holds for the field data, so we can get an estimator of σ_v from observed **Vv** and **Va**. The ratio of **Vv/Va** is roughly related to the standard deviation of wind direction, and the relation can be expressed as **Vv/Va** = $\exp(-0.5\sigma_d^2)$ for $\sigma_d < 20$ deg.

1. Introduction

The wind is a vector quantity, so it is (physically) natural to regard the vector mean wind as a measure of the mean wind, its magnitude as the mean wind speed, and its direction as the mean wind direction. However, in usual surface weather observations, the scalar mean wind speed is used as the mean wind speed, and the mean wind direction estimated from wind direction data only is used as the mean of wind direction. Means of wind speed and direction are taken over a time period of interest; for example, 10-min means are usually adopted in Japan.

Such a treatment of wind requires consideration of the following subjects in wind statistics: 1) the relationship between the direction of the vector mean wind and the scalar mean wind direction; 2) the relationship between the vector mean wind speed and the scalar mean wind speed; and 3) the discontinuity in wind direction scale at 0 and 360 deg. The third item requires special treatment for estimating the mean and the standard deviation of wind direction. Mardia (1972) has discussed, theoretically, statistics of directional data and proposed single-pass estimators of the mean and the standard deviation of wind direction. Ackermann (1983) and Yamartino (1984) have also proposed single-pass estimators of the standard deviation of wind direction.

The purpose of the present paper is to discuss the above problems in wind statistics and to evaluate the proposed single-pass estimators by using field data.

2. Methods for estimating the mean and the standard deviation of wind direction and some problems in wind statistics

Before discussing the above problems by using field data, we summarize the proposed methods and some theoretical derivations in wind statistics. Let V_i and D_i be wind speed and wind direction, respectively, at some instance. Let the east–west and the south–north components of wind speed be the x- and y-components, respectively. From a sequence of n observations of V_i and D_i for some period of time, a mean of the east–west component, Vx, and a mean of the south–north component, Vy, are defined as

$$Vx = -n^{-1} \sum V_i \sin D_i \quad (1a)$$

$$Vy = -n^{-1} \sum V_i \cos D_i, \quad (1b)$$

where the meteorological definition of D_i is assumed; D_i is measured clockwise from the north point. Then a vector mean wind speed **Vv** and a vector mean wind direction **Dv** can be computed as

$$\mathbf{Vv} = (Vx^2 + Vy^2)^{1/2}, \quad (2)$$

$$\mathbf{Dv} = \tan^{-1}(Vx/Vy). \quad (3)$$

Let the component parallel to **Dv** be the u-component and the component perpendicular to **Dv** be the v-component in the present paper.

a. Means of wind direction

If the discontinuity point does not lie within the range of fluctuations in wind direction, a mean of wind direction $\langle D \rangle$ can be calculated as

$$\langle D \rangle = n^{-1} \sum D_i. \tag{4}$$

Here we call $\langle D \rangle$ the arithmetic mean wind direction. If some data are beyond the discontinuity point, then it requires special treatment of the data to apply (4). This is why $\langle D \rangle$ cannot be computed in a single pass.

Mardia (1972) defines the mean direction as follows. Let P_i be the point on the circumference of the unit circle corresponding to the angle D_i and let O be the center of the circle. Then the mean direction Da is defined to be the direction of the resultant of the unit vectors $\overline{OP}_i$, $i = 1, \cdots, n$. Thus,

$$Da = \tan^{-1}(Sa/Ca), \tag{5}$$

where the average value of $\cos D_i$ is defined as

$$Ca = n^{-1} \sum \cos D_i, \tag{6}$$

and the average value of $\sin D_i$ as

$$Sa = n^{-1} \sum \sin D_i. \tag{7}$$

Here we define Da as the mean wind direction, so there are three definitions of the mean of wind direction:

a) the direction of the vector mean wind, **Dv**;
b) the arithmetic mean wind direction, $\langle D \rangle$; and
c) the mean wind direction, Da.

b. Standard deviations of wind direction

If the discontinuity point does not lie within the range of fluctuations in wind direction, the standard deviation of wind direction σ_d can be calculated as

$$\langle \sigma_d \rangle^2 = n^{-1} \sum D_i^2 - \langle D \rangle^2. \tag{8}$$

Here we define $\langle \sigma_d \rangle$ as the arithmetic standard deviation of wind direction. By similar reasoning in the case of the calculation of the arithmetic mean wind direction, this cannot be calculated in a single pass.

By assuming that the values of D_i have a normal distribution, Mardia (1972) shows that a suitable definition of the standard deviation in radian measure is

$$\sigma_{dm} = (-2 \ln R)^{1/2}, \tag{9}$$

where

$$R = (Sa^2 + Ca^2)^{1/2}. \tag{10}$$

For the sake of the following discussion, if we introduce the following notation (Yamartino, 1984) as

$$\epsilon^2 = 1 - R^2, \tag{11}$$

then σ_d can be written as

$$\sigma_{dm} = [-\ln(1 - \epsilon^2)]^{1/2}. \tag{12}$$

Yamartino (1984) has developed three estimators for σ_d. The first estimator is

$$\sigma_{d1} = \epsilon. \tag{13}$$

This estimator in radian measure can be derived analytically by using the small-angle approximation, so that it underpredicts σ_d above 30 deg. Yamartino then develops the second estimator as

$$\sigma_{d2} = \sin^{-1}(\epsilon). \tag{14}$$

However, the underprediction of this estimator at large σ_d is still large. Yamartino develops the third estimator as

$$\sigma_{d3} = \sin^{-1}(\epsilon)(1.0 + b\epsilon^3), \tag{15}$$

where $b = (2/3^{1/2}) - 1$. This estimator is the best interpolative function derived empirically.

Ackermann (1983) presents an estimator for a "vector σ_d" based on the total derivative of the vector mean wind direction **Dv**. He implicitly assumes that perturbations of the wind components are small. Using the present notation, his estimator σ_{da} is written as

$$\sigma_{da} = (Vy^2\sigma_x{}^2 + Vx^2\sigma_y{}^2 - 2VxVy\sigma_{xy})^{1/2}/\mathbf{Vv}^2, \tag{16}$$

where $\sigma_x{}^2$, $\sigma_y{}^2$ and σ_{xy} are a variance of the x-component, a variance of the y-component, and a covariance of the x- and the y-components, respectively. All terms in the right-hand side can be calculated in a single pass. Let σ_v be a standard deviation of the v-component of wind speed. Ackerman (1983) suggested that σ_{da} is approximately equivalent to a ratio $\sigma_v/\mathbf{Vv}$, but he did not prove this. Yamartino (1984) has proven, however, that Eq. (16) can be transformed not approximately but exactly into the following expression:

$$\sigma_{da} = \sigma_v/\mathbf{Vv}. \tag{17}$$

Then, Ackermann's estimator σ_{da} means the turbulent intensity of the v-component (Yamartino, 1984).

c. A problem in the definition of wind direction

In the above discussion (as in Yamartino, 1984) wind direction data are treated implicitly as random-angle data distributed in the range of 0–360 deg without considering the time history of its variation. In such a treatment, we cannot obtain an arithmetic mean wind direction or an arithmetic standard deviation of wind direction in a single pass, as previously mentioned. When the fluctuation of wind direction is small, such a treatment causes no physical problem. However, when it is large and exceeds the range of ±180 deg, the situation is changed.

We consider now a simple case as shown in Fig. 1. Considering that the wind direction rotates, a mean of wind direction can be estimated as 240 deg using the first dataset. On the other hand, when we neglect the time history of the variation of wind direction, the mean can be estimated as 60 deg using the second da-

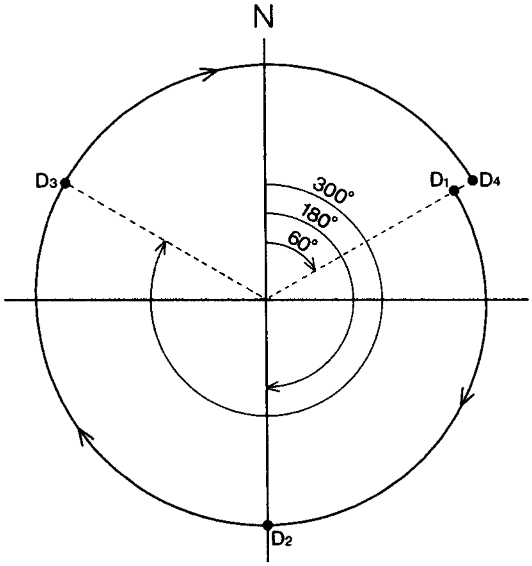

FIG. 1. Example of angle data (deg). In this case, two datasets can be considered as follows: 1) $D_1 = 60$, $D_2 = 180$, $D_3 = 300$, $D_4 = 420$; 2) $D_1 = 60$, $D_2 = 180$, $D_3 = -60$, $D_4 = 60$.

taset. In this case, different mean values result, depending on whether or not we consider the time history of the variation of direction.

Wind direction is a continuous quantity, so it is physically preferable to treat direction data considering its time development process. If we treat wind direction thusly, we can remove wind direction scale discontinuities and calculate means and standard deviations in a single pass as follows. The following method was suggested by Y. Mitsuta (personal communication, 1985).

We define the $(i + 1)$th value of direction as

$$D_{i+1} = D_i + \delta_i \tag{18}$$

where δ_i is the amount of the variation in direction for a time interval of the ith and the $(i+1)$th observations and is defined positive for clockwise rotation and negative for counterclockwise rotation. If we define all the observation data in this way, even though some data exceed the discontinuity point of scale, they all are represented continuously and can be applied directly to (4) and (8). In digital data acquisition systems, we can easily estimate δ_i as follows.

If an averaging time is chosen which is short enough that the following relation holds:

$$|\delta_i| < 180 \text{ deg}$$

and the observed values of $\hat{D}_{i+1}$ and $\hat{D}_i$ are initially given in the range of 0–360 deg, then δ_i is defined as

$$|\hat{D}_{i+1} - \hat{D}_i| < 180 \text{ deg}, \quad \text{then} \quad \delta_i = \hat{D}_{i+1} - \hat{D}_i,$$

$$\hat{D}_{i+1} - \hat{D}_i < -180 \text{ deg}, \quad \text{then} \quad \delta_i = \hat{D}_{i+1} - \hat{D}_i + 360 \text{ deg},$$

$$\hat{D}_{i+1} - \hat{D}_i > 180 \text{ deg}, \quad \text{then} \quad \delta_i = \hat{D}_{i+1} - \hat{D}_i - 360 \text{ deg}.$$

By using (18), all the data can be defined continuously, one after another.

d. The relationship between the vector and the scalar mean wind speeds

In this subsection we discuss theoretical relationships between the vector and the scalar mean wind speeds. Assuming that the components of the turbulent velocity are isotropic and are distributed according to a normal law, Hesselberg and Björkdal (1929) find the probability density function $f(Z)$ for normalized scalar wind speed as

$$f(Z) = ZT^{-2} \exp[-(1+Z)/(2T^2)] I_0(Z/T^2), \tag{19}$$

where $Z = V/\mathbf{Vv}$, V is scalar wind speed, $T = \sigma_u/\mathbf{Vv} = \sigma_v/\mathbf{Vv}$ the intensity of turbulence and I_0 the Bessel function of an imaginary argument of the zero order. This can be derived as a special case of $f(Z)$ for the nonisotropic turbulence found by Frenkiel (1951). From (19) we can calculate the relationship between Z and T. The result will be described later.

Assuming that the intensity of turbulence is small, Frenkiel (1951) shows an approximate relation for **Va** and **Vv** in terms of σ_v as

$$\mathbf{Va}/\mathbf{Vv} = 1 + \frac{1}{2}(\sigma_v/\mathbf{Vv})^2. \tag{20}$$

He noticed that this equation was derived by Kampé de Fériet (1946). From Kampé de Fériet–Frenkiel's equation, the standard deviation of the v-component σ_v can be expressed as a function of **Va** and **Vv** as

$$\sigma_v = [2\mathbf{Vv}(\mathbf{Va} - \mathbf{Vv})]^{1/2}. \tag{21}$$

Examining (17) and without using the small angle approximation, an intuitively appealing approximation is assumed as

$$\sigma_v = \mathbf{Vv} \tan\sigma_d. \tag{22}$$

When Eq. (22) is combined with (20), we obtain

$$\mathbf{Vv}/\mathbf{Va} = \left(1 + \frac{1}{2}\tan^2\sigma_d\right)^{-1}. \tag{23}$$

For small σ_d, Eq. (23) reduces to

$$\mathbf{Vv}/\mathbf{Va} = 1 - \frac{1}{2}\sigma_d^2. \tag{24}$$

Of course, this equation can be obtained directly from (17) and (20).

If it is assumed that the fluctuations in wind speed about the mean are uncorrelated with fluctuations in wind direction, then Yamartino (1984) shows

$$(\mathbf{Vv}/\mathbf{Va})^2 = 1 - \epsilon^2 = R^2. \tag{25}$$

When Eq. (25) is combined with (9), we obtain

$$\mathbf{Vv}/\mathbf{Va} = \exp\left(-\frac{1}{2}\sigma_d^2\right). \tag{26}$$

Combining (25) with (13), Yamartino (1984) derives an expression as

$$\mathbf{Vv}/\mathbf{Va} = (1 - \sigma_d^2)^{1/2}. \qquad (27)$$

Further, using (25) Yamartino suggests an expression related to (15) as

$$\mathbf{Vv}/\mathbf{Va} = \{1 - \sin^2[\sigma_d(1 - g\sigma_d)]\}^{1/2}, \qquad (28)$$

where $g = 3^{1/2}[1.0 - 0.5(3)^{1/2}]/\pi$. It should be noticed that this equation is not derived exactly from (15), but also derived as the interpolative inversion formula, and that in his paper this equation is not exactly presented as (28), but also can be obtained from two equations [(32) and (33) in his paper].

Now the ratio **Vv**/**Va** is expressed in four different types of function of σ_d: Eqs. (23), (26), (27) and (28). These equations can be transformed into (24) for $\sigma_d \ll 1$, so that for small σ_d these equations are equivalent to each other. Evaluations of these equations should be done based on field data.

3. Observations and results

By using field data we evaluate the following problems:

1) the relationship between the observed $\langle D \rangle$ and the single-pass estimators of *Da* and **Dv**:

2) the relationship between the observed $\langle \sigma_d \rangle$ and the single-pass estimators of σ_d;

3) the relationship between **Vv** and **Va**.

a. Observations

Mitsuta et al. (1973) investigated some of the foregoing problems using an analog observation system. In the present study, observations were made by using a microprocessor-controlled data acquisition system. The wind sensor used was a two-dimensional sonic anemometer (Kaijodenki, SA-200). Wind observations were made at Kagawa University at a height of 19 m above the ground. This site is located in an urban area. A sampling duration was about 10 min (exactly 9 min 56 sec) and an instantaneous value was sampled every 2 sec; one record was about 10 min long with 296 points. For each observation, means and standard deviations of wind speed and direction and other additional quantities were calculated. Statistical quantities were obtained continuously every 10 min. An observation period was 7 days, from 10 February to 16 February 1985. Some observations, which include wind direction data beyond the discontinuity point, were removed, and a number of observations used in the following analysis were reduced to 698. The following discussions are based on the 698 samples of each statistical quantity.

The Mitsuta approach presented in section 2c was suggested after this observation, so this approach has not been evaluated in the present study.

b. Results

1) THE RELATIONSHIP BETWEEN THE VECTOR AND THE SCALAR MEAN WIND SPEEDS

The foregoing theoretical treatment of the wind is described, assuming that the components of the turbulent velocity and the wind direction angles are normally distributed. In order to test normality of these quantities, values of the skewness, *S*, and the kurtosis, *K*, are calculated for each observation. For a normal distribution, $S = 0$ and $K = 3$.

Means of the observed values of *S* and *K* for each component are calculated and shown in Table 1 with standard deviations. The means of *S* and *K* for the *x*- and *y*-components are nearly 0 and 3, respectively. It is assumed that the *x*- and *y*-components are roughly distributed according to a normal law. On the other hand, the means of *S* and *K* for a wind direction angle are different from those expected from a normal law. The values of *K* are more scattered than those for the *x*- and *y*-components. If we plot the observed set of *S* and *K* in the *S*–*K* plane, we find that some plots are scattered around the point of the normal distribution, but some others are far away from this point. It cannot necessarily be assumed that the wind directions are distributed according to a normal law.

In order to test isotropy of the components of the turbulent velocity, a comparison between σ_u and σ_v is shown in Fig. 2. The plots are scattered but distributed systematically around a line of $\sigma_u = \sigma_v$. It is considered that the turbulent components are roughly isotropic.

Normality and isotropy of the components of the turbulent velocity hold roughly for the present data; thus, the assumptions in (19) are satisfied. By using (19), we can calculate theoretical values of the mean, the skewness, and the kurtosis of $Z = V/\mathbf{Vv}$ as a function of *T*. It should be noted that the values of the skewness and the kurtosis for *V* do not change if we normalize *V* with respect to **Vv**. The results are shown in Table 2. In this table, the theoretical values of **Vv**/**Va** calculated from (20) are also shown for comparison. The values of **Vv**/**Va** estimated from (19) and (20) agree well for each value of *T*. The ratio **Vv**/**Va** decreases as *T* increases and, for example, it becomes 0.65 for $T = 1$. The skewness of *Z* increases as *T* increases and is positive. The values of the kurtosis of *Z* are near 3 for the range of *T* listed in the table.

TABLE 1. Means of the skewness and the kurtosis for the *x*- and *y*-components, scaler wind speed *V* and wind direction angle *D*. Numbers in parentheses denote the standard deviations for each quantity.

Parameter	*x*	*y*	*V*	*D*
S	−0.13	−0.10	0.35	0.28
	(0.34)	(0.35)	(0.34)	(0.54)
K	3.03	3.08	3.01	4.02
	(0.60)	(0.70)	(0.68)	(1.82)

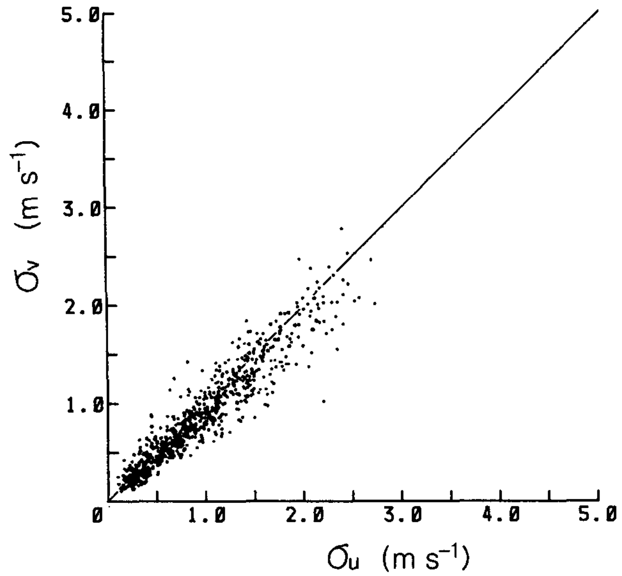

FIG. 2. A comparison of the observed σ_u and the observed σ_v. The solid line represents a relation of $\sigma_u = \sigma_v$.

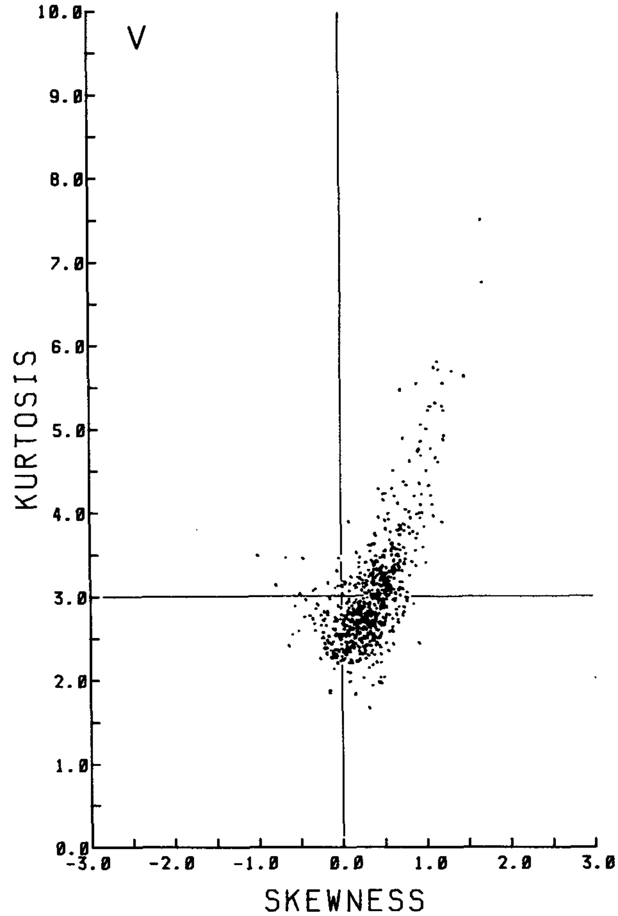

FIG. 3. Plots of the skewness and the kurtosis for scalar wind speed.

The observed sets of S and K for scalar wind speed are shown in Fig. 3. The plots tend to be scattered systematically away from the point of the normal distribution. Most of S are positive and a mean of the observed S is 0.35, as shown in Table 1. The values of K spread widely, and K tends to increase as S increases. For the present dataset, most of Tv are distributed in the range of 0.1–0.8, as shown in Fig. 6. If the turbulent components of wind speed are isotropic, then from Table 1 we expect theoretically that S and K change with T and that S are in the range of 0–0.4 and K are in the range of 2.8–3.0. Some of the plots are scattered around this area, but not all of the observed plots are necessarily distributed around this area.

Figure 4 shows the result when we calculate S and K of the scalar wind speed for cases when the distributions of the x- and y-components are near normal. The figure shows that the plots are still scattered but distributed around the expected area. This supports roughly the validity of the function (19). When we plot the observed sets of S and K for relatively high mean winds, the distribution shows the same tendency as in Fig. 4. However, it should be noticed that some of the plots for light mean winds are also included in Fig. 4.

A comparison of the vector mean wind speed **Vv** and the scalar mean wind speed **Va** is shown in Fig. 5. Each value of **Vv** is smaller than that of **Va**, as expected, and the ratios of **Vv**/**Va** are in the range of 88%–100% for **Va** > 2 m s^{-1}. It should be noticed that the data are for 10-min periods and this ratio is sensitive to the sampling duration.

Kobayashi and Senshu (1977) investigated Kampé de Fériet–Frenkiel's relation given by (20) and found that it *almost* holds for observed data. However, the number of samples obtained by them was only 10, and

TABLE 2. Theoretical values of the mean, the skewness and the kurtosis of Z as a function of T. Note that $Z = V/\mathbf{Vv}$, and T is the intensity of isotropic turbulence; $T = \sigma_v/\mathbf{Vv} = \sigma_u/\mathbf{Vv}$.

T	Mean of Z	Skewness	Kurtosis	Vv/Va	Vv/Va given by (20)
0.1	1.01	0.00	3.01	1.00	1.00
0.2	1.02	0.01	2.99	0.98	0.98
0.3	1.05	0.04	2.95	0.96	0.96
0.4	1.09	0.11	2.87	0.92	0.93
0.5	1.14	0.21	2.82	0.88	0.89
0.6	1.20	0.30	2.82	0.83	0.85
0.7	1.28	0.38	2.86	0.78	0.80
0.8	1.36	0.44	2.91	0.74	0.76
0.9	1.45	0.48	2.97	0.69	0.71
1.0	1.55	0.52	3.02	0.65	0.67
1.1	1.65	0.54	3.06	0.61	0.62
1.2	1.76	0.56	3.09	0.57	0.58
1.3	1.86	0.58	3.12	0.54	0.54
1.4	1.97	0.59	3.14	0.51	0.51
1.5	2.08	0.60	3.16	0.48	0.47
1.6	2.20	0.60	3.18	0.46	0.44

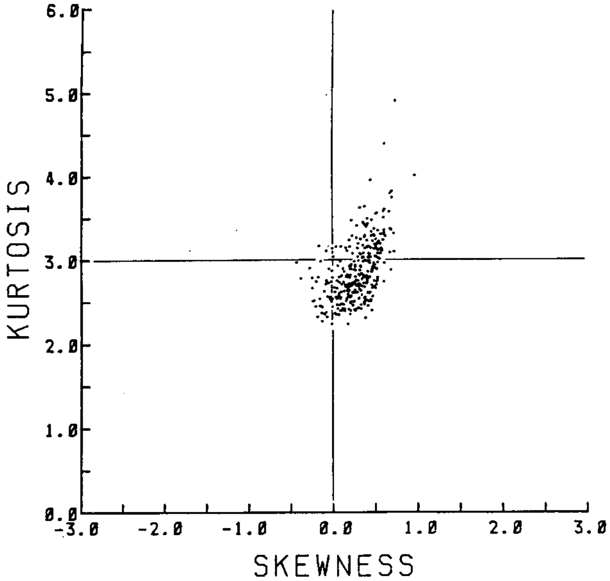

FIG. 4. As in Fig. 3, but for the cases when the skewness, Sx and Sy, and the kurtosis, Kx and Ky, of the x- and y-components are in the following ranges: $-0.5 < Sx, Sy < 0.5$, $2.5 < Kx, Ky < 3.5$.

it seems too small to confirm its adequacy in the natural wind. The relationship between the observed ratio **Vv/Va** and the observed Tv is shown in Fig. 6. A curve corresponding to (20) is also shown in the figure. The observed ratio **Vv/Va** is closely related to Tv and the data fit the curve well.

This means that Eq. (20) holds well for the field data, and using (21) we can obtain an estimate of σ_v from the observations of **Vv** and **Va**. Actually, as in Fig. 7, the observed σ_v and the estimated $\hat{\sigma}_v$ from **Vv** and **Va** agree quite well, except for a few extreme data.

Equations (23), (26), (27) and (28) suggest that the ratio **Vv/Va** is related to σ_d. The relation between the observed **Vv/Va** and $\langle\sigma_d\rangle$ is shown in Fig. 8. We find that the observed **Vv/Va** is roughly related to $\langle\sigma_d\rangle$. Curves corresponding to these equations are also shown in the figure. The curve corresponding to (26) is almost equivalent to the curve corresponding to (28) for $\sigma_d < 60$ deg. For small σ_d, these equations hold for the field data. Among these equations, Eqs. (26) and (28) represent the observed data relatively well. However, for $\langle\sigma_d\rangle > 20$ deg, Eqs. (26) and (28) systematically underpredict σ_d. For $\langle\sigma_d\rangle > 40$ deg, though a number of data are small, Eqs. (26) and (28) again tend to fit the observed data.

As mentioned before, Eq. (26) is derived from (12) and (25), and Eq. (28) is derived from (15) and (25). Further, as described later, Eqs. (12) and (15) represent the observed data well. It is assumed that the systematic underpredictions by (26) and (28) for $\sigma_d > 20$ deg are caused by the inadequacy of (25). Yamartino (1984) suggested that Eq. (25) must be evaluated in coastal and complex terrain before its uncertainty limits are understood.

In connection with this, a comparison between the observed ratio **Vv/Va** and the observed R is shown in Fig. 9. In this figure the abscissa denotes $1 - R$ and the ordinate denotes $1 - \mathbf{Vv/Va}$. If $\mathbf{Vv/Va} = R$, then the plots lie on the solid line. However, the plots are scattered systematically away from the line. The discrepancy becomes large at large $1 - R$; it should be noted that $0 < R < 1$ from the definition of R. This means that Eq. (25) does not hold for the observed data at small R and also that the assumption that the fluctuations in wind direction and speed are uncorrelated with one another does not hold.

The observed relation between **Vv/Va** and R is expressed as

$$\mathbf{Vv/Va} = (1 - a) + aR, \tag{29}$$

where $a = 0.86$. When Eq. (29) is combined with (9), we obtain

$$\mathbf{Vv/Va} = (1 - a) + a\exp(-0.5\sigma_d^2). \tag{30}$$

A curve corresponding to (30) is also shown in Fig. 8. We found that the curve fits the observed data for $\sigma_d < 40$ deg well.

2) THE MEAN AND THE STANDARD DEVIATION OF WIND DIRECTION

We compare the arithmetic mean wind direction $\langle D\rangle$ with the mean wind direction Da. The degree to which $\langle D\rangle$ and Da agree may depend on the magnitude of fluctuations in wind direction. Figure 10 shows the

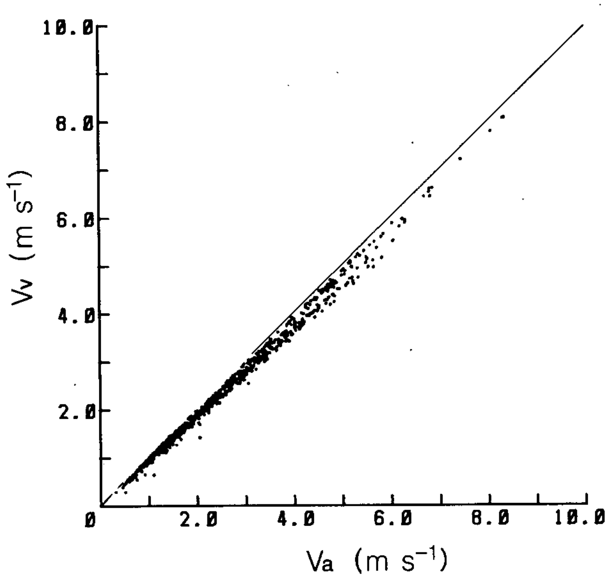

FIG. 5. A comparison of the vector mean wind speed **Vv** and the scalar mean wind speed **Va**. The solid line represents a relation of **Vv** = **Va**.

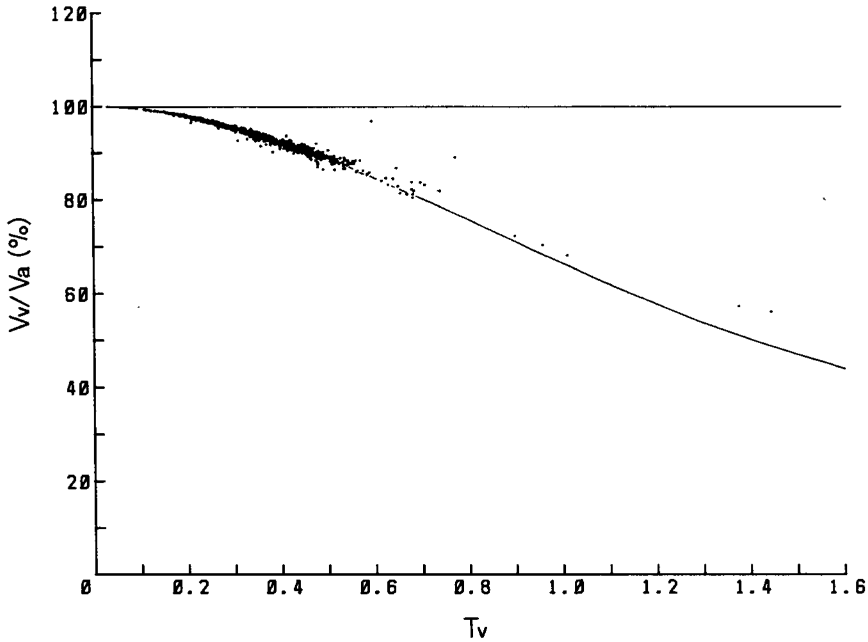

FIG. 6. The ratio of **Vv/Va** as a function of *Tv*. $Tv = \sigma_v/\mathbf{Vv}$. The curve corresponds to (20).

relationship between the difference of $Da - \langle D \rangle$ and $\langle \sigma_d \rangle$. The scatter tends to increase as $\langle \sigma_d \rangle$ increases. For the present data, the difference angles are restricted in the range of ±5 deg except for few extreme data and the values of *Da* agree closely with those of $\langle D \rangle$.

The relationship between the difference of $\mathbf{Dv} - \langle D \rangle$ and $\langle \sigma_d \rangle$ is shown in Fig. 11. The scatter tends to increase as $\langle \sigma_d \rangle$ increases. However, the difference angles are restricted in the range of ±10 deg, except for a few extreme data.

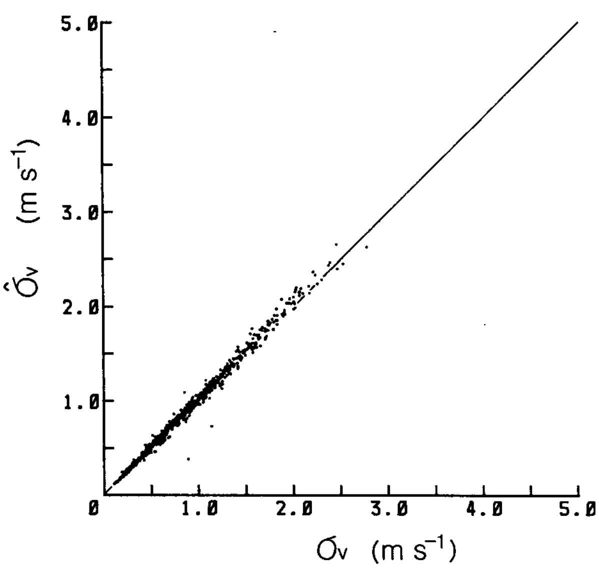

FIG. 7. A comparison of the estimated $\hat{\sigma}_v$ from **Vv** and **Va** with the observed σ_v.

Next we discuss the standard deviation of wind direction. Mardia's estimator (12) and Yamartino's estimator (15) for σ_d are expressed as functions of ϵ given by (12); the relationship between the observed ϵ and $\langle \sigma_d \rangle$ is investigated and shown in Fig. 12. Curves corresponding to (12) and (15) are shown in this figure. For comparison, a curve corresponding to the small angle approximation (14) is also shown. The figure shows that the observed ϵ are related to the observed $\langle \sigma_d \rangle$. Equation (14) does not represent the observed relation for large ϵ. Mardia's (12) and Yamartino's (15) well represent the observed relation.

Relative errors and rms errors for the estimators σ_{dm} (12) and σ_{d3} (15) are calculated and shown in Table 3 with those for other estimators for comparison. The rms errors for σ_{dm} and σ_{d3} almost agree with and are smaller than those for the others. On the other hand, the rms relative error for σ_{d3} is quite small and that for σ_{dm} is relatively large. This means that the error of σ_{dm} is larger for small σ_d and is smaller for large σ_d than that of σ_{d3}. However, because the rms relative error of σ_{dm} is smaller than 2%, it is assumed that both estimators of σ_d do not matter in practical application.

A comparison of σ_{dm} given by (12) with $\langle \sigma_d \rangle$ is shown in Fig. 13. We find that σ_{dm} and $\langle \sigma_d \rangle$ agree quite well, except for large σ_d.

A comparison of σ_{da} given by the "vector method" with $\langle \sigma_d \rangle$ is shown in Fig. 14. The scatter is larger than

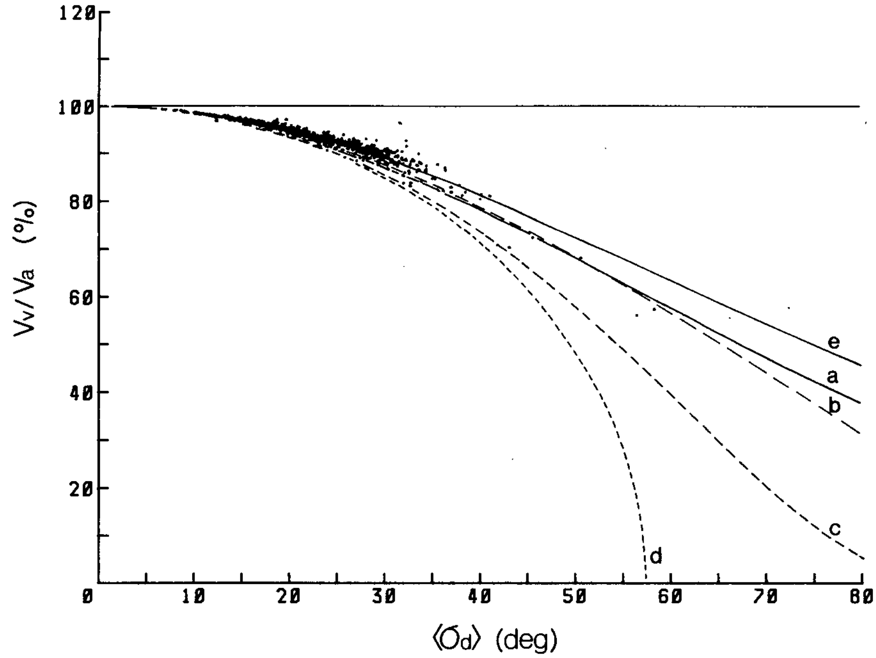

FIG. 8. The ratio of Vv/Va as a function of σ_d. Curves are labeled as follows. Curve a: author's (26); curve b: Yamartino's (28); curve c: author's (23); curve d: Yamartino's (27); and curve e: author's (30).

that in Fig. 13. A rms error and a relative error for σ_{da} are shown in Table 3. Both errors are larger than those of any other estimators, so this estimator appears less preferable than Yamartino's or Mardia's estimators.

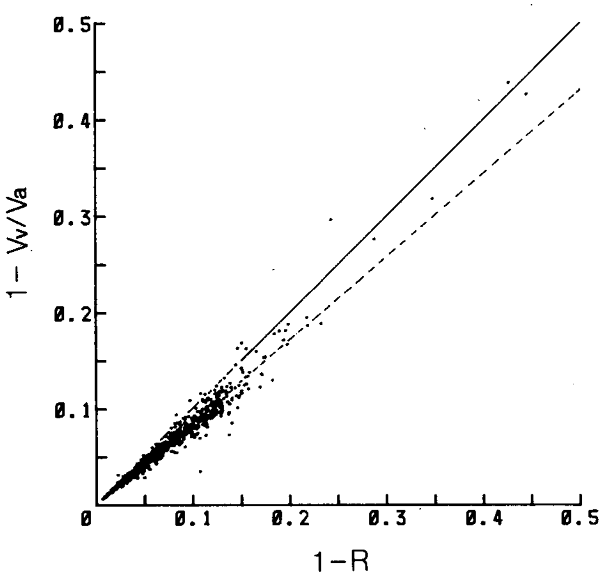

FIG. 9. A comparison of the observed Vv/Va and the observed R. The solid line represents a relation of $1 - \mathrm{Vv/Va} = 1 - R$, and the dashed line represents a relation of $1 - \mathrm{Vv/Va} = a(1 - R)$, $a = 0.86$, obtained by a least-squares method.

Verrall and Williams (1982) have presented a single-pass estimator of the standard deviation of wind direction, which is calculated from accumulations of $\sin D$, $\cos D$, $\sin^2 D$, $\cos^2 D$ via a transform formula. Their method, however, is rather complicated. Using (6) and (7) in their paper, we can easily derive Mardia's formulation given by (9) in the present paper. Fisher (1983) commented that the estimator in their paper appears less preferable to the standard deviation proposed by Mardia (1972) and they agreed with Fisher's comment in their reply.

Using data samples created with a Gaussian random number generator, Yamartino (1984) has evaluated estimators of σ_d proposed by himself, Ackermann (1983) and Verrall and Williams (1982) and has found that his estimator (15) provides the best estimate of σ_d. However, he did not evaluate Mardia's estimator. The present evaluation made by using the field data suggests that both estimators of Yamartino's (15) and Mardia's (12) provide quite good estimates of σ_d in the range of $\sigma_d < 60$ deg.

Yamartino (1984) has suggested that his estimator (15) differs quite radically from the theoretical result of Mardia's (12) at the limit $\epsilon \rightarrow 1$, Yamartino's $\sigma_{d3} \rightarrow \pi/\sqrt{3}$, and Mardia's $\sigma_{d3} \rightarrow \infty$. This difference is derived from different definitions of wind direction distribution. As mentioned before, Yamartino has assumed that wind direction data are distributed in the range of 0 to 360 deg. The values of σ_{d3} are in the

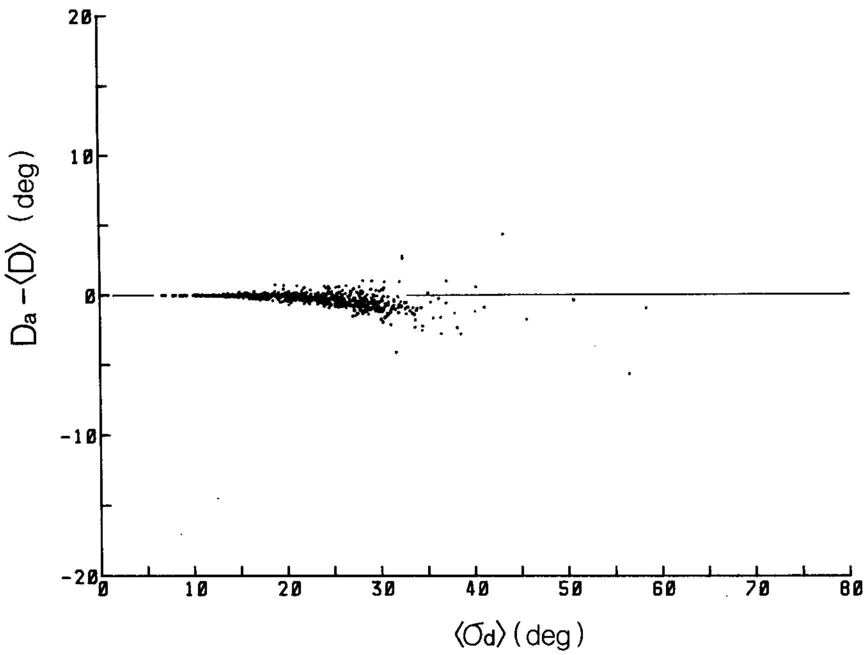

FIG. 10. The difference of $Da - \langle D \rangle$ as a function of $\langle \sigma_d \rangle$.

limited range, and in this case the maximum value of σ_{d3} can be estimated theoretically as previously shown. Yamartino's Eq. (15) interpolates between the small angle approximation and the theoretical limit of σ_d and its functional form is determined empirically.

On the other hand, Mardia (1972) has derived his estimator (9) [or (12)] assuming the wrapped normal distribution of wind direction (see Mardia, 1972, pp. 53–55 and 74). The σ_{dm} is the ordinary standard deviation on the line whose range is (0, ∞). For large σ_d,

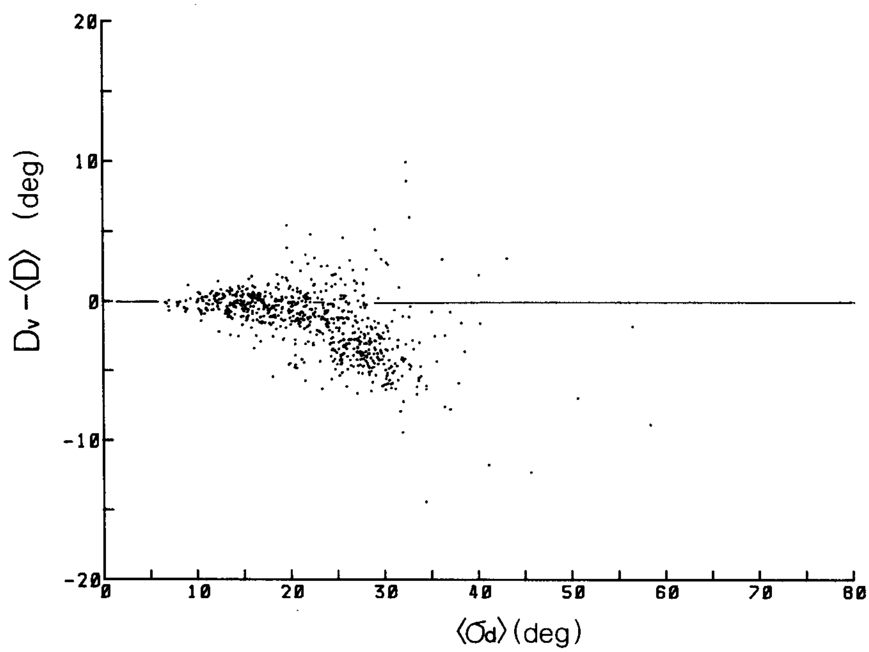

FIG. 11. The difference of **Dv** − ⟨D⟩ as a function of $\langle \sigma_d \rangle$.

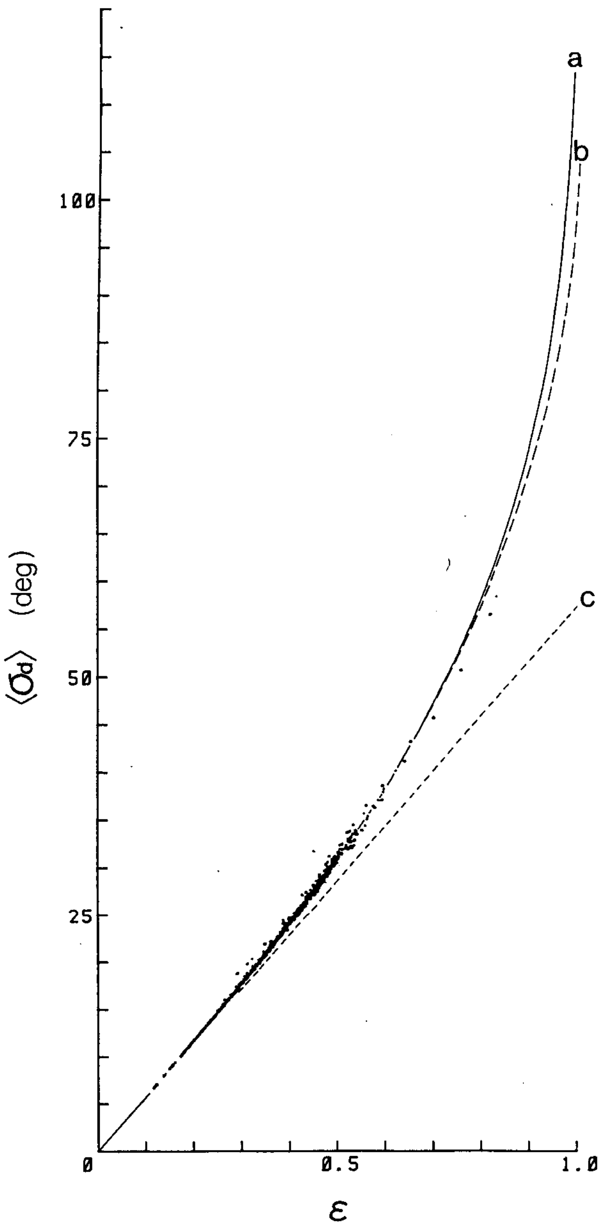

FIG. 12. The standard deviation of wind direction $\langle\sigma_d\rangle$ as a function of ϵ. Curves are labeled as follows. Curve a: Mardia's (12); curve b: Yamartino's (15); and curve c: the small angle approximation (13).

it is meaningless to make a comparison of both estimators, and evaluations for them should be made separately by using σ_d observations based on the different definitions.

The assumption of the normal distribution of wind direction is not necessarily satisfactory, as mentioned before (Table 1). However, the deviation of the distribution of wind direction from a normal law has little influence on the estimates of σ_d in practice, as shown in Fig. 13. There is no more appropriate theoretical distribution of wind direction.

TABLE 3. Root-mean-square errors and relative errors for estimators of σ_{dm} (12), σ_{d1} (13), σ_{d2} (14), σ_{d3} (15) and σ_{da} (16). The error is denoted as $\sigma_{dm} - \langle\sigma_d\rangle$, the relative error as $(\sigma_{dm} - \langle\sigma_d\rangle)/\langle\sigma_d\rangle$, and so on.

	Estimators for σ_d				
	σ_{dm}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{da}
rms error (deg)	0.43	1.60	0.70	0.47	3.42
rms relative error (%)	1.40	5.28	0.07	0.03	13.96

Though there is a marked difference between Mardia's estimator and Yamartino's one in the definitions of large σ_d, both estimators agree quite well in the range of $\epsilon < 0.8$, as shown in Fig. 12. Considering that Mardia's estimator has a theoretical basis with a relatively simple formulation, it can be concluded that Mardia's method provides the best single-pass estimator of σ_d in practical application.

4. Conclusion

Single-pass methods for estimating the mean and the standard deviation of wind direction have been evaluated using extensive field data. It can be concluded that Mardia's methods for estimating the mean and the standard deviation of wind direction provide good estimators. Yamartino's Eq. (15) also provides a good estimator of σ_d.

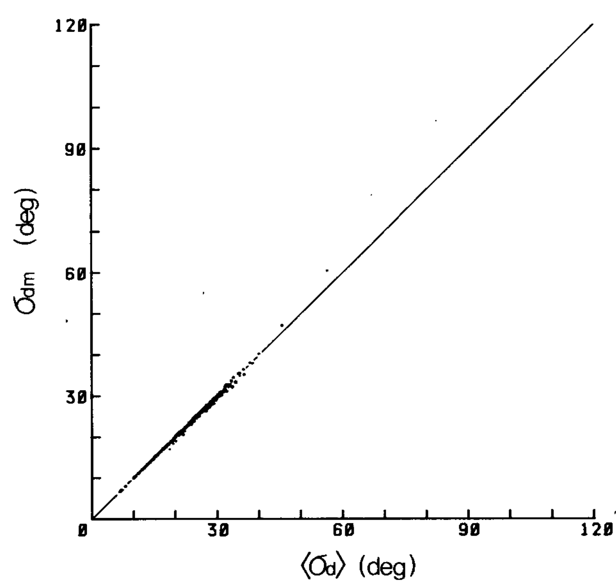

FIG. 13. A comparison of σ_{dm} with $\langle\sigma_d\rangle$.

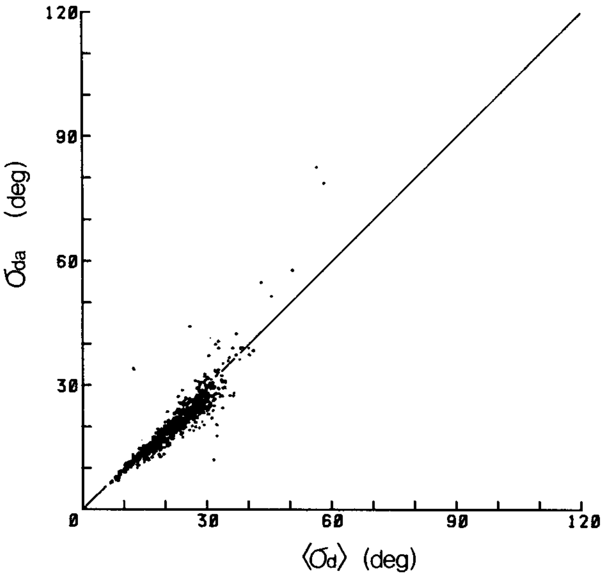

FIG. 14. A comparison of σ_{da} with $\langle\sigma_d\rangle$.

Kampé de Fériet–Frenkiel's Eq. (20) holds for field data, so we can get an estimate of σ_v from the observed values of **Vv** and **Va**. The ratio **Vv**/**Va** is roughly related to $\langle\sigma_d\rangle$ and the relation can be expressed as **Vv**/**Va** $= \exp(-0.5\sigma_d^2)$ for $\langle\sigma_d\rangle < 20$ deg. For the present data, the observed **Vv**/**Va** is closely represented by an empirical expression, **Vv**/**Va** $= (1 - a) + a \exp(-0.5\sigma_d^2)$ where $a = 0.86$, for $\sigma_d < 40$ deg.

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