NOTES

Wavelength Dependence of Aerosol Extinction Coefficient for Stratospheric Aerosols

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ABSTRACT

A simple empirical formula for the wavelength dependence of the aerosol extinction coefficient is proposed. The relationship between the constants in the formula and the variable parameter in the aerosol size distribution is explicitly expressed. Good agreement is found between the extinction coefficients calculated from the proposed formula and that calculated from Mie theory. The proposed expression is shown to be better than the Ångström formula commonly used by atmospheric scientists.

The wavelength dependence of aerosol extinction has been of interest to atmospheric scientists for quite a long time due to its radiative effect and its application to remote sensing techniques. In general, the exact relationship between aerosol extinction coefficient β_λ and wavelength λ cannot be expressed as a simple analytical function. Ångström (1964) proposed that β_λ varies with λ within the visible and near infrared range according to the empirical formula:

$$\beta_\lambda = c/\lambda^\alpha, \tag{1}$$

where the exponent α is called the Ångström coefficient or exponent, and c is a constant. For Rayleigh scattering, $\alpha = 4$, but for common aerosol particles, α is smaller than 4 and in some cases may even be negative (Tomasi et al., 1983). Obviously, there is no simple relation between α and the average size of aerosol particles. In addition, Lenoble and Pruvost (1983) have shown that α is actually not a constant, but a function of λ for a given aerosol size distribution.

The major factor that affects the relationship between β_λ and λ is the aerosol size distribution. In a recent paper by Nicholls (1984), the wavelength dependence of the aerosol extinction coefficient is approximated as a summation of a power series. In that paper, the anomalous diffraction approximation is used, and the aerosol size distribution is represented by the power law as first suggested by Junge:

$$n(r) = c_v/r^\nu, \tag{2}$$

where $n(r)$ is the number of aerosol particles per unit volume of air with radii between r and $r + dr$, and c_ν and ν are constants. Nicholls (1984) pointed out that even using the Van de Hulst anomalous diffraction assumption, the possible analytically closed forms of β_λ for integer values of ν are cumbersome involving the sine integral or cosine integral functions. Tomasi et al. (1983) have thoroughly discussed the relationship between the Ångström exponent α and the Junge exponent ν. They found that the proposed linear relationship

$$\alpha = \nu - 2$$

(Van de Hulst, 1957; Bullrich, 1964) is not valid in most cases.

There are many reasonable size-distribution types for the stratosphere. Russell et al. (1981), for example, used nine size-distribution types to describe the inner stratospheric layer. Pinnick et al. (1976) suggested using an exponential type of the form:

$$n(r) = A \exp\left[-\left(\frac{r}{r_0}\right)\right], \tag{3}$$

where A is a constant related to the aerosol number concentration and r_0 is a variable parameter related to the effective radius of the whole aerosol size spectrum. A value of $r_0 = 0.075\ \mu$m is recommended for nonvolcanic stratospheric aerosol particles. Rosen et al. (1978) even suggested that a stratospheric aerosol generated by a growth process may be better described by an exponential size distribution than by a power law size distribution.

The power law used by Russell et al. (1981) to describe stratospheric aerosols is a truncated power law:

$$n(r) = Ar^{-P} \tag{4}$$

where $P = 0$ for $r < 0.1\ \mu$m and $r > 0.5\ \mu$m. The recommended value of P is 4.0.

Another size-distribution type used by Russell et al. (1981) to describe the stratospheric aerosol is a modified gamma function of the form:

$$n(r) = Ar^a \exp(-br), \tag{5}$$

where b is a variable parameter, and $a = 1$ in the AFGL background aerosol model (Shettle and Fenn, 1979). A value of $b = 18\ \mu m^{-1}$ is recommended for the unperturbed aerosols in the stratosphere (McClatchey et al., 1980).

The most widely used size-distribution model for stratospheric aerosols is probably the lognormal expression:

$$n(r) = \frac{A}{r} \exp\left[-\frac{\ln^2(r/r_g)}{2\ln^2\sigma}\right] \tag{6}$$

where r_g is a variable parameter called mode radius and $\sigma = 1.86$ for background aerosols. The recommended value of r_g is 0.0725 μm.

The general equation describing the wavelength dependence of β_λ, the aerosol extinction coefficient is given by

$$\beta_\lambda = \int_{r_1}^{r_2} n(r) Q\left(\frac{2\pi r}{\lambda}, m\right) \pi r^2 dr, \tag{7}$$

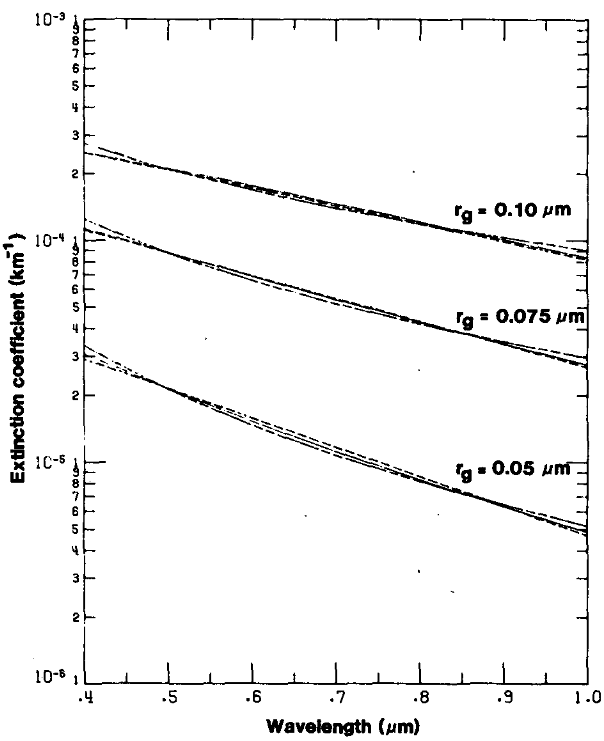

FIG. 1. Variation of aerosol extinction coefficient with wavelength for three values of r_g in the lognormal size distribution expression. Solid lines are results calculated from the Mie theory. Short dashed lines are results calculated from the empirical formula proposed in this paper. Short and long dashed lines are results calculated from the Ångström formula.

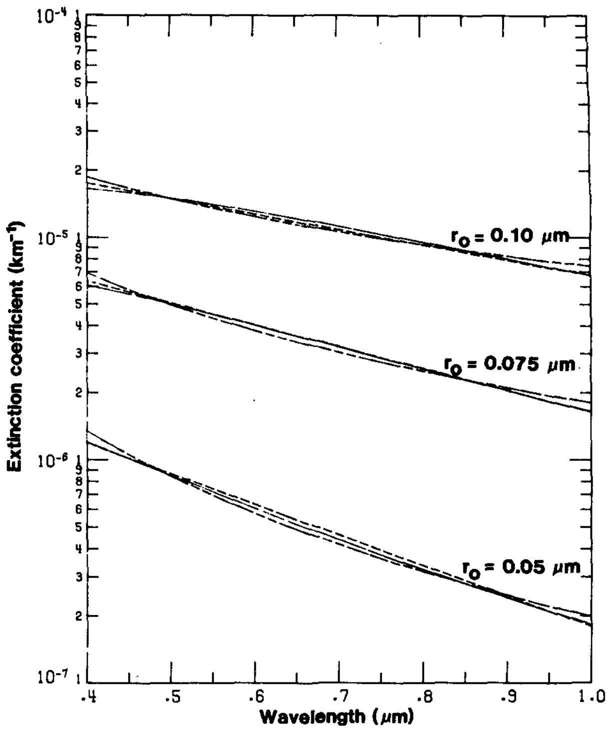

FIG. 2. As in Fig. 1 but for r_0 in the exponential size distribution expression.

where $Q(2\pi r/\lambda, m)$ is the extinction efficiency which is the function of the Mie parameter $X = 2\pi r/\lambda$, m is the index of refraction, and r_1 and r_2 are the lower and upper limits of the radii of aerosol particles under consideration, respectively. In this paper we use $m = 1.40$, $r_1 = 0.01\ \mu$m, and $r_2 = 1.0\ \mu$m. In general, the index of refraction m is a function of the wavelength, but in the short wavelengths ranging from 0.4 to 1.0 μm considered in this paper, the variation of m with λ is very small and we assume it is a constant.

Plots of β_λ as a function of λ for different variable parameters in the lognormal, exponential, truncated power, and modified gamma formulas are shown as solid lines in Figs. 1–4, respectively. Each line is generated by 31 data points with an increment of 0.02 μm in wavelength. The value of A is taken as 1 in all our calculations. It can be seen that in all cases $\ln\beta_\lambda$ varies almost linearly with λ. Hence it is proposed that

$$\beta_\lambda = \exp(C_0 - C_1\lambda), \tag{8}$$

where C_0 and C_1 are constants independent of λ but dependent on the variable parameter governing the aerosol size distribution.

In order to determine the value of C_0 and C_1, we need only to know $\beta_{\lambda 1}$, $\beta_{\lambda 2}$, the aerosol extinction coefficient at two reference wavelengths λ_1, and λ_2. The values of C_0 and C_1 are given by

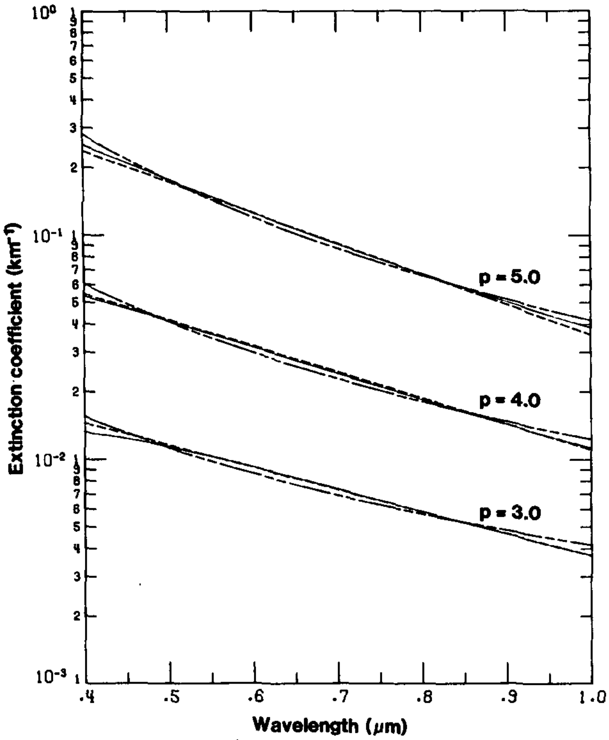

FIG. 3. Variation of aerosol extinction coefficient with wavelength for three values of P in the truncated power size distribution expression. Solid lines are results calculated from the Mie theory. Short dashed lines are results calculated from the empirical formula proposed in this paper. Short and long dashed lines are results calculated from the Ångström formula.

$$C_1 = \frac{1}{\lambda_2 - \lambda_1} \ln(\beta_{\lambda 1}/\beta_{\lambda 2}), \tag{9}$$

and

$$C_0 = \ln\beta_{\lambda_1} + c_1\lambda_1 . \tag{10}$$

We chose $\lambda_1 = 0.55$ and $\lambda_2 = 0.85$ μm as the reference wavelengths. The variations of $\ln(\beta_{\lambda_1}/\beta_{\lambda_2})$ and $\ln\beta_{\lambda_1}$ with the variable parameter r_g in the lognormal size distribution are shown in Figure 5. The solid line represents the calculated values of $\beta_{\lambda_1}/\beta_{\lambda_2}$ or β_{λ_2} from Mie theory, and the dashed line represents the best quadratic fit to the actual values of $\beta_{\lambda_1}/\beta_{\lambda_2}$ or β_{λ_2}. Since the solid and dashed lines are almost identical, both $\ln(\beta_{\lambda_1}/\beta_{\lambda_2})$ and $\ln\beta_{\lambda_1}$ can be represented by an expression of the form $a_0 + a_1 r_g + a_2 r_g^2$. It can be shown that for exponential, truncated power and modified gamma aerosol size distributions, the values of $\ln(\beta_{\lambda_1}/\beta_{\lambda_2})$ or β_{λ_1} can also be represented by a quadratic equation of the variable parameter in the size distribution. Consequently we have

$$C_0 = a_0 + a_1 p + a_2 p^2, \tag{11}$$

$$C_1 = b_0 + b_1 p + b_2 p^2, \tag{12}$$

where a_0, a_1, a_2 and b_0, b_1, b_2 are constants, and p is the variable parameter in the aerosol size distribution. More specifically, for the exponential size distribution, p is r_0 in Eq. (3); for the truncated power law size distribution, p is P in Eq. (4); for the modified gamma size distribution, p is b in Eq. (5); and for the lognormal size distribution, p is r_g in Eq. (6). The values of a_0, a_1, a_2 and b_0, b_1, b_2 for each of these size distributions are listed in Table 1. The values of β_λ calculated from the empirical formula (8) are plotted as short dashed lines in Figs. 1–4. It can be seen that in all cases, the differences between the solid line and dashed line are extremely small and are always less than 10%. In most cases, the differences are less than 2%.

It would be of interest to compare the β_λ values calculated from the Ångström formula and the Mie theory. For each set of β_λ values calculated from the Mie theory with a given aerosol size distribution, we have calculated the slope and intercept of the regression line in the $\ln\beta_\lambda$ vs $\ln\lambda$ plot. These values are the values of α and c in Eq. (1). The calculated values of β_λ from Eq. (1) are plotted as short and long dashed lines in Figs. 1–4. It can be seen that in general, at wavelengths close to 0.4 or 1.0 μm, the difference between β_λ (1),

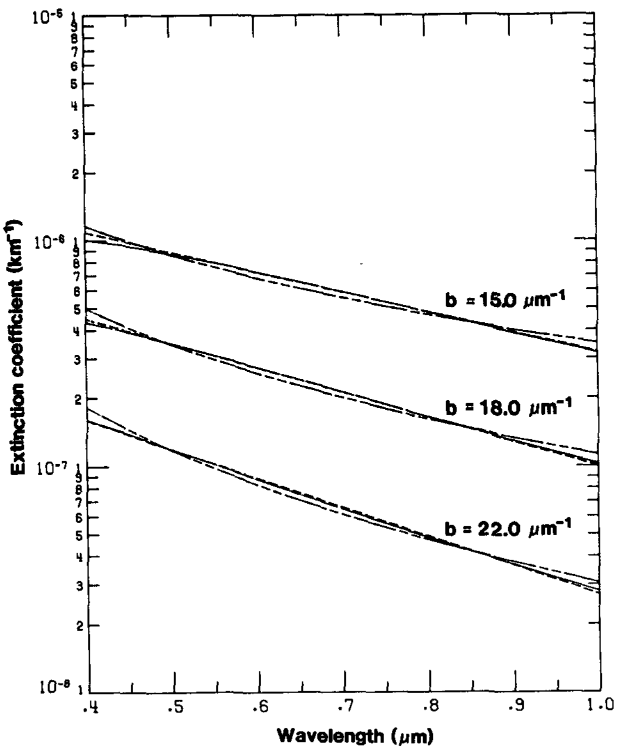

FIG. 4. Variation of aerosol extinction coefficient with wavelength for three values of b in the modified gamma size distribution expression. Solid lines are results calculated from the Mie theory. Short dashed lines are results calculated from the empirical formula proposed in this paper. Short and long dashed lines are results calculated from the Ångström formula.

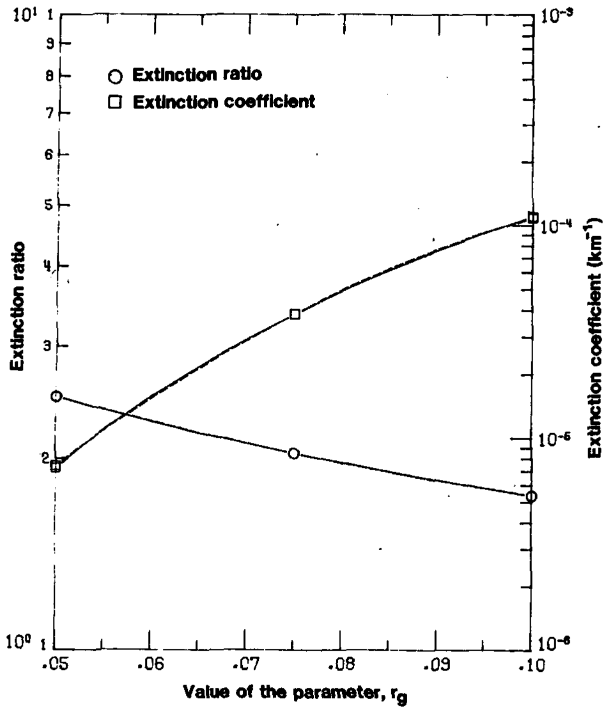

FIG. 5. Values of $\beta_{0.55}/\beta_{0.85}$ and $\beta_{0.85}$ as a function of r_g in the lognormal aerosol size distribution. Solid lines are the calculated result from the Mie theory, and the dashed lines are the quadratic best fit approximation to the calculated results.

the value of β_λ calculated from Eq. (1) and β_λ (7), the value of β_λ calculated from Eq. (7), is larger than the difference between β_λ (8), the value of β_λ calculated from Eq. (8) proposed in this paper and β_λ (7). In order to find out whether Eq. (1) or Eq. (8) is a better description of the wavelength dependence of β_λ, we have calculated the standard error of estimate for the Ångström expression defined as

$$E_A = \left[\frac{\sum(\beta_\lambda(1) - \beta_\lambda(7))^2}{n}\right]^{1/2}, \qquad (13)$$

where $n = 31$ in our calculation, and the standard error of estimate for the expression (8) defined as

$$E_Y = \left[\frac{\sum[\beta_\lambda(8) - \beta_\lambda(7)]^2}{n}\right]^{1/2}. \qquad (14)$$

The values of E_A and E_Y for different aerosol size distributions used in generating Figs. 1–4 are listed in Table 2. It can be seen that in all cases, E_Y is less than E_A, and in most cases E_Y is less than one-half of E_A. It should be noted that in each case the values of c and α used to calculate β_λ (1) are those that minimize the value E_A, but the values of C_0 and C_1 used to calculate β_λ (8) do not necessarily minimize the value E_Y since they are constrained by Eqs. (11) and (12). The fact

TABLE 1. Values of constants in Eqs. (11) and (12) for four types of aerosol size distributions.

Constants	Lognormal	Exponential	Truncated power	Modified gamma
a_0	−12.7	−16.8	−6.73	−8.63
a_1	88.5	111.0	0.875	−0.329
a_2	−366.0	−466.0	8.68×10^{-2}	2.89×10^{-3}
b_0	4.93	5.57	1.58	−1.82
b_1	−44.5	−57.0	0.109	0.344
b_2	139.0	171.0	4.07×10^{-2}	-5.77×10^{-3}

that E_Y is still less than E_A in all cases demonstrated that the empirical formula (8), which suggests that $\ln\beta_\lambda$ varies linearly with λ, is better than the Ångström formula, which suggests that $\ln\beta_\lambda$ varies linearly with $\ln\lambda$. In addition, we have shown that the constants C_0 and C_1 in Eq. (8) can be expressed as simple functions of the variable parameter in the aerosol size distribution, but the constants c and α in the Ångström formula are more complicated functions of the variable parameter in the aerosol size distribution.

It should be noted that if $A \neq 1$, we still have

$$\beta_\lambda = \exp(C_0' - C_1\lambda), \qquad (15)$$

where

$$C_0' = \ln A + C_0, \qquad (16)$$

or

$$C_0' = (\ln A + b_0) + b_1 p + b_2 p^2. \qquad (17)$$

By solving Eqs. (12) and (15), the variable parameter p and the constant related to aerosol number concentration A can be retrieved easily from the plotting of the wavelength dependence of β_λ obtained in the experiment.

Recently, a large amount of extinction coefficient data for stratospheric aerosols at two wavelengths has been collected by the SAGE satellite experiment (McCormick, 1983). Another data set of global aerosol extinction coefficients at four wavelengths ranging from

TABLE 2. Comparison of standard errors of estimate for the Ångström formula and the empirical expression proposed in this paper.

Size distribution	Variable parameter	E_A	E_Y
	0.05	7.5×10^{-7}	6.0×10^{-7}
Lognormal	0.075	3.2×10^{-6}	6.1×10^{-7}
	0.10	7.8×10^{-6}	2.6×10^{-6}
	0.05	3.8×10^{-7}	1.6×10^{-7}
Exponential	0.075	2.3×10^{-7}	7.6×10^{-8}
	0.10	6.9×10^{-7}	3.4×10^{-7}
	3.0	6.4×10^{-4}	3.4×10^{-4}
Truncated power	4.0	2.0×10^{-3}	4.7×10^{-4}
	5.0	7.3×10^{-3}	4.7×10^{-3}
	15.0	4.4×10^{-8}	2.2×10^{-8}
Modified gamma	18.0	1.7×10^{-8}	3.6×10^{-9}
	22.0	5.7×10^{-9}	7.9×10^{-10}

0.385 to 1.02 μm is being collected by the SAGE II satellite experiment (Mauldin et al., 1984). The simple empirical expression of β_λ suggested in this note may be useful in analyzing these extinction coefficient data as well as other aerosol extinction data sets collected by other experiments.

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