Attenuation of Microwaves by Spherical Hail

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(Manuscript received 29 July 1985, in final form 1 February 1986)

ABSTRACT

Calculations have been made of the radar backscattering and attenuation cross sections of dry and spongy ice spheres. One set of calculations was the cross sections of spheres with diameters exponentially distributed. As expected, attenuation cross sections are greater at a wavelength of 3.21 cm than at 5.05 and 10.0 cm. Calculations were also made of attenuation by monodisperse distributions of spheres composed of spongy ice and having diameters as large as about 8 cm. Attenuation of 3-cm radiation by dry ice and spongy ice spheres can be very large. At most diameters and water volume fractions, the one-way attenuation of 10-cm radiation by monodisperse spheres, in concentrations giving a radar reflectivity of 60 dBZ, is negligibly small (i.e., <0.1 dB km^{-1}), but at a few diameters and water fractions, attenuation can be substantially larger. Although in most circumstances attenuation increases as wavelength decreases, there are exceptions at some diameters and water volume fractions. These calculations may explain observations that C-band attenuation in hailstorms is not always larger than S-band attenuation.

1. Introduction

Although significant progress has been made in recent years in the development of techniques for discriminating between the radar signals of rain (or snow) and hail, the problem cannot be regarded as solved. Various hail detection techniques have been investigated. McCormick et al. (1979), Barge and Humphries (1980), and others have used polarization diversity measurements; Bringi et al. (1984) have proposed that large hail produces high equivalent radar reflectivities Z[1] and very small differential reflectivities Z_{DR}. A hail detection procedure that has received considerable attention involves comparison of the backscattered signals at two wavelengths, usually S-band (10 cm) and X-band (3 cm) or C-band (5 cm). Over the last few years, Rinehart and Tuttle (1982) and Tuttle and Rinehart (1983) have made important contributions to this dual-wavelength technique of hail detection. They showed how critical it is to have matched beams or to accurately correct for mismatches, and they presented a procedure for taking attenuation into account when calculating a hail signal from measurements of the dual-wavelength ratio (10 $\log Z_s/Z_x$).

Tuttle and Rinehart (1983) assumed that attenuation was negligible at the 10.7-cm wavelength that they used and that in regions of a cloud without hail, attenuation can be obtained from an equation of the form

$$A = 0.00048 Z_s^{0.6} \tag{1}$$

where A is the two-way attenuation in dB km^{-1}, Z_s the S-band reflectivity in mm^6 m^{-3}, and the coefficient and exponent depend on radar wavelength and meteorological circumstances. The values in Eq. (1) are averages obtained by Tuttle and Rinehart (1983) from an analysis of observations made on two days. Similar expressions have been derived by others. (See Battan, 1973, p. 73.)

Equation (1) can be used to calculate X-band attenuation from measured S-band reflectivity at each range along any ray through the storm. This procedure appears to be satisfactory in regions of rain, but, as noted by Tuttle and Rinehart (1983), it does not adequately account for attenuation in regions containing moderate to large hail sometimes mixed with rain. They proposed a procedure to deal with such circumstances, but it acts to prevent an overestimate of hail attenuation rather than to make an estimate of its actual value. Perhaps this is the best that can be done at this time, but it seems worthwhile to estimate how much attenuation hail is likely to cause.

2. Observations of attenuation in hailstorms

Empirical estimates of attenuation by hail have yielded conflicting results (McCormick, 1970; Wilson,

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[1] It should be understood throughout this article that the term radar reflectivity, designated by Z, refers to the equivalent radar reflectivity, normally written as Z_e.

1978). Wilson made such estimates by comparing simultaneous S-band and C-band observations of storms having radar reflectivities exceeding 60 dBZ and therefore likely to contain hail. Downrange of one such high S-band reflectivity area, the C-band reflectivity was so low as to indicate two-way attenuation values exceeding 5 dB km^{-1} within the storm center. On the other hand, a second region of high S-band reflectivity within the same storm did not indicate C-band attenuation. Wilson speculated that perhaps the heavily attenuating cell contained wet hail while the nonattenuating cell contained dry hail. In two other storms, he found that C-band signals in the shadow of regions of high S-band reflectivity ranged from 15 dB greater to 10 dB less than the maximum S-band reflectivities.

Rinehart (1984) analyzed simultaneous S-band and X-band observations of a storm that had a maximum S-band reflectivity of 77 dBZ and that produced hail which, at the ground, had maximum diameters of about 10 cm. It was estimated from dual-wavelength comparisons that the storm contained hailstones having diameters of 8 cm or larger which were monodisperse or nearly so. Rinehart estimated that, at low elevation angles, S-band signals were attenuated by "as much as 5 to 6 dB," while the X-band attenuation was much larger.

3. Calculations of attenuation by exponentially distributed spherical hail

The backscattering and attenuation properties of hailstones depend to varying degrees on the following factors:

a) *Hailstone shape.* When making calculations of the scattering properties of hailstones, it is usually assumed that they are smooth spheres, but it is well known that hailstones seldom have these properties. Often hailstones have the shape of rough cones, or ellipsoids, or are "shapeless" masses of ice.

b) *Orientation of nonspherical hailstones.* In general, raindrops tend to be oblate and fall preferentially with their axes of symmetry nearly vertical. But how do large nonsymmetrical hailstones tend to be oriented, and if they oscillate or rotate, what are the characteristics of these motions?

c) *The size distribution of the hailstones.* Most often it is taken to be exponential and limited by minimum and maximum diameters, but observations reveal that sometimes showers of large hail are nearly monodisperse (Carte and Held, 1978).

d) *Composition of hailstones and relevant refractive indices.* Hailstones, depending on growth and environmental conditions, have a variety of compositions: dry ice, wet ice, spongy ice of various mixtures of water and ice, a solid ice core coated with spongy ice, layers of ice of differing densities, etc. Each of these physical states is associated with a different refractive index. The problem is further complicated by uncertainties—in the case of spongy ice—about the correct procedure for calculating the refractive index (Bohren and Battan, 1982).

Most published calculations of the backscattering and attenuation cross sections of "hailstones" have been made assuming them to be smooth spheres that are monodisperse or exponentially distributed in size and have refractive indices that can be specified accurately.

As shown by Atlas and Ludlam (1961) and Battan (1971), attenuation by small, dry ice spheres is small, a result to be expected because of the low absorption coefficient of ice. It seems reasonable to expect a similar result for small, nonspherical dry hailstones. On the other hand, a thin layer of water or spongy ice substantially increases attenuation by ice spheres. The attenuations in Table 1 (a condensed version of a table in Battan, 1971) were calculated assuming that ice spheres were distributed in the form similar to one given by Douglas (1964):

$$N_D = 9.0e^{-3.09D}, \tag{2}$$

where the diameter D is in centimeters and N_D is the number per cubic meter per millimeter. The liquid water contents represented by the distributions shown in Table 1 are 1.0, 2.3 and 2.7 gm m^{-3} for $D_{\max}$ equal to 0.97, 1.93, and 2.89, respectively.

At a wavelength of 3.2 cm, wet-hail spectra with maximum diameters of about 2 cm or greater yielded calculated attenuations exceeding 3 dB km^{-1}. The calculated ice-sphere reflectivities for the 3.2-cm attenuations exceeding 3.0 dB km^{-1} in Table 1 ranged from 54.3 to 63 dBZ. (See Battan, 1971.)

In this paper we present results of calculations, by means of the well-known Mie equations, of some scat-

TABLE 1. One-way attenuation (dB km^{-1}) of spherical dry and wet ice spheres at a temperature of 0°C (from Battan, 1971).

	λ = 3.2 cm			λ = 5.5 cm			λ = 10.0 cm		
	Water-shell thickness (cm)								
$D_{\max}$ (cm)	Dry	0.01	0.10	Dry	0.01	0.10	Dry	0.01	0.10
0.97	0.12	0.91	1.50	0.02	0.19	0.94	0.002	0.05	0.08
1.93	1.21	3.01	3.49	0.18	0.79	2.30	0.017	0.15	0.89
2.89	1.66	3.46	3.79	0.33	1.12	2.60	0.034	0.19	1.18

TABLE 2. One-way attenuation (dB km^{-1}) and radar reflectivity (dBZ) (in parentheses) of uniform spongy ice spheres distributed exponentially in diameter to the indicated maximum diameters.

	λ = 3.21 cm		λ = 5.05 cm		λ = 10.0 cm	
	Water volume fraction					
D_{max} (cm)	0.1	0.5	0.1	0.5	0.1	0.5
0.97	0.88	1.83	0.22	0.65	0.05	0.05
0.97	(54)	(60)	(55)	(58)	(56)	(58)
1.93	3.38	4.00	1.52	2.58	0.18	0.60
1.93	(57)	(62)	(63)	(69)	(67)	(70)
2.89	3.72	4.32	1.96	2.91	0.28	0.88
2.89	(58)	(62)	(64)	(69)	(69)	(74)

tering properties of spheres composed wholly or partly of spongy ice, i.e., a mixture of water and ice at a temperature of 0°C. The refractive indices of the spongy ice were obtained from a theory formulated by Maxwell Garnett (1904) and extended by Bohren and Battan (1982) to the case of spongy ice composed of small, randomly oriented ice ellipsoids in a water matrix. Best agreement between calculations and experimental data was found when spongy ice was assumed to be composed of small ice inclusions suspended in a water matrix.

In the new set of calculations, the same sphere-size distribution was used as in an earlier paper (Battan, 1971), in order to examine the differences in attenuation between spongy ice spheres and those composed of wet and dry ice. Clearly the attenuation depends on the sizes and composition of the ice spheres and on the wavelength. Some typical values were selected in order to obtain some notion of the values of attenuation likely to be caused by spongy ice, spherical hailstones.

Table 2 gives the one-way attenuations and radar reflectivities (in parentheses) of spheres composed entirely of spongy ice having the indicated volume fractions of water. Table 3 gives the attenuations and radar reflectivities of spheres with solid ice cores (density 920 kg m^{-3}) having spongy ice coatings 0.1 or 0.2 cm thick and the indicated water contents. A comparison of the data in Table 1, 2 and 3 shows that the attenuation by spheres composed of spongy ice and those composed of ice cores coated with a layer of spongy ice cause attenuations not very much different from those of wet ice spheres. It is seen, for example, that for particles distributed exponentially in size with maximum diameters of 2.89 cm, attenuations at a wavelength of 3.21 cm are 2.9–4.4 dB km^{-1}; at a wavelength of 5.05 cm, attenuations are 1–3 dB km^{-1}; and at a wavelength of 10 cm, attenuations are 0.1–1.2 dB km^{-1}. As expected, attenuation coefficients increase as the water content of the ice sphere increases. Most of the attenuation is caused by spheres having diameters less than about 2 cm.

The size distributions of ice spheres shown in Tables 2 and 3 have the expected reflectivities, generally 50 to 70 dBZ. The higher the water content, the higher the reflectivities.

4. Attenuation by large, monodisperse ice spheres

It has been observed that, on occasion, hail showers are composed of hailstones whose size distributions are more nearly monodisperse than exponential. Calculations were made of the attenuation that would be caused by monodisperse, spongy ice spheres having number densities that yield an effective radar reflectivity of 60 dBZ (10^6 mm^6 m^{-3}). The number densities were obtained from calculated values of backscattering cross sections as functions of wavelength, diameter and refractive index.

Table 4 gives one-way attenuations for monodisperse distributions of spongy ice spheres where the ice (as

TABLE 3. One-way attenuation (dB km^{-1}) and radar reflectivities (dBZ) (in parentheses) of ice spheres covered with layers of spongy ice having water volume fractions of 0.1 and 0.5.

	λ = 3.21 cm				λ = 5.05 cm				λ = 10.0 cm			
	Spongy ice thickness (cm)											
	0.1		0.2		0.1		0.2		0.1		0.2	
	Water volume fraction											
D_{max} (cm)	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5
0.97	0.59	1.61	0.79	1.69	0.17	0.41	0.21	0.59	0.04	0.05	0.05	0.05
0.97	(52)	(58)	(53)	(60)	(54)	(56)	(55)	(57)	(55)	(57)	(55)	(58)
1.93	2.44	3.88	3.03	4.04	0.71	2.33	1.03	2.49	0.11	0.23	0.14	0.35
1.93	(56)	(62)	(57)	(63)	(61)	(67)	(61)	(70)	(64)	(66)	(65)	(66)
2.89	2.92	4.22	3.49	4.37	1.04	2.72	1.43	2.83	0.14	0.36	0.19	0.66
2.89	(57)	(63)	(58)	(63)	(61)	(68)	(63)	(70)	(66)	(67)	(67)	(71)

TABLE 4. One-way attenuation (dB km^{-1}) by monodisperse spongy ice spheres at concentrations yielding an effective radar reflectivity of 60 dBZ.

Diameter (cm)	λ = 3.21 cm			λ = 5.05 cm			λ = 10.0 cm		
	Water volume fraction								
	0.0	0.25	0.50	0.0	0.25	0.50	0.0	0.25	0.50
1.1	1.8	2.4	2.0	0.17	0.74	0.28	0.01	0.04	0.06
2.1	5.8	6.0	4.1	0.79	0.93	0.51	0.01	0.07	0.01
3.1	1.5	11.1	8.5	0.95	1.69	1.01	0.02	0.02	0.02
4.1	0.70	14.7	10.5	0.66	1.43	0.92	0.04	0.08	0.03
5.1	0.22	11.5	7.4	0.32	1.20	0.88	0.22	0.15	0.22
6.1	0.29	8.7	5.6	0.07	1.73	1.13	0.06	0.16	0.08
7.1	0.68	7.8	5.2	0.03	1.13	0.82	0.08	0.03	0.03
8.1	0.68	8.1	5.7	0.03	1.76	1.20	0.04	0.12	0.05

before) is assumed to be composed of ice inclusions in a water matrix. Attenuations generally are greatest at the shortest wavelength and at the water volume fraction of 0.25, but there are exceptions to these generalizations. Note, for example, that at a diameter of 5.1 cm and water volume fraction (f) equal to zero, attenuation at a wavelength of 5.05 cm exceeds that at 3.21 cm. Because of the complicated nature of the scattering of electromagnetic waves by particles having diameters of the order of, or greater than, the wavelength, it is expected that, at some diameters other than the ones shown in the table, attenuations can differ substantially from those of nearly the same size and water contents.

Calculations were made of attenuation by monodisperse spheres composed of a solid ice core surrounded by shells of spongy ice (ice inclusions in a water matrix) at number densities yielding an effective reflectivity of 60 dBZ. Tables 5, 6 and 7 show the one-way attenuations of spheres coated with layers of spongy ice 0.10 cm thick and fractional water volumes from 0 to 0.75.

In general, the greater the wavelength the smaller the attenuation, but there are some exceptions. Note that at a diameter of 4.1 cm and $f = 0.10$, attenuation at a wavelength of 10 cm is 0.47 dB km^{-1}, but at 5.05 cm it is only 0.28 dB km^{-1}. At a diameter of 4.3 cm and $f = 0.10$ (not shown), 10-cm attenuation is a very large 14.4 dB km^{-1}, while at wavelengths of 3.21 cm and 5.05 cm, it is 0.66 dB km^{-1} and 0.25 dB km^{-1}, respectively.

It is evident from Tables 5 and 6 that spheres 2.1 cm in diameter which are coated with a thin layer of spongy ice can produce extremely high attenuations. These results again show the highly complex nature of Mie scattering and indicate that, at diameters intermediate between those shown in the tables, there could be unexpectedly high or low attenuations.

Tables 5, 6 and 7 also show that changes of attenuation with water content, in the spongy ice shell, depend on the sphere diameter and wavelength.

The tabulated quantities show that, in most circumstances, the attenuation of 10-cm radiation by monodisperse ice spheres yielding a reflectivity of 60 dBZ is negligibly small and about one order of magnitude smaller than the attenuation at 5.05 cm, but there are exceptions. These data indicate that it is possible under some—presumably rare—circumstances for 10-cm attenuation to be nearly the same as or greater than the attenuation at 3.21 and 5.05 cm. This result may account for the variable patterns of reflectivity reported by Wilson (1978).

For a monodisperse distribution, the number density of spheres is proportional to the effective radar reflectivity Z. Therefore, attenuations associated with values of Z different from 60 dBZ can easily be obtained by multiplying the values in Table 5–7 by the ratio of any value of Z divided by 10^6 mm^6 m^{-3}. As a consequence, if the reflectivity of monodisperse ice spheres were 70 dBZ, the attenuations would be 10 times the values in Tables 5–7.

5. Conclusions

Because of the effects of nonsphericity, orientation, rotation, roughness and composition of hailstones, it is not possible to be certain about the attenuation pro-

TABLE 5. One-way attenuation of 3.21-cm radiation (dB km^{-1}) by monodisperse spheres composed of ice cores surrounded by a shell of spongy ice 0.10 cm thick. Number of concentrations are those that yield an effective radar reflectivity of 60 dBZ.

Diameter (cm)	Water volume fraction				
	0.0	0.10	0.25	0.50	0.75
1.1	1.79	6.41	3.78	1.52	1.21
2.1	5.82	54.90	5.75	2.88	2.34
3.1	1.54	3.74	2.54	2.11	2.09
4.1	0.70	1.26	1.58	2.02	2.48
5.1	0.22	1.01	1.49	1.50	1.79
6.1	0.29	1.06	1.44	1.64	2.03
7.1	0.68	1.73	1.38	1.80	3.13
8.1	0.68	0.93	0.98	2.28	12.44

duced by hailstones. This article deals only with the attenuation of ice spheres that are composed of dry ice and spongy ice and that are exponential or monodisperse in diameter distributions.

In the case of spheres distributed exponentially in diameter, attenuations by spheres composed of spongy ice having reasonable water contents and spheres composed of ice cores coated with layers of spongy ice are not very different from the attenuations of water-coated ice spheres. Attenuation decreases with increasing wavelength and increases with increasing water contents.

When monodisperse spongy ice spheres exist in sufficient number densities to yield an effective reflectivity of 60 dBZ, attenuation generally decreases with increasing wavelength. Monodisperse spheres composed entirely of spongy ice, at the diameters shown in Table 4, and water volume fractions of 0.0, 0.25 and 0.50 cause one-way attenuations ranging from about 1 to 15 dB km^{-1}, 0.03 to 1.8 dB km^{-1}, and 0.01 to 0.22 dB km^{-1} at wavelengths of 3.21, 5.05 and 10.0 cm, respectively. Attenuation varies in a somewhat unpredictable fashion as sphere diameter varies. Attenuations at a wavelength of 10 cm are negligibly small when reflectivity is 60 dBZ, but at reflectivities 10 times greater (70 dBZ), one-way 10-cm attenuation can, in the case of large (i.e., diameter exceeding 5 cm) monodisperse, spongy ice spheres, be as large as 1–2 dB km^{-1}.

At most diameters, attenuation by monodisperse spheres, composed of ice cores surrounded by shells of spongy ice 0.10 cm thick and yielding a radar reflectivity of 60 dBZ, decreases as the wavelength increases from 3.21 to 10.0 cm. One-way attenuation at a wavelength of 10 cm usually is less than 0.1 dB km^{-1}, i.e., negligibly small, but at some sphere diameters and water contents, attenuation can be much larger. Depending on diameters and water content, 10-cm attenuation, although usually less than 5.05-cm attenuation, may at some diameters and water contents be about the same as, or greater than, 5.05-cm attenuation. For example, at diameters of 4.1 cm, 4.3 cm and 7.1 cm and water volume fraction $f = 0.10$, attenuation of 10-cm radiation is greater than that of 5.05-cm radiation. These results may account for Wilson's (1978) findings that downrange of one region of high radar reflectivity, C-band two-way attenuation was 5 dB km^{-1} greater than S-band attenuation, while downrange of another high-reflectivity region there was no discernible C-band attenuation.

TABLE 6. One-way attenuation of 5.05-cm radiation (dB km^{-1}) by monodisperse spheres composed of ice cores surrounded by a shell of spongy ice 0.10 cm thick. Number concentrations are those that yield an effective radar reflectivity of 60 dBZ.

Diameter (cm)	Water volume fraction				
	0.0	0.10	0.25	0.50	0.75
1.1	0.17	0.40	0.67	0.99	0.32
2.1	0.79	7.57	0.47	0.29	0.29
3.1	0.95	0.15	1.84	1.12	0.68
4.1	0.66	0.28	0.21	0.28	0.41
5.1	0.32	0.24	0.14	0.14	0.17
6.1	0.07	0.10	0.18	0.60	0.76
7.1	0.03	0.04	0.07	0.20	1.08
8.1	0.03	0.07	0.11	0.15	0.20

TABLE 7. One-way attenuation of 10-cm radiation (dB km^{-1}) by monodisperse spheres composed of ice cores surrounded by a shell of spongy ice 0.10 cm thick. Number concentrations are those that yield an effective radar reflectivity of 60 dBZ.

Diameter (cm)	Water volume fraction				
	0.0	0.10	0.25	0.50	0.75
1.1	0.011	0.048	0.050	0.054	0.081
2.1	0.011	0.019	0.029	0.075	0.081
3.1	0.015	0.027	0.093	0.044	0.012
4.1	0.045	0.467	0.081	0.016	0.018
5.1	0.221	0.085	0.032	0.034	0.119
6.1	0.063	0.066	0.060	0.055	0.053
7.1	0.082	0.081	0.022	0.012	0.013
8.1	0.043	0.022	0.010	0.014	0.018

Acknowledgments. This material is based upon work supported by the National Science Foundation under Grants ATM-8302225 and ATM-8303629.

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