Methods for Estimating the Mean and the Standard Deviation of Wind Direction

YUKIHIRO MORI

Earth Science Laboratory, Kagawa University, Kagawa 760 Japan

3 January 1987 and 13 March 1987

ABSTRACT

In a recent paper, the author reviewed methods for estimating the mean and the standard deviation of wind direction. Some comments on this subject are now added. If large sporadic eddy fluctuations do not occur in a sampling duration, we can exactly calculate an arithmetic mean and standard deviation of wind direction by using an approach suggested by Mitsuta. This is via a single-pass procedure without considering the discontinuity in wind direction scale. By using the Mitsuta approach it is found that Mardia's method provides a good estimator of the standard deviation of wind direction even in the case when the range of fluctuations of wind direction is wide.

1. Introduction

Calculating statistical quantities of wind direction has been a long-standing problem because of the discontinuity in wind direction scale at 0° and 360°. In the last 10 yr, microprocessor-controlled data acquisition systems have come into standard use in wind observations. Using these systems, new approaches for estimating the standard deviation of wind direction, σ_d, via a single-pass procedure have been proposed (Verrall and Williams, 1982; Ackermann, 1983; Yamartino, 1984; Nelson, 1984). Fisher (1983) has noted that the circular standard deviation of direction defined by Mardia (1972) can be also calculated via a single-pass procedure.

The author summarized and evaluated the proposed single-pass estimators of σ_d (Mori, 1986). The purpose of this note is to add some comments on methods for estimating the mean and the standard deviation of wind direction.

2. Additional comments

Turner (1986) noted that Irwin (1980) presented a method for calculating σ_d straightforwardly via a two-pass procedure. In Irwin's method, the calculation of σ_d is made by using a mean of wind direction defined by Mardia (1972), which will be described later. Thus, Irwin's σ_d is not exactly equal to the arithmetically calculated σ_d.

The mean and the standard deviation of wind direction can be calculated from the wind rose. Mintz and Dean (1952) obtained climatological means of the wind direction by the vector addition of the branches (more exactly, the lengths of petals) of the wind rose. This method for calculating the mean of wind direction, Dr, can be applied for short-period wind data. On the other hand, Mardia (1972) treated each wind direction as a vector of unit length and defined the resultant vector as the mean of wind direction, Dm. The difference in the definitions of Dr and Dm lies in the order of taking summations, sines and cosines of wind direction angles. The statistical meaning of Dr is then the same as that of Dm; Dr is a rough estimate of Dm. Suppose that the wind rose is calculated for K sectors of the compass. If $K \rightarrow \infty$, then Dr $\rightarrow$ Dm.

The standard deviation of wind direction σ_{dr} can also be calculated from the wind rose (Nelson, 1984). Then, Dr and σ_{dr} can be computed via a single-pass procedure, but this procedure requires a relatively large amount of memory in a microprocessor to store summations.

3. The Mitsuta approach

Previously proposed methods, either via a single- or a two-pass procedure, do not provide an arithmetic mean or an arithmetic standard deviation of wind direction. Mitsuta (personal communication, 1985) suggested a method for continuously estimating wind directions (for details see Mori, 1986). The Mitsuta approach is as follows:

We define the $(i + 1)$th value of direction as

$$D_{i+1} = D_i + \delta_i, \tag{1}$$

where δ_i is the amount of the variation in direction for a time interval of the i-th and the $(i + 1)$th observations and is defined as positive for clockwise rotation and negative for counterclockwise. If we define all the observation data in this way, even though some data exceed the discontinuity point of scale, they all are represented continuously and the arithmetic mean and standard deviation can be calculated straightforwardly via a single-pass procedure. In a digital data-acquisition system, a time interval of instantaneous sampling, Δt, should be chosen which is short enough that the following relation holds:

$$|\delta_i| < 180°. \quad (2)$$

In Mori (1986), this approach has been shown, but not evaluated. In practical application of this approach, a problem arises as to how to set the time interval. Setting of Δt depends on the characteristics of the dynamic response of an anemometer to wind fluctuations, the wind itself, and on the actual frequency content of wind direction fluctuations.

In order to get a criterion of setting Δt, observations were made at the same site by using the same observation system as shown in Mori (1986). The observation site is located in an urban area and the surroundings are rough. The wind sensor used was a two-dimensional sonic anemometer; the frequency response is 10 Hz. In observations, it is difficult to check perfectly whether condition (2) is satisfied or not. The frequency distribution of $|\delta_i|$ was investigated and the observational condition was set such that observed values of $|\delta_i|$ were made as small as possible.

In the present study, a sampling duration was 10 min and an instantaneous value was sampled every 1 sec; one record was 10 min long with 600 points. Observations were made first for the x- and y-components of the wind, and then scalar wind speeds and directions were calculated in a microprocessor.

When outputs of the wind speed components from the sonic anemometer were sampled directly, it was found that many observations included data of $|\delta_i|$ beyond about 160°. It is clear that the shorter the sampling interval we adopt, the smaller the value of each $|\delta_i|$ becomes. Therefore, if we adopt a very short sampling interval, it may be possible to satisfy (2). However if we do so, we cannot avoid the problem that the sampling of D_i follows short-time scale eddy rotations; this problem will be discussed later. In order to remove short-time period wind fluctuations, the outputs of the wind components were then passed through a low-pass filter which consists of a CR-circuit with a time constant of 1 s, and further, through a three-term Hamming numerical filter.

Under these conditions, observations were made. The records were obtained every 30 min. An observation period was 11 days, from 17 to 27 November 1986. The total number of observations were 451. The frequency distribution of total data of $|\delta_i|$ was estimated for eight sectors, which were constructed by dividing 180° into eight equal parts. The samples of observation were classified according to maximum values of $|\delta_i|$ for each observation. The results are listed in Table 1. The number of the observed $|\delta_i|$ beyond 90° is only 0.06%. On the other hand, 10% of the samples of observation include data of $|\delta_i|$ beyond 135°. This means that these samples include a small number of large $|\delta_i|$ observations which are caused by large sporadic eddy disturbances.

When the sampling of D_i follows few sporadic rotations, a 10 min mean of wind direction is significantly affected and shows a value which is quite different from that expected. Such cases were observed even when wind speeds were relatively high. How we consider such a case is a problem of the definition. When we consider a mean wind direction at some point only, we have no reason to reject such a case. However, when we intend to estimate a mean wind direction at some point as a representative of that in the relatively large surrounding area, it seems to be preferable that the sampling of D_i does not follow sporadic rotations.

It is thought that most of the samples of observation satisfy (2) and do not include sporadic rotations. When some samples include few sporadic rotations, it is possible to check such cases.

In these observations, arithmetic standard deviations estimated by the Mitsuta approach $\langle\sigma_d\rangle$ and those by Mardia's method σ_{dm} were also calculated. A comparison between $\langle\sigma_d\rangle$ and σ_{dm} is shown in Fig. 1. In this figure, when the values of $\langle\sigma_d\rangle$ exceed 120° or the samples of observation include the maximum value of $|\delta_i|$ beyond 115.5°, relationships are not plotted. Thus, the number of observations plotted were reduced to 375. This figure shows that the σ_{dm} agrees well with the $\langle\sigma_d\rangle$ in the range of $\langle\sigma_d\rangle < 60°$. In the range of large $\langle\sigma_d\rangle$ beyond about 60°, the relationships are rather scattered.

4. Concluding remarks

In microprocessor-controlled data acquisition systems, when instantaneous fluctuations of wind direc-

TABLE 1. Frequency distribution of observed $|\delta_i|$ for the total 451 samples (the first row) and of the samples classified according to maximum values of $|\delta_i|$ for each sample.

	Sectors*							
	1	2	3	4	5	6	7	8
$\lvert\delta_i\rvert$	265 950	3947	437	124	56	34	24	28
samples	128	138	62	33	29	18	18	25

* sector 1: 0°–22.5°; sector 2: 22.5°–45°; sector 3: 45°–67.5°; sector 4: 67.5–90°; sector 5: 90°–112.5°; sector 6: 112.5°–135°; sector 7: 135°–157.5°; sector 8: 157.5°–180°.

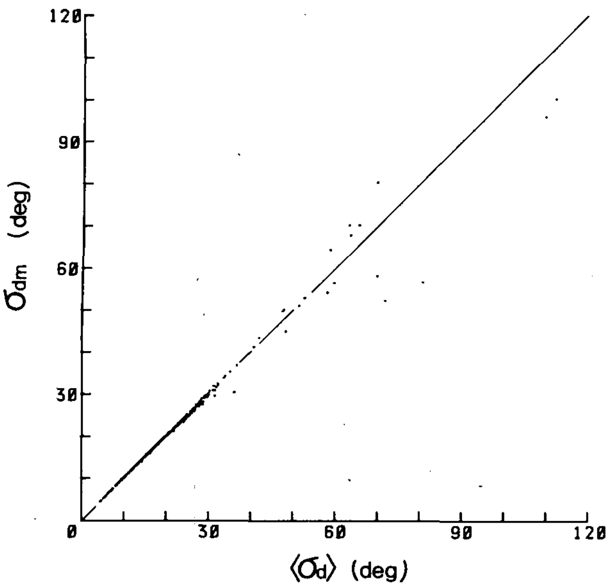

FIG. 1. A comparison between $\langle \sigma_d \rangle$ calculated by the Mitsuta approach and Mardia's estimator σ_{dm}.

tion in a time interval of sampling are relatively small, we can calculate exactly the arithmetic mean and standard deviation of wind direction by using the Mitsuta approach. This is via a single-pass procedure without considering the discontinuity in wind direction scale. However, when large sporadic eddy fluctuations occur, observations sometimes follow small-scale eddy rotations, and means and standard deviations of wind direction are different from those expected. So, in some conditions, it is necessary to reject the data.

Using the Mitsuta approach, we can obtain arithmetic standard deviations of wind direction even when the range of fluctuations of wind direction is wide. Mardia's method for estimating the standard deviation of wind direction has been evaluated in the range of large $\langle \sigma_d \rangle$. It is found that this method provides a good estimator of σ_d even in the case when the range of fluctuations of wind direction is wide.

Acknowledgments. I would like to express my thanks to Professor Y. Mitsuta, Kyoto University, for his valuable suggestions. I am also indebted to the reviewers for their valuable comments. This study was supported in part by a grant from the Ministry of Education, Science and Culture of Japan.

REFERENCES

Ackermann, G. R., 1983: Means and standard deviations of horizontal wind components. *J. Climate Appl. Meteor.*, **22**, 959–961.

Fisher, N., 1983: Comment on "A method for estimating the standard deviation of wind directions." *J. Climate Appl. Meteor.*, **22**, 1971.

Irwin, J. S., 1980: Dispersion Estimate Suggestion #9: Processing of wind data. (document for internal distribution) Environmental Application Branch, Meteorology and Assessment Division, ESRL, U.S. Environmental Protection Agency.

Mardia, K. V., 1972: *Statistics of Directional Data.* Academic Press, 357 pp.

Mintz, Y., and G. Dean, 1952: The observed mean field of motion of the atmosphere, Geophysical Research Papers, No. 17, Air Force Cambridge Research Center, 1–65.

Mori, Y., 1986: Evaluation of several "single-pass" estimators of the mean and the standard deviation of wind direction. *J. Climate Appl. Meteor.*, **25**, 1387–1397.

Nelson, E. W., 1984: A simple and accurate method for calculation of the standard deviation of the horizontal wind direction. *J. Air Pollut. Control Assoc.*, **34**, 1139–1140.

Turner, D. B., 1986: Comparison of three methods for calculating the standard deviation of the wind direction. *J. Climate Appl. Meteor.*, **25**, 703–707.

Verrall, K. A., and R. L. Williams, 1982: A method for estimating the standard deviation of wind direction. *J. Appl. Meteor.*, **21**, 1922–1925.

Yamartino, R. J., 1984: A comparison of several "single-pass" estimators of the standard deviation of wind direction. *J. Climate Appl. Meteor.*, **23**, 1362–1366.