The Sampling Variations of Hailstone Size Distributions

RAYMOND K. W. WONG, NORMAN CHIDAMBARAM, LAWRENCE CHENG AND MARIANNE ENGLISH

Resource Technologies Department, Natural Resources Division, Alberta Research Council, Edmonton, Alberta, Canada

(Manuscript received 8 May 1987, in final form 31 August 1987)

ABSTRACT

The use of a shifted gamma size distribution for hailstone samples is proposed. This is shown to provide a better fit than the usual exponential form, using time-resolved Alberta data. It is also concluded that there is a dependence of the shape of hailstone size distributions on the duration of sampling time. Such shape variations are associated with the sampling efficiency of the smaller size categories. The importance of the smaller sizes to the common hail integral estimates is also investigated. The minimum sizes required for sampling accuracy of these integral estimates are also obtained.

1. Introduction

It is usually assumed that the exponential distribution is an adequate representation of hailstone spectra at the ground (Douglas, 1963; Federer and Waldvogel, 1975, 1978; Cheng et al., 1985) and aloft (Ulbrich, 1974, 1977). For many general applications, this is an acceptable assumption. However, under certain conditions, significant departures from the exponential form have been observed (e.g., Federer and Waldvogel, 1978). Such departures appear as curvature on the usual semilogarithmic plot of particle number versus size. Physically, this can be caused by such factors as wind sorting, melting of the smaller stones, and the location of the sampling site within the storm. One common observation is that there are fewer particles in the smaller sizes than predicted by the exponential function. The smaller sizes are difficult to measure in the field. The present study aims at investigating certain sampling aspects of these smaller sizes in the hail size spectrum, examining ways to describe or parameterize them and assessing their effects in the computation of the integral measures of hail. In the present study, the gamma size distribution (Ulbrich, 1983; Wong and Chidambaram, 1985) is used in describing hailstone size distributions and also in parameterizing the size distribution shape. This is because the gamma size distribution has a flexible shape parameter, which can be estimated from the sample. Time-resolved hailstone samples from the Alberta Hail Project are fitted with the gamma size distribution, and the variations of the size distribution shape due to the sampling efficiency of the smaller sizes are examined. The effects of the smaller sizes on useful hailfall measures, such as the total concentration, hail water mass, hail impact energy, and radar reflectivity are also investigated.

Corresponding author address: Dr. Raymond K. W. Wong, Resource Technologies Department, Alberta Research Council, P.O. Box 8330, Station F, Edmonton, Alberta, Canada T6H 5X2.

2. The hailstone samples

A time-resolved hailstone sampling system was operated as part of the Alberta Hail Project. The hailstone samples used in the present study were obtained using mobile sampling vehicles from two storms in 1980 (27 July and 2 August) and three storms in 1982 (30 June, 21 July and 11 August).

Two instrumented vehicles were used in 1980 and three were used in 1982. For details of these sampling vehicles, see Cheng et al. (1985). Also, the sampling area (hail catcher aperture) was increased from 0.21 to 1.0 m^2 in 1982. Most samples were obtained to the southwest of the zone of maximum radar reflectivity, within a distance of a few kilometers. Climatologically, much of the hailfall in Alberta hailstorms occurred to the southwest of the maximum reflectivity core.

The hailstones were first photographed and their equivalent volume diameters in size intervals of 1 mm were computed. Each size distribution was examined for representativeness and samples with less than 100 stones or maximum size less than 10 mm were combined with subsequent samples collected at the same location to form samples of adequate size. A total of 84 samples were obtained.

3. The gamma size distribution

It is known that hailstone samples collected over short durations do not follow the usual exponential distribution (Federer and Waldvogel, 1978). For the present study, the gamma size distribution is used because of its ability to parameterize size distribution shape (Wong and Chidambaram, 1985).

The gamma size distribution is a more flexible representation of size distributions of which the usual exponential size distribution is a special case. Ulbrich (1983) has presented methods for the estimation of the parameters of the gamma size distribution using remote measurables, and demonstrated the ability of the distribution function to describe curvature of size distributions on the semilogarithmic plots of particle number versus size. Wong and Chidambaram (1985) used a version of the gamma size distribution based on the gamma probability density:

$$N(D)dD = \frac{N^*}{\Gamma(\alpha)\beta^\alpha} D^{\alpha-1} \exp(-D/\beta)dD, \qquad (1)$$

where $D \geqslant 0, \alpha > 0, \beta > 0$, and $\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t)dt$; $N(D)dD$ is the particle number per unit volume. The particle concentration parameter N^* is to be estimated from the sample, after estimates of α the shape parameter, and β the scale parameter, have been obtained. The procedures for obtaining the parameters N^*, α and β were described in Wong and Chidambaram (1985).

However, a modification of the procedure is necessary when applied to ground measurement of hailstones. There are two important considerations regarding the ground measurement of hailstone size distributions. First, hailstones are typically larger than 5 mm in diameter (Pruppacher and Klett, 1978) and have a finite maximum size, say $D_{\max}$ in actual sampling situations. Therefore, the range of possible sizes is not from zero to infinity as usually assumed for the gamma probability density. Rather, it should be from 5 mm to $D_{\max}$. This needs to be taken into consideration in the use of the gamma size distribution (1). For positively skewed, long tailed hailstone size distributions, the upper truncation limit $D_{\max}$ has minimal effects on the estimates of the distribution parameters because of the relatively few observations. But the nonexistence of observations in the 0–5 mm range has serious effects on fitting equation (1) to the observed hailstone size distributions, particularly when maximum likelihood techniques are used. Second, the observed hailstone samples at ground are volume integrated. In other words, the total number of stones observed, N_t is represented by the integral

$$N_t = \int_a^{D_{\max}} N(D)V(D)dD, \qquad (2)$$

where $V(D)$ is the size dependence of sampling volume and $V(D) = V_0D^b = AtkD^b$; a is the minimum size of 5 mm. Here, A is the area of the sampling device (m^2), t is the duration of sampling time (sec), and $v(D) = kD^b$ ($m\ s^{-1}$) is assumed to be an adequate description of the size dependence of terminal fall speed. The constant k is 4.43 ($m\ s^{-1}\ mm^{-b}$) and b is 0.5 (e.g., Waldvogel et al., 1978).

4. The estimation procedure

The use of the gamma probability density presupposes that data exist in the range $[0, \infty)$. (A left square bracket (right parenthesis) is used here to indicate closed (open) interval on the real number line). For the purpose of the present study, a transformation of variable in the form of an axis translation was used to accommodate the fact that hailstone samples were truncated below the minimum size. The following procedure was therefore proposed to fit the gamma size distribution to observed samples of hailstones. Each observed sample was first adjusted for the sampling volume by dividing the observed particle number using $V(D) = AtkD^b$. This gives the size distribution per unit volume. Define

$$D' = D - a, \qquad (3)$$

where a is the lower truncation point, say 5 mm. This is, in effect, a variable transformation, which shifts the size axis so that the origin coincides with the minimum size for hailstones. This permits a better fit to model (1) and eliminates the effect of zero observation in the 0 to 5 mm range. The size distribution function is now determined by the size range that is actually observed.

Model (1) is applied to the transformed data set. Note that in the present case, it is assumed that the transformed variable D' follows a gamma size distribution, which implies that the original variable D does not. This should not create any difficulties in most applications as long as there is an improved goodness-of-fit and the estimate of the particle number concentration for a given size interval is accurate. The parameters α and β were obtained by the method of maximum likelihood (Wong and Chidambaram, 1985). The parameter N^* can be computed by noting that the total particle concentration per unit volume is

$$n_t = \int_0^{D'_{\max}} N(D')dD' \qquad (4)$$

or

$$n_t = \frac{N^*}{\Gamma(\alpha)\beta^\alpha} \int_0^{D'_{\max}} D'^{\alpha-1} \exp(-D'/\beta)dD'$$
$$= \frac{N^*}{\Gamma(\alpha)} \gamma\left(\alpha, \frac{D'_{\max}}{\beta}\right). \qquad (5)$$

Thus, one has

$$N^* = \frac{n_t\Gamma(\alpha)}{\gamma(\alpha, D'_{\max}/\beta)}, \qquad (6)$$

where $\gamma(a, x) = \int_0^x t^{a-1} \exp(-t)dt$.

For $D'_{\max}/\beta$ sufficiently large, N^* converges to n_t. The rate of convergence depends on the shape parameter. This dependence is shown in Fig. 1. For a given value of $D'_{\max}/\beta$, the smaller the parameter α, the closer the ratio

$$\frac{\gamma(\alpha, D'_{\max}/\beta)}{\Gamma(\alpha)}$$

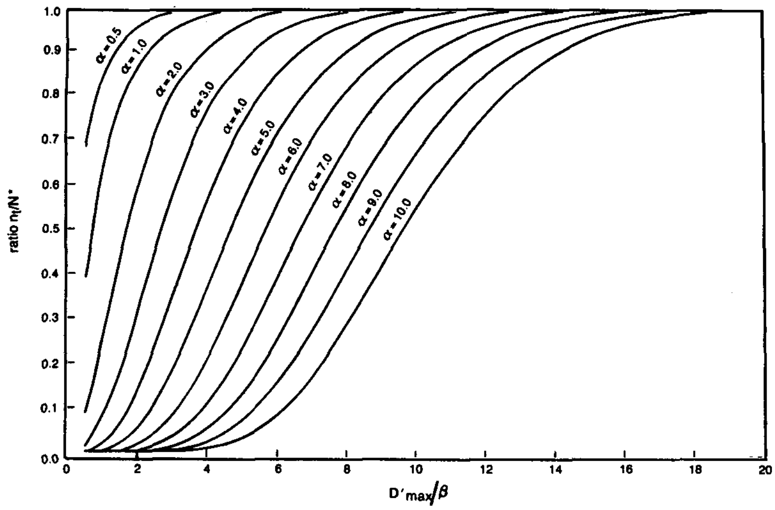

FIG. 1. The relationship between n_t/N^* and D'_{max}/β for different values of the shape parameter α.

is to unity. From Eq. (6), this means n_t is closer to N^*. For a small shape parameter $\alpha = 0.5$, say, n_t/N^* is $\geqslant 0.95$ if D'_{max}/β is $\geqslant 1.92$, whereas for a large shape parameter, $\alpha = 10.0$, for example, n_t/N^* does not exceed 0.95 until D'_{max}/β exceed 15.71. In practice, D'_{max}, α, β, and n_t are all known prior to the computation of N^*. The relationships in Fig. 1 delineate the range of α and D'_{max}/β values for which n_t approximates N^* with a certain accuracy. Conversely, one can trace the values of α and D'_{max}/β for a given ratio of n_t/N^*. One can see in Fig. 2 that for any given ratio of n_t/N^*, α increases almost linearly with D'_{max}/β. Figure 2 also shows the manner in which the relationship between the concentration parameter N^* and the observed concentration n_t changes with the variation of size distribution shape and the maximum size obtained.

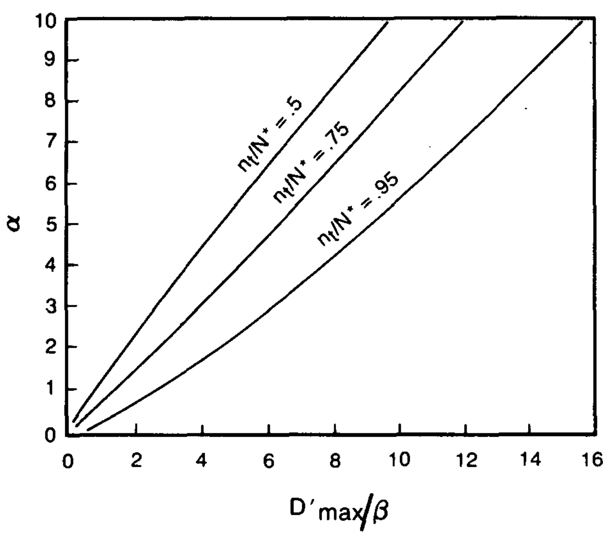

FIG. 2. The relationship between α and D'_{max}/β for different values of n_t/N^*.

The above procedure was used in the estimation of distribution parameters. Note that the axis translation does not affect the usefulness of the fitted function in describing the distribution of hailstones with size. Since there is a one-to-one correspondence between D and D', the size distribution for D' can replace that for D for all practical purposes. It will be shown that the axis translation in fact provides better fit of observed hailstone size distributions using model (1).

To illustrate the effects of the transformation (3), a hailstone sample was fitted using model (1) with and without the transformation. For the purpose of comparison, the commonly used exponential model

$$N(D)dD = \frac{N^*}{\lambda} e^{-\lambda D} = N_0 e^{-\lambda D} \qquad (7)$$

was also used. Equation (7) is equivalent to fixing the shape parameter at unity. Figure 3 shows that the gamma model was able to provide a better fit both before and after the transformation than the exponential model (7).

There was also a significant improvement in the goodness-of-fit due to the axis translation. This was shown by first fitting the Alberta hailstone samples using (1). The total of absolute difference between the observed and fitted frequencies over the given size ranges was then obtained for each sample before and after the translation (3). The values obtained after the translation were found to be significantly smaller (P-value ≈ 0.000 based on the Mann-Whitney test), indicating a significant improvement in fit.

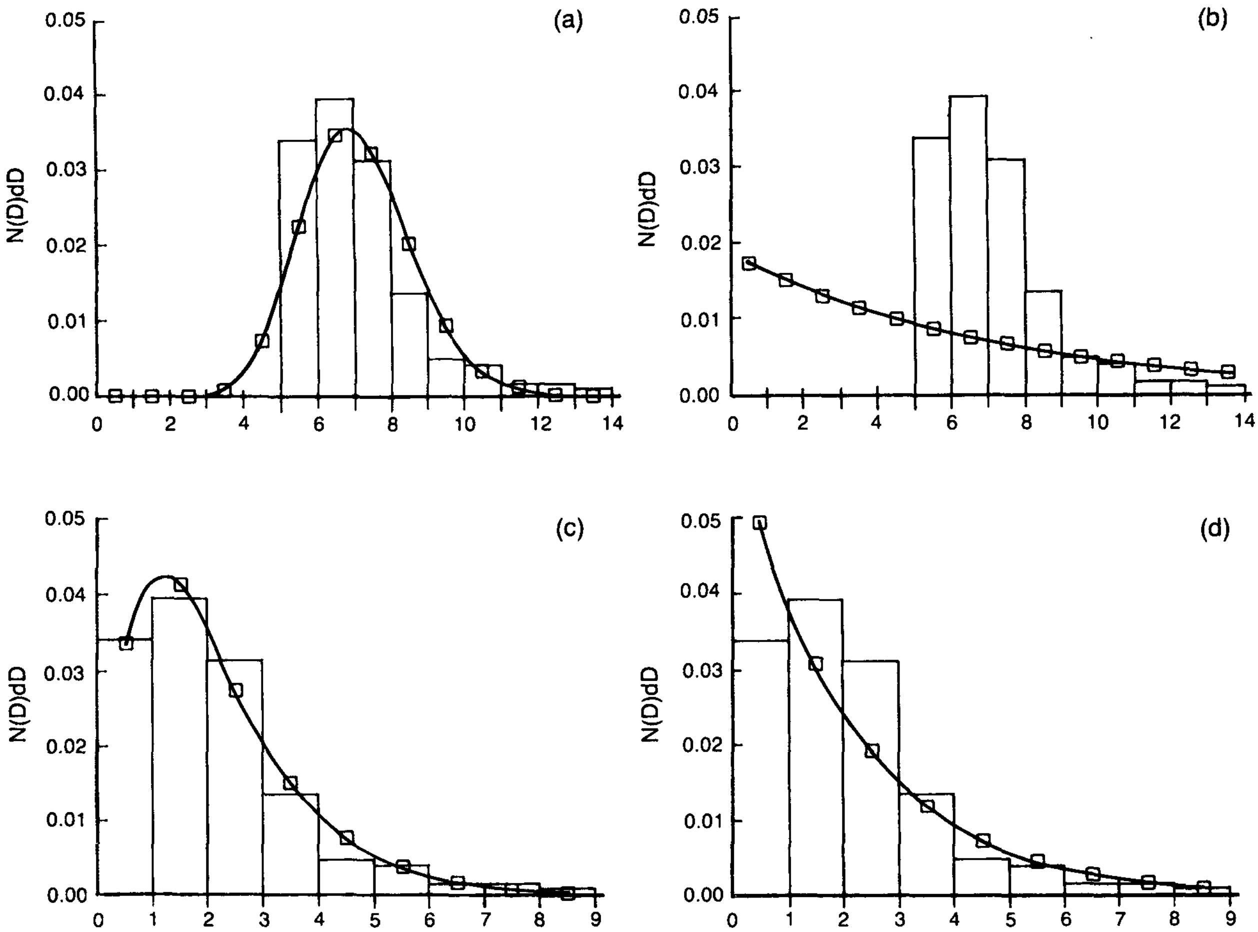

FIG. 3. Comparison of the effect of variable transformation on size distribution model fitting. Panels (a) and (b) are for the gamma and exponential distribution, respectively, but without variable transformation. Panels (c) and (d) are the same but with the transformation.

5. The variation of shape with sampling duration

The parameter α obtained using the procedure described in section 4 represents the size distribution shape. Since the samples were time resolved, it was possible to examine any relationship between size distribution shape and sampling duration.

The variation of size distribution shape with sampling duration was investigated for raindrops by Joss and Gori (1978). Using data from widespread rain and thunderstorms, it was found that the shape of raindrop size distributions changed from approximately monodispersed to closely exponential with increasing sampling duration. The hail samples used for the present study had much higher variability than those collected by Joss and Gori from uniform widespread rain. Therefore, for each sampling duration, the median shape factor from the samples was computed as a measure of central tendency. The relationship between the median shape factor and sampling duration is shown in Fig. 4. The regression line with a negative slope was tested to be significant with a P-value of 0.0234, using a recently developed distribution-free, least absolute deviations (LAD) regression technique. This method was chosen because of the resistance to outlier effects and the geometric consistency between the interpretation of data and the analysis performed. This means results are more intuitive than those obtained from the usual least-squares procedures.

One may conclude from Fig. 4 that there is a negative relationship between α and sampling duration. This means that the longer the sampling time, the more skewed the size distribution becomes. The trend shown in Fig. 4 for the Alberta hail samples is very similar to that found by Joss and Gori (1978) for rain, except that the time scale over which the variation occurred is much smaller due to the very nature of hailfall. Hailfall at a given location seldom lasts for more than a few minutes. Yet, within the small time range for hail samples, Fig. 4 shows that the sampling variations of the hailstone size distribution is towards the exponential with increasing sampling time. (Recall that $\alpha = 1$ represents the exponential distribution, and $\alpha \gg 1$ represents a symmetric bell shape.) It must be stressed

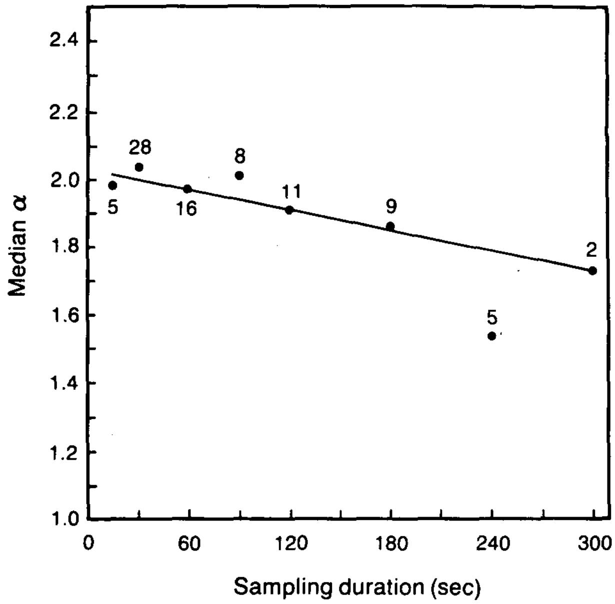

FIG. 4. Variation of the median shape factor with sampling duration.

that there is no evidence from the present data that exponentiality can ever be reached. Joss and Gori (1978) found that rain samples of different intensity must be combined in order to obtain true exponentiality with increasing sampling duration. In the present case, the available data set did not permit a more detailed analysis involving stratification of the data based on the characteristics of the storms. However, the tendency towards increased positive skewness agrees with the rain results of Joss and Gori (1978). The range of variation in α can be shown by plotting the density functions in Fig. 5. Here, a range of α from 2.05 to 1.75 was assumed for the range of sampling durations from 15 to 300 sec. It can be seen that the greatest difference between the long and the short sampling durations is in the relative frequencies of the smaller stones.

The above results imply that longer sampling time improves the sampling efficiency of the smaller sizes. This should be considered in sampling situations when accuracy in the smaller sizes is required. It is known that for an integral estimate, $X(n)$, based on a hail size distribution,

$$X(n) = \int_a^{D_{max}} CN(D)V(D)D^n dD, \tag{8}$$

where a is the 5 mm minimum size described previously, C the appropriate coefficient, and $N(D)$ the exponential distribution, the maximum contributing size depends on the value of the exponent n (Joss and Gori, 1978). One may therefore expect that the importance of the smaller sizes to the sampling accuracy of an integral estimate will also be dependent on the exponent n for the particular estimate. In section 6, the contribution of the smaller sizes to several integral estimates will be examined empirically using the Alberta hail samples. These estimates are total number concentration n_t ($n = 0$), hail water content HWC ($n = 3$), hail impact energy IE ($n = 4$), and hail radar reflectivity Z ($n = 6$).

Incidentally, an interesting observation was that the median shape factor was approximately 2.0. This implies that a simplification of the gamma size distribution with $\alpha = 2.0$ is likely to work well in the general description of hail size distributions. When α is set constant at 2.0, the model becomes

$$N(D')dD' = \frac{N^*}{\beta^2} \exp(-D'/\beta)D'dD'. \tag{9}$$

The only parameters needed to be estimated are β and N^*. In this case, the maximum likelihood estimates of β is simply $\overline{D'}/2$, where $\overline{D'} = \sum_{i=1}^{n_t} D'_i/n_t$ is the mean of D', and N^* can be approximated by n_t, the total number of stones, for $D'_{max}/\beta > 5$ (Fig. 1).

6. The empirical assessment of the effects of the smaller sizes on integral estimates

Integral estimates based on hailstone samples, are often required for various purposes, including the test of cloud-seeding hypotheses, the assessment of damage by hail and the calibration of hail sensors. Since effective sampling of a hail size distribution is difficult because of the elusive nature of the smaller hailstones, it is of interest to examine their contribution to the statistics of these integral estimates.

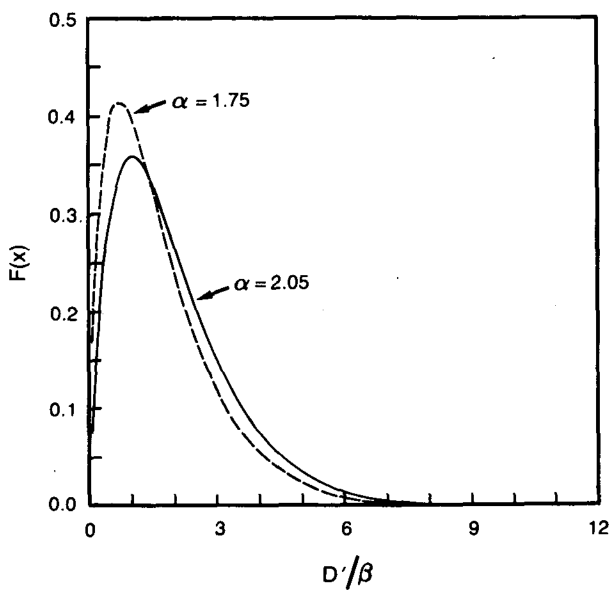

FIG. 5. The gamma density function with α = 1.75 and 2.05.

The estimates n_t, HWC, IE, and Z are defined for unit volume of air and computed using the following:

$$\left.\begin{aligned}
&1)\quad n_t = \sum_{i=1}^{k} f(D_i)\Delta D_i, && (\mathrm{m}^{-3})\\
&2)\quad \mathrm{HWC} = \frac{\rho\pi}{6\times 10^3}\sum_{i=1}^{k} f(D_i)D_i^3\Delta D_i, && (\mathrm{g\ m}^{-3})\\
&3)\quad \mathrm{IE} = \frac{(4.43)^2\rho\pi}{12\times 10^6}\sum_{i=1}^{k} f(D_i)D_i^4\Delta D_i, && (\mathrm{J\ m}^{-3})\\
&4)\quad Z = \sum_{i=1}^{k} f(D_i)D_i^6\Delta D_i, && (\mathrm{mm}^6\ \mathrm{m}^{-3})
\end{aligned}\right\} \quad (10)$$

where

ρ (=0.9 g cm^{-3}), the assumed density of hailstone,
$f(D_i)$ the particle number per cubic meter of air per size interval for size D_i,
D_i the midpoint of the ith size class in mm,
ΔD_i the interval of the ith size class.

The summation is over the k classes from 5 mm to the largest detectable size. One value for each integral estimate was computed for each sample. A frequency distribution for each estimate can thus be constructed over all of the samples obtained. This frequency distribution will be labeled as the 5 mm distribution, which is the basic distribution to be compared. It is formed by including the contribution of all sizes from 5 mm to the largest.

Similarly, the 6 mm distribution can be formed by recomputing the estimates without the 5–6 mm class. The summations in Eq. (10) are therefore over the $k - 1$ classes larger than 6 mm. Comparing the 5 and 6 mm distributions for each estimate will show the contribution of the 5 to 6 mm category to the particular integral estimate. Similar comparisons can be made between the 5 and 7 mm distribution (for the contribution of the 5 to 7 mm range) and the 5 and the 8 mm distribution (for the contribution of the 5 to 8 mm range) and so on.

The comparisons were made using a permutation test which is the one-dimensional case of the multiresponse permutation procedure (MRPP). This test is based on the Euclidean distance measure and is for testing whether the two samples are clustered differently in the data space. (Mielke et al., 1976, 1981). This procedure was selected because it is distribution-free and geometrically consistent, which means more intuitive results (Mielke, 1985, 1986). The results are shown in Table 1. The P-values shown are for the comparisons between the estimates based on the 5 mm distribution and those of the other distributions, where a portion of the smaller sizes have been removed. The hypothesis

TABLE 1. Comparisons between the samples using the entire size spectrum and those using different truncated spectra for integral estimates of hail. The entries are P-values from comparing the frequency distributions of the integral estimates from various levels of truncation. The values are rounded off to three decimal places.

Comparisons of 5 mm distribution versus	Integral estimates			
	$n = 0$ n_t	$n = 3$ HWC	$n = 4$ IE	$n = 6$ Z
6 mm distribution	0.302	1.000	1.000	1.000
7 mm distribution	0.002	0.370	0.654	1.000
8 mm distribution	0.000	0.027	0.142	0.553
9 mm distribution	0.000	0.000	0.006	0.119
10 mm distribution	0.000	0.000	0.000	0.013

was that there is no difference between the samples of estimates. Here, the sampling difficulty of the smaller sizes was simulated by the complete elimination of the size range. This may be an exaggerated scenario, but it serves to provide an approximate lower limit for reference when measurements are to be taken in the field. Consider the first column of P-values in Table 1. It shows that the contribution from hailstones in the 5 to 6 mm category is insignificant to the estimates of n_t. A significant difference was only detected when the 5 and 7 mm distributions were compared. In this context, a P-value of less than 0.03 can be considered as significant. A lower limit of 6 mm appears to be important to the accurate sampling for estimates of n_t. Similarly, the lower limit for the accurate sampling of HWC, IE and Z estimates are 7, 8 and 9 mm, respectively, based on the results of Table 1.

Such empirical lower limits for sampling accuracy can be compared using the concept of modal or maximum contributing diameter introduced by Joss and Gori (1978). The modal or maximum contribution diameter $D_m(n)$ for an integral estimate $X(n)$, based on the exponential size distribution for $N(D)$, is defined as

$$D_m(n) = \frac{\int_0^\infty N(D)D^n dD}{\int_0^\infty N(D)D^{n-1} dD}. \quad (11)$$

Joss and Gori (1978) stressed that $D_m(n)$ indicates the true location of the maximum contribution for exponential size distributions only. Although the $D_m(n)$ for gamma size distributions have been used in the parameterization of size distribution shape (Ulbrich, 1983; Wong and Chidambaram, 1985), the characteristics of these have not been examined. It appears from the present analysis that $D_m(n)$ corresponds closely to the lower limits for sampling accuracy of the gamma size distribution used in the general description of hail.

It can be shown that for the case of the gamma size distribution (1),

$$D_m(n) = \beta(\alpha + n - 1). \tag{12}$$

For $n = 0$, $D_m(0) = \beta(\alpha - 1)$; for $n = 1$, $D_m(1) = \beta\alpha$; and for $n > 1$, $D_m(n) = \beta\alpha + (n - 1)\beta$. Respectively, these are the mode, the mean, and $(n - 1)$ times the scale parameter above the mean, of the gamma size distribution involved. This is true for hailstone size distributions with $\alpha > 1$. In such cases, the usual exponential size distribution overpredicts in the smaller size categories. For $\alpha < 1$, $D_m(0) < 0$, the mode is not defined. Hence $D_m(n)$ for the gamma size distribution is different from that of the exponential size distribution, where $D_m(n)$ is simply equal to n times the mean. This is obvious from Eq. (12) with α set equal to unity for the exponential case. Estimates of $D_m(n)$ can be obtained using the median values of the parameters α and β from the Alberta samples (1.94 and 0.56 mm, respectively) and transformation (3). The results are listed in Table 2. The close correspondence shown in Table 2 suggests that $D_m(n)$, as defined by Eq. (12), can be considered as an approximation to the lower size limit for the sampling accuracy of integral quantities of hail.

7. Conclusions

In the present study, a shifted gamma size distribution was proposed as an appropriate model for describing hailstone size distribution, and parameterizing size distribution shape. The shape of measured hailstone size distributions often appears as curvature on the usual semilogarithmic plot of particle number versus size. This means that the exponential distribution may not be a good approximation. The shifted gamma size distribution can also accommodate the fact that hailstones are at least 5 mm in size. It is shown that the shifted gamma size distribution provides better fit to the size distributions of hail.

Using time-resolved hailstone samples from the Alberta Hail Project, the variation of the shape of hailstone size distributions was examined. A significant relationship was found between the median shape factor and the sampling duration. This is in agreement with that found for rain by Joss and Gori (1978). There is a tendency towards greater positive skewness with increasing sampling time. Much of this variation is due to the sampling inefficiency of the smaller sizes.

The effects of sampling inaccuracy for the smaller sizes on integral estimates was examined empirically, using the Alberta hailstone samples. The minimum sizes required for the accurate representation of the various integral estimates were obtained. There is a dependence of these minimum sizes on the exponent n in the definition of the integral estimates. It also appears that these minimum sizes are closely related to $D_m(n)$ as represented by Eq. (12).

TABLE 2. The correspondence between $D_m(n)$ and the empirically determined lower limits for sampling accuracy. See text for further details.

	Integral estimates			
	$(n = 0)$ n_t	$(n = 3)$ HWC	$(n = 4)$ IE	$(n = 6)$ Z
Empirical limit (mm)	6.0	7.0	8.0	9.0
$D_m(n)$ (mm)	5.5	7.2	7.8	8.9

Acknowledgments. The authors wish to thank their colleagues at the Alberta Research Council for their assistance in this work, which was part of the research activities of the Alberta Hail Project. The Alberta Hail Project was funded by the Alberta Department of Agriculture and the Alberta Research Council. The preparation of the manuscript by Ms. Donna Hipkin, using the LATEX word processor, is also much appreciated.

REFERENCES

Cheng, L., M. English and R. Wong, 1985: Hailstone size distributions and their relationship to storm thermodynamics. *J. Climate Appl. Meteor.*, **24**, 1059–1067.

Douglas, R. H., 1963: Size distributions of Alberta hail samples. *Sci. Rep. MW-36*, Stormy Weather Research Group, McGill University, 55–70.

Federer, B., and A. Waldvogel, 1975: Hail and raindrop size distribution from a Swiss multicell storm. *J. Appl. Meteor.*, **14**, 91–97.

——, and ——, 1978: Time-resolved hail analyses and radar structure of Swiss storms. *Quart. J. Roy. Meteor. Soc.*, **104**, 69–90.

Joss, J., and E. G. Gori, 1978: Shapes of raindrop size distributions. *J. Appl. Meteor.*, **17**, 1054–1061.

Mielke, P. W., 1985: Geometric concerns pertaining to applications of statistical tests in the atmospheric sciences. *J. Atmos. Sci.*, **42**, 1209–1212.

——, 1986: Non-metric statistical analysis: Some metric alternatives. *J. Statist. Plann. Inference*, **13**, 377–387.

——, K. J. Berry and E. S. Johnson, 1976: Multi-response permutation procedures for a priori classifications. *Commun. Statist.-Theor. Meth.*, **A5**, 1409–1424.

——, —— and G. W. Brier, 1981: Application of multi-response permutation procedures for examining seasonal changes in monthly mean sea-level pressure patterns. *Mon. Wea. Rev.*, **109**, 120–126.

Pruppacher, H. R., and J. D. Klett, 1978: *Microphysics of Cloud and Precipitation*, D. Reidel, 714 pp.

Ulbrich, C. W., 1974: Analysis of doppler radar spectra. *J. Appl. Meteor.*, **13**, 387–396.

——, 1977: Doppler radar relationships for hail at vertical incidence. *J. Appl. Meteor.*, **16**, 1349–1359.

——, 1983: Natural variations in the analytical form of the raindrop size distribution. *J. Climate Appl. Meteor.*, **22**, 1764–1775.

Waldvogel, A., W. Schmid and B. Federer, 1978: The kinetic energy of hailfall, Part I: Hailstone spectra. *J. Appl. Meteor.*, **17**, 515–520.

Wong, R. K. W., and N. Chidambaram, 1985: Gamma size distribution and stochastic sampling errors. *J. Climate Appl. Meteor.*, **24**, 568–579.