A Simple Parameterization Scheme for Joint Statistics of Cloud Field Morphology and Physical Parameters

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(Manuscript received 28 January 1989, in final form 19 February 1990)

ABSTRACT

A simple parameterization scheme for the joint statistics of cloud field morphology and other cloud parameters is discussed. The statistics of the cloud field morphology are obtained from the NOAA-9 AVHRR using the "Hit or Miss" image transformations. The parameterization scheme is used to reproduce the joint statistics of two aspects of the cloud field morphology and the cloud top radiance, brightness temperature and optical depth. The accuracy of the scheme is discussed.

1. Introduction

One of the aims of the ISCCP program is to determine the fractional cloud amount from satellite imagery because current numerical weather prediction models rely upon this quantity to parameterize the radiative processes that occur in broken cloud. However, this parameter provides only a first approximation for calculating the shortwave and longwave fluxes transmitted through the atmosphere. The geometry and physical characteristics of the cloud affect the transmission, emission and absorption of radiation and also affect the amount of scattering through the sides of the cloud. The degree of fragmentation of a cloud field also has a significant impact on the shortwave and longwave fluxes transmitted by the field as a result of interactions between clouds, e.g., scatter between clouds, shading effects, etc. (Harshvardan and Weinman 1982; Welch and Wielicki 1984). To improve the radiation transfer calculations, information concerning these factors is needed. Since cumulus clouds exist typically in the size range 50 m to several thousand meters, very high resolution imaging is required to accurately assess the amount of cloud and its properties. The amount of data that would be generated from such a high resolution imager is overwhelming and would present enormous data storage problems.

Rather than collecting entire high resolution images it has been proposed (O'Brien 1987) that the statistics relating to the cloud field be collected in real time on board the satellite and these statistics used to supplement lower resolution imagers, such as on the NOAA series of satellites. Consider the following simple example of the types of statistics that will be required. Note that we are considering the next order of approximation above that which uses only the cloud fraction. We wish to calculate the number of photons in a given energy range that reach the ground in the presence of cloud. If \( P_s \) is the probability of a photon reaching the ground and \( N_0 \) is the number of photons entering the atmosphere, then the number of photons reaching the ground is

\[
N = N_0 \cdot P_s = N_0 [(1 - P_c) + P_c \cdot P_t] \tag{1}
\]

where \( P_c \) is the probability of a photon striking a cloud and \( P_t \) is probability that the photon is then transmitted through the cloud. If the photon enters a cloudy region of optical thickness in the range \( \sigma, \sigma + d\sigma \) then the probability that it is transmitted is \( \exp(-\sigma) \). The probability that it strikes such a region of cloud is \( p(\sigma)\,d\sigma \). However, if scattering processes are included then the probability for transmission is modified by a function that depends on both the optical thickness and the size, \( \lambda \), of the cloud (Davies 1978). Let the modifying function be \( m(\lambda, \sigma) \). Here \( \sigma \) may be some weighted average, perhaps, over all the clouds of size \( \lambda \) to enable \( m(\lambda, \sigma) \) to be deduced, for example, from the work by Davies. Then the probability for transmission is given by sums over all cloud sizes and optical thicknesses of the probability \( p(\lambda) \cdot d\lambda \) of the photon striking a portion of a cloud in the size range \( \lambda, \lambda + d\lambda \) which has a probability \( p(\sigma|\lambda)\,d\sigma \) of having an optical thickness in the range \( \sigma, \sigma + d\sigma \) and a probability \( m(\lambda, \sigma) \exp(-\sigma) \cdot d\sigma \cdot d\lambda \) for transmission.

\[
P_t = \int \int p(\lambda) \cdot p(\sigma|\lambda) \cdot m(\lambda, \sigma) \exp(-\sigma) \cdot d\sigma \cdot d\lambda \tag{2}
\]
Note that \( p(\lambda) \cdot p(\sigma|\lambda) = p(\lambda, \sigma) \). This is the joint probability distribution. Then the number of photons reaching the ground can be written as

\[
N = N_0 \left( (1 - P_e) + P_e \cdot \int \int p(\lambda, \sigma) \times m(\lambda, \sigma) \cdot \exp(-\sigma) \cdot d\sigma \cdot d\lambda \right).
\]  

(3)

In this example, no account of emission of radiation from the cloud has been considered. To account for this, data relating to cloud temperature or the radiance from the top of the cloud are required.

To obtain the sort of statistics described above, O'Brien describes an instrument to generate cloud size distributions in real time on board a satellite. This image processor uses "Hit or Miss" transformations to generate the size distribution. In addition to the size distribution, the joint or conditional distributions are required. However, providing all the statistical distributions increases the amount of data to be collected and stored. It was discovered that for the Hit and Miss transformations and for a number of cloud parameters (cloud top temperature, optical thickness and radiance from the cloud top) obtained from NOAA AVHRR, there are simple relationships between the morphology statistics of the cloud field and the physical parameters. In this paper, these relationships are discussed and they are used to reconstruct the distributions from parameters obtained from the original distributions. This parameterization allows a substantial reduction in the amount of data required for storage. In section 2, there is a brief discussion of Hit or Miss transformations and the method of sizing by openings. In section 3 a set of conditional probability density functions are introduced and the joint statistic parameterization scheme is discussed. The scheme is applied to cloud images obtained from the NOAA-9 AVHRR (section 4) and a comparison between the original and the reconstructed distributions is made.

2. The hit or miss transformation

All image transformations described in this paper are performed on a binary image obtained by thresholding the original image at some value \( s = s_0 \) of the cloud parameter \( s \). All points of the image belonging to the cloud are given a value \( b \) and all others are given the value \( \bar{b} \). The Hit or Miss transformation employs a structuring element \( B \), which consists of an array of pixel values. The structuring element is translated over the entire image by centering it at each point \( x \) in the image. If the image exactly matches the structuring element at \( x \), then \( x \) is assigned a particular value in the transformed image, otherwise it is assigned the complement value.

For example, consider some object within an image and let \( b = 1, \bar{b} = 0 \) and \( B = (1, 1, 1) \). If \( x \) resides within the object and not on the edge then the pixel values about \( x \) will be \((1, 1, 1) = B \) and thus \( x \) is a part of the transformed image and is given the value \( b = 1 \). If \( x \) resides at the edge of the object, then the pixel values about \( x \) will be \((0, 1, 1) \) or \((1, 1, 0) \) which do not match the structuring element. Likewise for all positions \( x \) not in the object. These positions are assigned the value \( \bar{b} = 0 \). Therefore, this particular Hit or Miss transformation removes all pixels residing at the edge of the object. This is termed an erosion of the object by \( B \). Note that an object in the image consisting of a single pixel is entirely removed by this erosion. Larger objects are reduced in size. The complement transformation is called a dilation whereby a structuring element of the form \( B = (0, 0, 0) \) is used and all matches are assigned the value \( \bar{b} = 0 \) and all others are given the complement value \( b = 1 \). This has the effect of expanding (dilating) the object, or eroding the background.

Sequences of Hit or Miss transformations can be used to gather statistics of many morphology characteristics of cloud fields. The two characteristics used in this paper are the distances of pixels from the edges of the clouds and the size of the clouds. Both characteristics are obtained by using a two dimensional structure element \( B \) to erode and dilate regions in the two dimensional image. The edge-distance distribution \( d(r) \) can be obtained using a sequence of erosions by a square structure element of side length equal to 3 pixels. Each successive erosion removes the outer layer of pixels from the remaining clouds. Thus the \( r \)th erosion will remove pixels which lie distances \( r \) from the edges of the clouds, where the distance is measured in units of pixel widths. The number of pixels removed by each erosion gives the edge-distance distribution \( d(r) \).

The size distribution is obtained by a sequence of openings using structuring elements of different sizes. An opening of an image consists of an erosion by an element \( B \) followed by a dilation by \( B \). Any region in the image smaller than the structuring element is entirely removed by the erosion. All regions larger than \( B \) are reduced by the erosion and then restored by the dilation and are thus unaffected by the opening. The size distribution \( n(\lambda) \) is the number of pixels removed by the opening by structure element \( B \) of side length \( 2\lambda + 1 \).

The joint distributions \( h(\lambda, s) \) or \( h(r, s) \) are obtained by examining the pixels removed after each image transformation. These distributions represent the numbers of pixels associated with cloud parameter values in the ranges \((s, s + \delta s)\) and \((\lambda, \lambda + \delta \lambda)\) or \((r, r + \delta r)\), where \( \delta \lambda \) and \( \delta r \) represent the increments in the morphology parameters between each transformation. Usually \( \delta \lambda = 1 \) and \( \delta r = 1 \) in units of pixel widths. In practice, \( \delta s \) represents the bin width of the histogram generated during the image transformations.

The shape of the structuring element strongly influences the image transformations and the resultant distributions. On the grid obtained with satellite images,
the simplest structural element is a square. The pixels within a square object, for example, will be associated with the one size, related to the side length of the structural element, which exactly covers the object. However, if the object was circular, then the pixels would be assigned different sizes depending on their positions within the circle because the square structural element would not fit exactly within it. Note that similar arguments apply to the application of circular structural elements to square objects, and to the application of any regularly shaped structural element to the irregular shape of a cloud. Thus the pixels within a single cloud will be associated with a range of sizes characteristic of the smallest dimensions across each feature or protuberance of the cloud. It has been argued (O’Brien 1987) that this feature makes the procedure of cloud sizing by openings appropriate for radiation transfer calculations.

3. Distributions and parameterization

We begin by defining a set of joint probability density functions which lead to relationships between the morphology distribution and the cloud parameter distribution. Here y will be used to represent any morphology parameter (such as size or distance from the cloud edge) and s will represent the cloud parameter (such as radiance from the cloud top). Let \( p(s) \) be the probability density function for \( s \) and let \( p(y|s) \) be the conditional probability density function for \( y \) given \( s \). Then the joint probability density of \( y \) and \( s \) is

\[
p(y, s) = p(y|s)p(s)
\]

\[
= p(s|y)p(y)
\]

\[
= p(s, y).
\]  

(4)

In addition, there are normalization conditions on the conditional probabilities.

\[
\int_0^\infty p(y|s)dy = 1
\]  

(5)

\[
\int_0^\infty p(s|y)ds = 1.
\]  

(6)

Together with Eqs. (4), these lead to Bayes’ Theorem:

\[
p(s|y) = \frac{p(y|s)p(s)}{\int_0^\infty p(y|s')p(s')ds'}.
\]  

(7)

It then follows that if \( p(y|s) \) and \( p(s) \) are known, the other distributions, \( p(y) \), \( p(s|y) \) and \( p(y, s) \), can be calculated.

It is useful to work with the conditional probability density function \( p(y|s) \) because this distribution does not depend directly on the number of pixels with a particular \( s \) value, although it still depends on \( s \), i.e. \( p(y|s) = p(y, s)/p(s) \). For the datasets examined it was discovered that the form of this distribution with \( s \) is insensitive to the choice of cloud parameter \( s \) and tends to a bell shape.

Now consider a sequence of image transformations, such as the sizing by openings. A sequence of transformations up to \( y \) will remove all pixels with morphology parameters in the range \((0, y)\). Since each pixel is associated with a parameter \( s \), then this value of \( s \) is removed from the available set of \( s \) parameters, described by \( p(s) \). The probability density for \( s \) after these transformations is \( p_y(s) \) given by

\[
p_y(s) = p(s) - \int_0^y p(y', s)dy'
\]

\[
= p(s) \int_y^\infty p(y'|s)dy'.
\]  

(8)

The conditional probability density function \( f(y|s) \) for this resultant distribution is given by

\[
f(y|s) = \frac{p(y, s)}{p_y(s)}
\]

\[
= \frac{p(y|s)}{\int_y^\infty p(y'|s)dy'}.
\]  

(9)

The function \( f(y|s) \) is the conditional probability of \( y \) given \( s \) taking account of the removal of some of the \( s \) values from the original distribution due to the Hit and Miss transformation. Note that \( f(0|s) = p(0|s) \).

In all cases observed, it is found that \( p(0|s) \) is a sharply peaked one-sided function, which can be approximated by a linear function of \( s \). The near linearity of \( p(0|s) \) is a consequence of the image threshold and the use of a regularly shaped structure element \( B \) on an irregularly shaped object (the cloud). The first Hit and Miss transformation will usually remove many pixels along the edges of the clouds. Since the clouds’ edges are defined by the threshold value \( s_r \), then these pixels are associated with \( s \) values close to \( s_r \). If the resolution of the cloud image was infinite, then all of these pixels would have the value \( s_r \) and \( p(0|s) \) would be a delta function at \( s = s_r \). The limited resolution of the image increases the width of the function which takes an approximately linear form. Since \( f(0|s) \) \( = p(0|s) \) then \( f(0|s) \) is approximately linear with \( s \). Furthermore, it was discovered that all the \( f(y|s) \) could be approximated by linear functions of \( s \) and could reproduce the Gaussian-like forms of the \( p(y|s) \).

The family of curves generated from linear approximations to the \( f(y|s) \) can be obtained by inverting Eq. (9). The variation of \( p(y|s) \) over a small range \( \delta y \) is

\[
p(y + \delta y|s) - p(y|s)
\]

\[
= f(y + \delta y|s) \int_{y+\delta y}^\infty p(y'|s)dy' - f(y|s)
\]


\[ \times \int_{y}^{\infty} p(y'|s)dy' = f(y + \delta y|s) \]

\[ \times \left[ \int_{y}^{\infty} p(y'|s)dy' - \int_{y}^{y+\delta y} p(y'|s)dy' \right] \]

\[ - f(y|s) \int_{y}^{\infty} p(y'|s)dy'. \quad (10) \]

Note that \( \int_{y}^{y+\delta y} p(y'|s)dy' \approx p(y|s)\delta y \). By dividing both sides by \( \delta y \) and taking the limits as \( \delta y \to 0 \) yields

\[ dp(y|s)/dy = [p(y|s)/f(y|s)]df(y|s)/dy \]

\[ - f(y|s)p(y|s) \quad (11) \]

where \( \int_{y}^{\infty} p(y'|s)dy' \) has been replaced by \( p(y|s)/f(y|s) \) using Eq. (9). The formal solution to this equation is

\[ p(y|s) = C f(y|s) \exp \left\{ - \int f(y|s)dy \right\} \quad (12) \]

where \( C \) is a normalization constant. Let \( f(y|s) = \beta(y) + \alpha(y)s \) where \( \alpha(y) < 0 \) then the conditional probability density function takes the form

\[ p(y|s) \sim (1 - B(y)s) \exp(A(y)s) \quad (13) \]

where \( A(y) \) and \( B(y) \) contain integrals over \( \alpha(y) \) and \( \beta(y) \). Plots of this function for a sequence of values of \( A(y) \) with \( B = 1 \) are shown in Fig. 1. The curves are terminated when \( f(y|s) < 0 \), which occurs in this case when \( s > 1 \). It can be seen from these plots that a range of Gaussian-like curves can be fitted by pairs of parameters \( \alpha(y) \) and \( \beta(y) \). The accuracy of this parameterization is considered in the following section where it is applied to distributions of radiance, cloud top temperature and optical depth obtained from size and edge-distance distributions of cloud field images from the NOAA-9 AVHRR.

### 4. The parameterization of datasets

The parameterization scheme has been applied to three datasets of cumulus clouds over the ocean obtained from the NOAA-9 AVHRR. To demonstrate the scheme, only one dataset will be discussed. The data was obtained from a region 256 km square. The scheme was tested using three cloud parameters that could be obtained from the NOAA-9 AVHRR channels 4 and 5: (i) the radiance \( L_4 \) from the cloud as computed from channel 4 of the AVHRR; (ii) the cloud brightness temperature \( T_4 \) computed from the channel 4 radiance by inverting the Planck function; and (iii) the cloud optical depth \( \sigma_4 \) at the channel 4 wavelength, at 10.8 \( \mu \)m.

The optical depth \( \sigma_4 \) was calculated using a crude cloud model and an assumed ratio for the optical depths, \( R = \sigma_4/\sigma_5 \), at the channel 4 and 5 wavelengths. The brightness temperature from a uniform cloud layer of optical depth \( \sigma_n \) at the wavelength of channel \( n \) is approximately

\[ T_n = T_w \exp(-\sigma_n) + T_c (1 - \exp(-\sigma_n)) \quad (14) \]

where \( T_w \) is the brightness temperature of the water beneath the cloud and \( T_c \) is the brightness temperature of the cloud. Equation 14 is obtained by expressing all radiances in terms of their equivalent blackbody temperatures and linearizing the Planck function. The use of brightness temperatures has the effect of removing the wavelength dependence associated with radiances. Here \( T_w \) is measured from the image in the regions free from cloud, \( T_4 \) and \( T_5 \) at the tops of the clouds are known from the AVHRR and \( \sigma_4 = \sigma_4/R \), leaving two unknown parameters \( \sigma_4 \) and \( T_c \). The ratio \( R = 0.73 \) has been obtained from calculations by Curran (1972) assuming a Deirmendjian (1969) cloud model C.1, which is applicable to cumulus clouds. Although this procedure for determining \( \sigma_4 \) is crude, it will serve the purpose of demonstrating the parameterization scheme.

Three images of the cloud field in the parameters \( s \in \{ L_4, T_4, \sigma_4 \} \) were formed. Thresholds were taken by
visually inspecting the images and estimating the values of the parameters at the edges of the clouds. During the image processing, the edges of an image were wrapped around to the opposite edge (i.e. the image was treated as a torus) to take care of clouds straddling the edge of the image. Each image was processed to form the joint distributions of the physical parameters with cloud size \( \lambda \) and edge-distance \( r \). The joint distribution \( h(y, s) \) is generated by dividing the maximum range of the cloud parameter over the image into 100 equal subranges (or histogram bins) of width \( \delta s \) and counting the numbers of pixels removed by each image transformation with parameters within the range of each bin (i.e. between \( s \) and \( s + \delta s \)). If the total number of cloudy pixels is \( N \), then the joint probability density function at each point is \( p(y, s) = h(y, s)/(N \delta y \delta s) \).

The conditional probability density functions \( p(\lambda | L_4), p(\lambda | T_4), p(\lambda | \sigma_4), p(r | L_4), p(r | T_4) \) and \( p(r | \sigma_4) \) are calculated by integrating the joint density functions to obtain \( p(L_4), p(T_4) \) and \( p(\sigma_4) \) and using Eq. (4). Then Eq. (9) is used to obtain the functions \( f \) to which linear regressions were applied. A typical set of functions, in this case \( f(\lambda | L_4) \), are shown plotted against \( L_4 \) in Fig. 2. These functions are only linear within a certain region. However, the nonlinear regions generally correspond to small values of the conditional probabilities and thus should not be given a significant weighting in the linear fit to the data. Thus, for all the

![Image of graphs showing probability density functions and their reconstructions for cloud parameters remaining after each set of image transformations: (a) \( p_L(L_4) \), (b) \( p_r(T_4) \), (c) \( p_{L\alpha}(\sigma_4) \), (d) \( p_r(L_4) \), (e) \( p_r(T_4) \), (f) \( p_r(\sigma_4) \).]
distributions \( f(y|s) \), the linear regression data were weighted by the values of the conditional probability densities \( p(y|s) \) at each point. The regressions for \( f(\lambda|L_a) \) are also shown in Fig. 2.

To gain a visual impression of how well the scheme reproduces the distributions, the probability densities \( p_r(s) \) remaining after transformations \( y \), as defined in Eq. (8), were reconstructed from the parameterizations and the original distributions \( p(s) \). This was done by iterating the formula

\[
p_{r+\delta y}(s) = p_r(s)(1 - f(y|s)\delta y)
\]

(15)

where \( \delta y \) is the change in the morphology parameter between each image transformation. Usually \( \delta y = 1 \). Equation (15) is obtained from Eq. (9) by writing

\[
p(y, s) = p_r(s)f(y/s)
\]

(16)

and then using Eq. (8) to write

\[
p_{r+\delta y}(s) = p(s) - \int_0^\infty p(y', s)\ dy'
\]

\[
= \left[ p(s) - \int_0^\infty p(y', s)\ dy' \right] - p(y, s)\delta y
\]

\[
= p_r(s) - p(y, s)\delta y
\]

(17)

The result follows by substitution from (16).

Some of the functions \( p_r(s) \) and the reconstructions are shown in Fig. 3. The edge-distance distributions are not reconstructed as well as the size distributions. This is because the nonlinear regions of the functions \( f(r|s) \) correspond to appreciable values of the conditional probabilities \( p(r|s) \). Note the “spikes” at the end of the distributions with optical depth, which arise because of the inability of the AVHRR to resolve optical depths greater than about 6. Clearly the parameterization scheme is unable to characterize this feature, which therefore requires an additional parameter set if it is to be reconstructed.

To obtain an objective measure of the accuracy of the parameterization scheme, the deviations of the reconstructed joint probability distributions, \( p_r(y, s) \), from the original distributions have been calculated using Eqs. (18) and (19).

\[
e_r(y) = \int_0^\infty |p(y, s) - p_r(y, s)|\ ds
\]

(18)

\[
e(y) = \left\{ \int_0^\infty |p(y, s) - p_r(y, s)|\ ds \right\}^{1/\int_0^\infty p(y, s)\ ds}
\]

(19)

The error \( e_r(y) \) gives the deviation of each fit as a fraction of the total probability distribution and it is equal to the sum of the deviations from the original joint histogram \( \hat{h}(y, s) \) expressed as a fraction of the total number of data points. This function has been plotted in Fig. 4 against the morphology parameter for all the data sets. The errors are typically less than 3% and diminish with increasing morphology parameter. The deviations have also been expressed as fractions, \( e(y) \), of the area under each joint distribution which give estimates of the accuracy of each reconstructed joint distribution relative to the original (Fig. 5). As one might expect, the fractional errors become larger with increasing morphology parameter as a result of the fewer numbers of data points.

5. Discussion

It is remarkable that the conditional probabilities \( f(y|s) \) are linear with \( s \) for a range of different parameters \( y \) and \( s \) and for markedly different distributions \( p(s) \). The reason for this apparent universal behavior is unclear. However, some insight into this problem may be gained by using Eqs. (4) and (9) to write

\[
p(s|y) = p_r(s)f(y|s)/p(y).
\]

(20)

In this equation, the variations of \( p(s|y) \) with \( s \) are similar to those of \( p_r(s) \) when \( f(y|s) \) varies slowly. Since \( p(s|y) \) is the distribution of \( s \) parameters over the smallest remaining morphology parameter \( y \) and

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Fig. 4. The deviation of each reconstructed joint probability density function from the original, \( p(y, s) \). This gives the error in the reconstructions relative to the total dataset: (a) size distribution errors, (b) edge-distance distribution errors.
The simple form of the conditional probability has enabled a simple linear parameterization scheme to be used to reconstruct joint distribution functions between cloud field morphology parameters and other physical parameters of clouds, where the morphology statistics are gathered using the Hit or Miss transformations. By retaining only the distribution \( b(s) = Np(s) \) in the cloud parameter \( s \) and the set of linear fit parameters \( \alpha(y) \) and \( \beta(y) \), all other distributions can be reconstructed. This represents a significant advantage where limited data storage capacity is a problem. The errors associated with the reconstructions are typically less than 3% of the total dataset.

As yet, the scheme has been tested only on cloud fields containing the one cloud type. Multiple layers of cloud will probably produce multimodal distributions in the cloud parameter \( s \). However, it has been demonstrated that the use of the conditional probability distributions in the parameterizations reduces the dependence of the scheme on the form of the distribution with \( s \), and it is expected that the scheme will fare well with multiple cloud layers. Further work is required to test this. The suitability of the parameterization scheme for radiation transfer studies will depend on the required accuracy. More complicated functions could, in principle, replace the linear functions \( f(y|s) \) to achieve greater accuracy but with a loss in the simplicity of this scheme.

**Acknowledgments.** The author wishes to acknowledge Dr. D. O’Brien for many invaluable discussions and Ms. J. Bathols for her assistance in collecting the NOAA-9 AVHRR images.

**REFERENCES**


