Frequency–Wavenumber Spectrum for GATE Phase I Rainfields

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ABSTRACT

The oceanic rainfall frequency–wavenumber spectrum and its associated space–time correlation have been evaluated from subsets of GATE Phase I data. The records, of a duration of 4 days, were sampled at 15 minute intervals in 4 × 4 km grid boxes over a 400 km diameter hexagon.

In the low frequencies–low wavenumber region the results coincide with those obtained by using the stochastic model proposed by North and Nakamoto. From the derived spectrum the inherent time and space scales of the stochastic model were determined to be approximately 13 hours and 36 km. The space–time correlation function evaluated from the frequency–wavenumber spectrum and that obtained directly from GATE Phase I records agreed.

The formalism proposed by North and Nakamoto was taken together with the derived spectrum to compute the mean square sampling error due to intermittent visits of a spaceborne sensor. The sampling error was estimated to be on the order of 10%, for monthly mean rainfall averaged over 500 × 500 km boxes, which meets the scientific requirements of the TRMM mission. This result is consistent with those previously reported in the literature.

1. Introduction

There is a renewed interest in the statistical properties of rainfields, because it appears now that observing systems will soon be in place to monitor rain rates on a globally distributed basis. Such monitoring is an integral part of any strategy to understand and characterize global change. In order to calculate the error structure of the various measurement designs proposed we must first have some preliminary estimates of the finer scale space–time structure of the rainfields. Collecting rain data for this purpose is so expensive and fraught with controversy that only a few attempts have been undertaken. One such collection effort has been the GATE (Global Atmospheric Research Program Atlantic Experiment) experiment. The purpose of this paper is to evaluate the space–time spectrum of rain on scales of hours to days in time and a few to a few hundred kilometers in horizontal distance.

North and Nakamoto (1989) have demonstrated that the space–time spectrum on the scales mentioned above is crucial to estimating the inevitable sampling errors to be expected in the usual designs contemplated such as satellite overpasses [e.g., see the study of the Tropical Rainfall Measuring Mission in Simpson et al. (1988)] or arrays of point gages. In this paper we estimate the spectrum and compare it qualitatively to a simple parametric form used in the North and Nakamoto study. We also perform some example computations of the errors expected in the satellite and raingage cases. While our study is specific to the tropical Atlantic we suspect that it is likely to hold at least qualitatively throughout the tropical oceans. Shin et al. (1990) have recently demonstrated that the gross parameters of the rainfield statistics are similar throughout the tropical Pacific.

2. Oceanic rainfall spectrum analysis

a. Introduction

The GATE experiment provided a comprehensive dataset of radar measurements of oceanic precipitation. During the GATE experiment detailed measurements from raingages and radars were made over an area called the B-scale covering a 400 km diameter hexagon and centered on 8°30′N, 23°30′W off the west coast of Africa. Arkell and Hudlow (1977) composited the radar measurements and presented an atlas of radar echos every 15 minutes. Patterson et al. (1979) converted the radar measurements to instantaneous rainrates averaged over 4 × 4 km pixels. The records used in our study were based on those from Patterson et al.

We did not analyze the whole set of records of GATE and limited ourselves to a smaller areal and time size subset of the Phase I records, which extend over the

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period 28 June to 16 July 1974. In this phase we found several occurrences of missing observations. The procedure that we followed was to linearly interpolate if the time interval among existing observations was less than 35 minutes. If the interval was larger we discarded that portion of the dataset. Due to this we obtained 11 subsets of observations sampled every 15 minutes from the original Phase I B-scale dataset with a spatial resolution of 4 × 4 km. In the final analysis we selected two subsets, whose total observation time interval exceeded 4 days. The first subset starts at 1217 LST on day 182 and ends at 1800 LST on day 186. The second subset starts at 0716 hours on day 193 and ends at 1830 hours on day 197. The main reason for the final selection was the total length in time of the subsets, in an attempt to resolve as closely as possible both diurnal and semidiurnal cycles.

b. Estimation procedure

To obtain the frequency–wavenumber spectrum three-dimensional (t, x, y) fast Fourier transforms of these two datasets were carried out. A Tukey–Hanning filter was applied to smooth the results. In the time domain, this filter has an equivalent lag window

$$\lambda(t) = [1 + \cos(\pi t/N)]/2 \quad \text{for} \quad |t| < N$$

and

$$\lambda(t) = 0 \quad \text{for} \quad |t| > N,$$

where t denotes the center point of the filtering and N is the total data length.

In the wavenumber domain a two-dimensional Tukey–Hanning filter was used with an equivalent lag window

$$\lambda(s) \times \lambda(s') = [1 + \cos(\pi s/M)][1 + \cos(\pi s'/M)]/4.$$

The spectral bandwidth of the filter was defined as $W = 1/[2N(1 - 2a + 1.5a^2)]$, where $a = 0.5$ (Blackman and Tukey 1958). Thus the frequency increment becomes $\Delta f = 4/(3N)$, where $N = 408$ for dataset 1 and $N = 428$ for dataset 2. Similarly the spectral bandwidth for the two-dimensional wavenumber domain was defined as $W = 1/[4XY(1 - 2a + 1.5a^2)^2]$, where X and Y are the total lengths in both x and y directions. Since spatial symmetry (homogeneity and isotropy) was assumed the wavenumber increment becomes

$$\Delta\nu = 16/(9X^2) \quad \text{where} \quad X = 70.$$

The ratio of the magnitude of the spectral estimate to the local magnitude of the true spectrum defined as χ^2 divided by degrees of freedom of the spectral estimate was used to evaluate 95% confidence limit (Mitchell 1966). A word of caution should be noted here. Preliminary results by Graves (1989) indicate that the confidence intervals for precipitation spectra should be wider than the ones used in our work due to the non-Gaussian structure of rainfall.

Since the observation lengths of each dataset were approximately 4 days the number of the rainfall realizations was too small to obtain a spatially homogeneous spectrum. Assuming that the underlying stochastic process is homogeneous we averaged the horizontally nonhomogeneous spectrum over a two-dimensional areal wavenumber plane to obtain a spatially homogeneous spectrum that only depends on the amplitude of the horizontal wavenumber. The advantage is that we obtained a spatially symmetric field from a limited number of observations and moreover the results could be compared with a simple spatially symmetric stochastic model that is feasible to construct. We believe that the assumption of a homogeneous stochastic process is not very restrictive in oceanic rainfall.

The space–time correlation may then be computed as the inverse fast Fourier transform of the frequency–wavenumber spectrum. The resulting spatial correlation function was compared with that evaluated by taking all products of the rainfall intensity for all discrete time and for random sampling in space (Bell 1987).

c. Frequency–wavenumber spectrum

In Fig. 1 the entire frequency–wavenumber spectra for each dataset are shown. The frequency spectra for different wavenumbers are shown in Fig. 2 in a logarithmic scale graph. Notice that for zero wavenumber the power in the lower frequency region ($f < 1$ cycle/

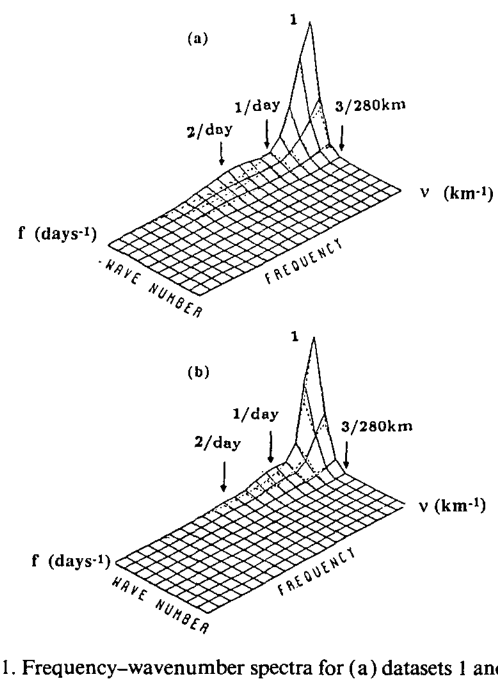

FIG. 1. Frequency–wavenumber spectra for (a) datasets 1 and (b) 2.

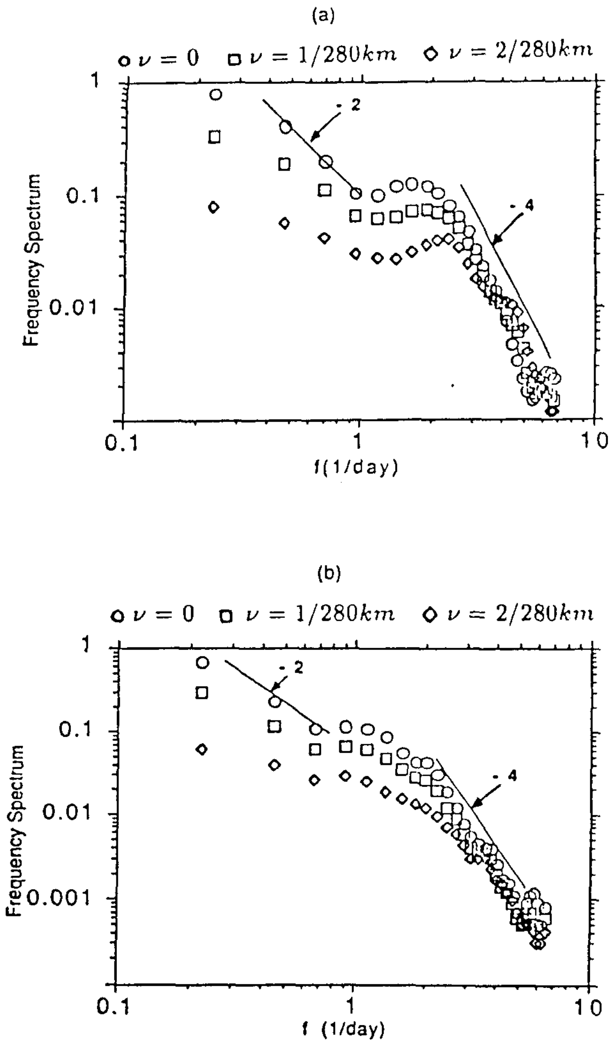

FIG. 2. Frequency spectra for fixed wavenumbers for (a) datasets 1 and (b) 2. For frequencies lower than 1 cycle/day, the spectrum follows a −2 power in both datasets. For frequencies higher than 3 cycles/day the slope is −4.

day) is approximately −2 but in the higher frequency region drops rapidly to −4. This is important for the stochastic characterization of the rainfall process since the above spectrum, which only represents the low frequency–low wavenumber region, is associated with an unbounded space–time correlation function at the origin.

It may also be observed that the fourth power becomes dominant for frequencies larger than 3 cycles/day for all wavenumbers. This will have an impact when the observed spectra are compared with those obtained from mathematical models of the rainfall process; e.g., Bell et al. (1989) and North and Nakamoto (1989). In particular, since the power of the frequency spectrum decays faster than −2 it may be expected that the inverse Fourier transform of the observed spectrum will lead to longer correlation times than those obtained from the mathematical models.

Three frequency regions may be distinguished from the figures: the second power region, the diurnal and semidiurnal cycle region, and the fourth power region. A significant interrelation between frequencies and wavenumbers up to the frequency 3 cycles/day and wavenumber range 2/280 km may also be observed. In contrast the slope is independent of wavenumber for frequencies higher than 3 cycles/day. This suggests that there may not exist length scale in the high frequencies case.

Notice that the diurnal and semidiurnal periodicities are noticeable only for very low wavenumbers. This could be explained by the fact that the diurnal and semidiurnal periodicities are affected by solar heating, which may be assumed to have an infinite length scale.

The frequency spectrum for zero wavenumber is shown in Fig. 3. In the figure it may be noticed that the diurnal and semidiurnal cycles exceed the 95% confidence limit for red-noise spectra as shown by the broken lines. The wavenumber spectra for both datasets are shown in Fig. 4. It may be noticed that the model fits well for frequencies lower than the diurnal cycle.

d. Space–time correlation function

The space–time correlation functions, obtained from the inverse Fourier transform of the observed spectra, are shown in Fig. 5. For dataset 2 a significant indication of wave propagation may be observed. The propagation speed was estimated to be 29 km h^{-1}. For dataset 1 a slight indication of wave propagation with a speed of 32 km h^{-1} may be seen.

In Fig. 6 the autocorrelation function is shown. In this case the observed autocorrelation has a longer tail than exponential decay for time lags larger than 1 hour. Zawadzki (1973) has found similar results in his analysis of precipitation patterns in which the autocorrelation follows a near-exponential form up to 1 hour time lags, but it had a longer tail for lags larger than 1 hour.

In Fig. 7 the spatial correlation functions with zero-time lag, obtained by the fast Fourier transform of the observed frequency wavenumber spectrum, are compared with the empirically obtained spatial correlation function (Bell 1987). The latter was obtained by fitting an empirical equation to the entire GATE Phase I dataset. It may be noticed that the departure from exponential decay in the space correlation becomes dominant for space lags greater than 20 km. This longer tail was also found by Zawadzki (1973), who used an optical device to directly estimate the space–time correlation of rainfall observed at the McGill FPS-18 weather radar without making any model assumptions. He also found that the space correlation follows an exponential decay up to 40 km and has a longer tail for greater distances and that the spatial correlation

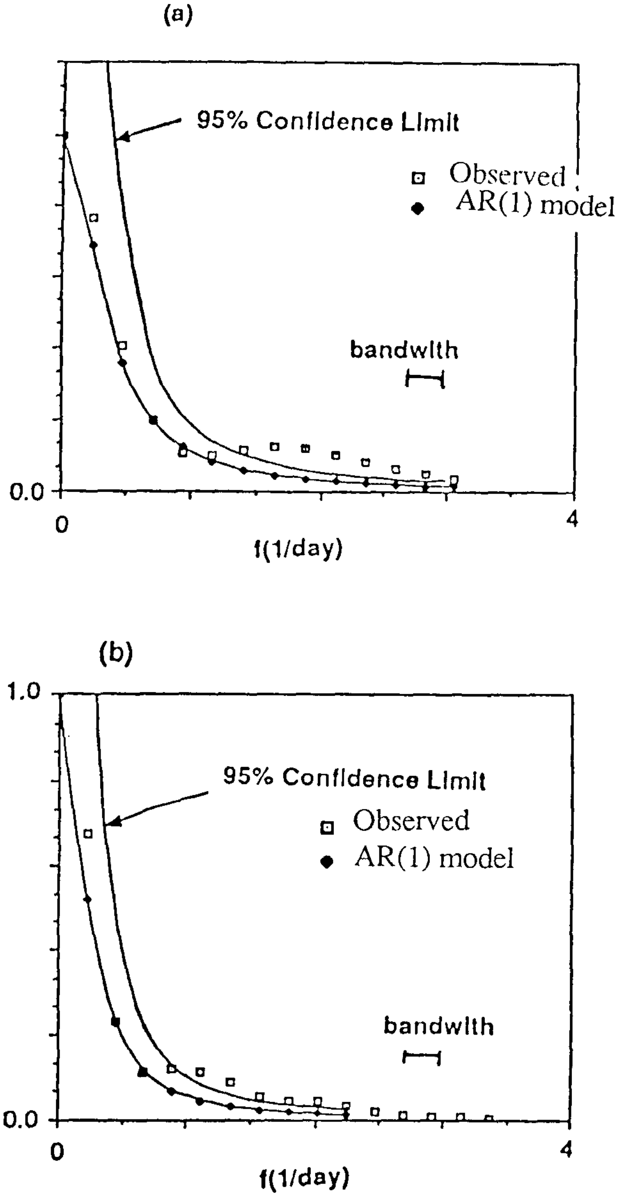

FIG. 3. Frequency spectra of zero-wavenumber case for (a) datasets 1 and (b) 2. The AR(1) continuum, its associated 95% confidence limit, and bandwidth are added.

distance defined as the *e*-folding time is between 20 to 35 km. This value is close to the value we obtained from the GATE records. Zawadzki (1973) observed that for time lags up to 40 minutes the autocorrelation of the rainfall intensity at a point is the same as the instantaneous spatial correlation between two spatial points separated by the translation of storms, which is known as Taylor's hypothesis in fluid turbulence (Taylor 1938). If a wave velocity of 29 km h^{-1}, which was found in the GATE data analysis, is used the translation distance by this wave is 20 km. This indicates that the rainfall process in GATE Phase I records, up to 20 km distance in space and 40 minutes in time, is similar to the precipitation process observed by Zawadzki.

3. Estimation of mean square sampling error

a. Introduction

Low-altitude satellite observations of precipitation will provide information about the time evolution of rainfall rates on a nearly global basis. They are, however, intrinsically constrained by their orbits to noncontinuous observations. These intermittent observations may cause errors in the estimates of areal and temporal averages of rainfall.

North and Nakamoto (1989) have proposed a technique to analytically estimate the mean square sampling error of measurements of the rainfall process due to intermittency either in space (raingages) or in time

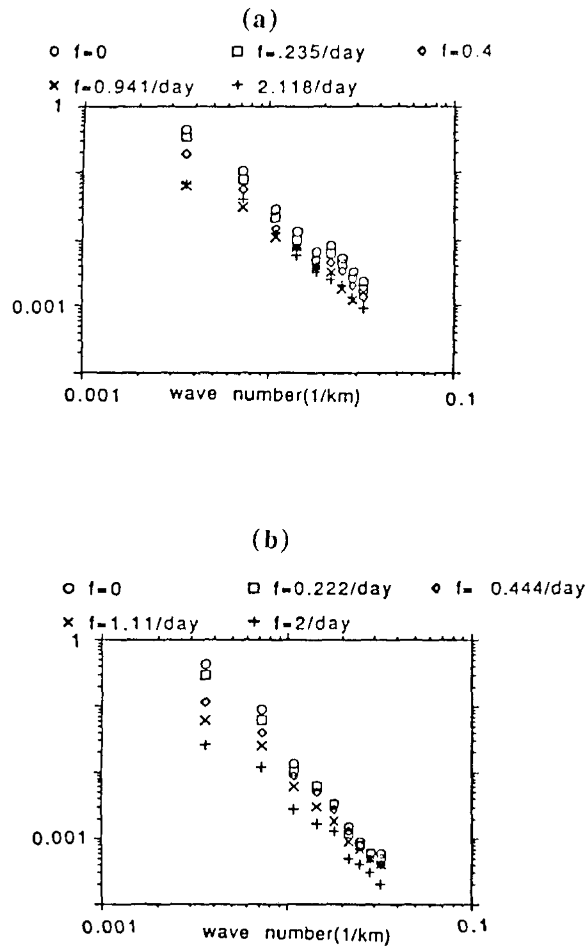

FIG. 4. Wavenumber spectra for fixed frequencies for datasets 1 and 2.

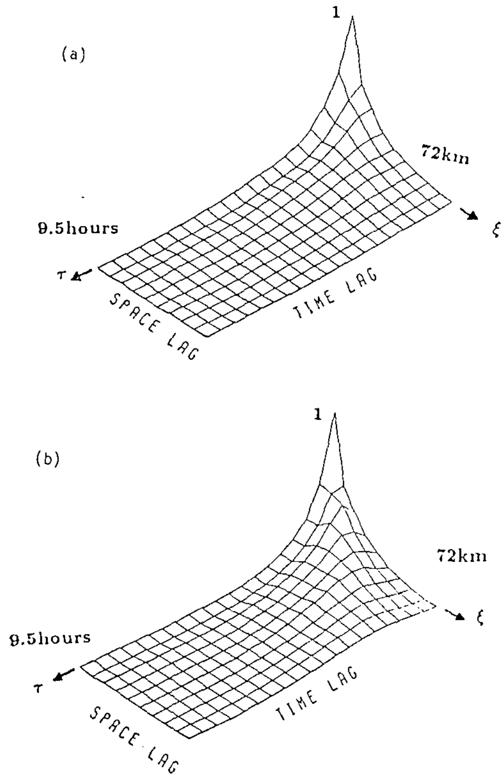

FIG. 5. Space–time correlation obtained by fast Fourier transform of the observed frequency–wavenumber spectrum for (a) datasets 1 and (b) 2. The propagation of wave with a speed of approximately 30 km h^{-1} is seen in (b) dataset 2.

(spaceborne sensors). In their work the mean sampling error is expressed as an integral over frequency and two-dimensional wavenumber with an integrand consisting of two factors, $|H(f, \nu_1, \nu_2)|^2$, a design-dependent filter, and $S(f, \nu_1, \nu_2)$, the frequency wavenumber spectrum of the precipitation field; i.e.,

$$\epsilon^2 = \iiint |H(f, \nu_1, \nu_2)|^2 S(f, \nu_1, \nu_2) df d\nu_1 d\nu_2. \quad (1)$$

From Eq. (1) it may be noticed that the information about the sampling design and the properties of the rainfield are clearly factored. Specifically for the case of satellite sampling the design-dependent filter becomes

$$H(f, \nu_1, \nu_2) = G(\pi\nu_1 L)G(\pi\nu_2 L) \times G(\pi f T)(1 - 1/G(\pi f \Delta t)), \quad (2)$$

where $G(\pi x)^2$ is the Bartlett filter, $G(\pi x)^2 = (\sin(\pi x)/\pi x)^2$.

North and Nakamoto (1989) also presented the case of a raingage network by representing it as a rectangular array of equally spaced gages; i.e., intermittent in space but continuous in time. The design-dependent filter is then

$$H(f, \nu_1, \nu_2) = G(\pi\nu_1 L)G(\pi\nu_2 L) \times G(\pi f T)[1 - 1/(G(\pi\nu_1\Delta l)G(\pi\nu_2\Delta l))], \quad (3)$$

where

Δt sampling time (e.g., 12 h)
Δl sampling length (e.g., 40 km)

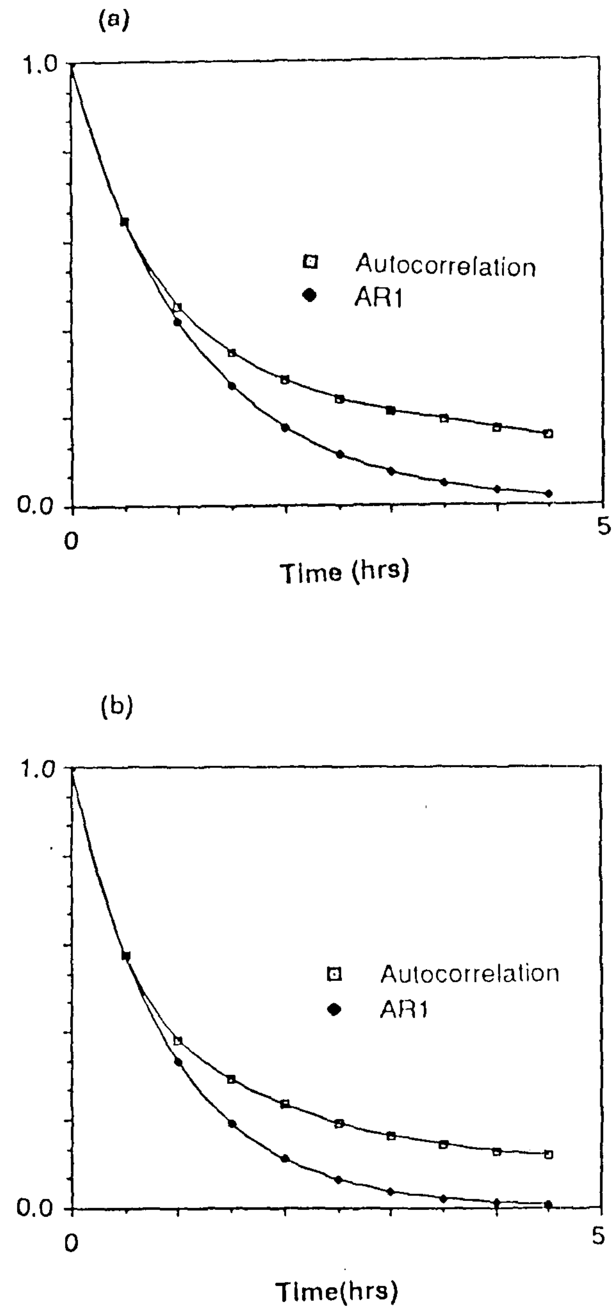

FIG. 6. Autocorrelation function obtained by fast Fourier transform of the observed frequency–wavenumber spectrum.

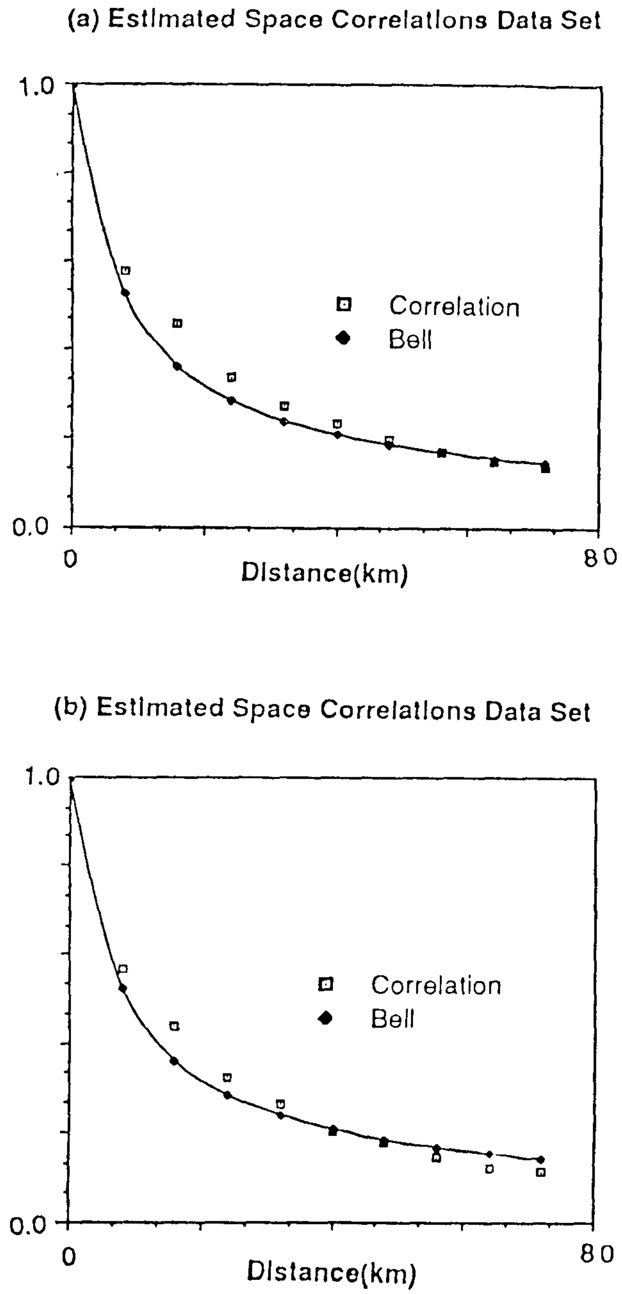

FIG. 7. Space correlation function obtained by fast Fourier transform of the observed frequency–wavenumber spectrum. The solid line is the empirical formula proposed by Bell (1987).

L length of sampling area (e.g., 280 km)
T length of sampling time (e.g., 1 month)

The Bartlett filter, $G(\pi x)^2$, converges to one when x approaches zero. Therefore, if the dimensions L of the averaging area vanish, the above formalism will predict the expected point results. Notice that the first zero of the Bartlett filter occurs at $1/L$ in the wavenumber domain. When $L \to \infty$, assuming that the spectrum $S(f, \nu_1, \nu_2)$ varies smoothly near the origin, the spectrum in the integrand may be replaced by $S(f, 0, 0)$ and the above integral over frequency domain leads the following expression for the sampling error for satellite measurement:

$$\epsilon^2 = \sigma_A{}^2 \times \frac{2\tau_0}{N\Delta t} \int \sum_{n \neq 0} \delta\left(f - \frac{n}{\Delta t}\right) S(f, 0, 0) df. \quad (4)$$

Notice that the sampling error for satellite measurements for large area-averaged random variates is affected by the frequency spectrum for the zero-wavenumber case. This result coincides with Shin and North's (1989) assumption that the area averaged precipitation may be represented by an AR(1) processes.

b. Mathematical models of the precipitation process

In the North and Nakamoto (1989) formalism estimates of the spectrum of the precipitation field were required. This spectrum may be evaluated as a function of a mathematical representation of the precipitation process like the models proposed by Bell (1987) and Waymire et al. (1984).

Bell (1987) introduced a space–time stochastic model that produces random spatial rainfall patterns similar to those observed in the GATE experiment. His model generates random fields that have non-Gaussian statistics at each grid point and are spatially and temporally correlated. The model can be tuned to fit reasonably well to GATE data. The model has recently been used for observing system simulation studies (Bell et al. 1989).

Another stochastic model of mesoscale rainfall was originally proposed by Gupta and Waymire (1979) and later generalized by Waymire et al. (1984) in the so-called WGR model. This model attempts to capture the general picture of the preferred organization observed in rainfall records (e.g., Houze 1981; Austin and Houze 1972). Although the WGR model was constructed to model extratropical storms, Valdés et al. (1990) have applied to GATE Phase I records, obtaining reasonable estimates of the parameters of the mathematical model. An analytical expression for the spectrum was derived in their work.

A simpler model than those mentioned above is the one used by North and Nakamoto (1989) and is an extension of the AR(1) process in time and in two-dimensional space which is called a Markov field (Bartlett 1978). The AR(1) model in time, or Markov process, has been widely used to model geophysical processes (e.g., Laughlin 1981; Trenberth 1984). Laughlin (1981) used an AR(1) process to model the time series of area-averaged rain rates for GATE dataset. He found a relationship between the autocorrelation time of an exponentially decaying correlation and the area-averaging size. Using the GATE dataset Shin and North (1989) found that area-averaged rainfall fits reasonably well to a Markov model for the areas 280 × 280 km and 500 × 500 km. Although these studies are instructive the temporal rainfall models can differ widely from one another just depending upon the time and space scales. This has been recently illustrated by Laughlin (1981), Rodriguez-Iturbe et al. (1984), North (1987), Bell et al. (1989), and Bell (1987).

North and Nakamoto (1989) assumed that the observations of the precipitation process may be regarded as being generated by a two-dimensional Markov field that evolves in time as a Markov process time series. Any assumptions of symmetry in the case of field quantities will pose special restrictions on both spectral and correlation statistics. If spatial symmetry in the two-dimensional field is assumed the switch from temporal process to spatial process in the directionality may arise: time marches forward only but there is no such directionality in a horizontal plane.

The proposed model follows a spatially two-dimensional damped diffusion with white noise forcing. The random variable representing instantaneous rainfall $\Psi(t, x, y)$ diffuses in a deterministic fashion through two dimensions in space (x, y) and in time t. The general form of the model is as follows:

$$\tau_0 \frac{\partial \Psi}{\partial t} - \lambda_0{}^2 \nabla^2 \Psi + \Psi = F(t, x, y), \qquad (5)$$

where τ_0 is an inherent time scale and λ_0 an inherent length scale for the above stochastic rainfall field, and F is a noise forcing function that might be white noise in both space and time up to certain frequency and wavenumber.

Using the notations from North and Nakamoto (1989) the Fourier transform of $\Psi(t, x, y)$, denoted as $\tilde{\Psi}(f, \nu_1, \nu_2)$, is easily computed as:

$$\tilde{\Psi}(f, \nu_1, \nu_2) = \frac{\tilde{F}(f, \nu_1, \nu_2)}{2\pi i \tau_0 f + (1 + 4\pi^2 \lambda_0{}^2 \nu^2)}, \qquad (6)$$

where $\tilde{F}(f, \nu_1, \nu_2)$ is the Fourier transform of the forcing F and $\nu = |\vec{\nu}|$ is the amplitude of the horizontal wavenumber vector. The power spectrum is then computed as

$$S(f, \nu_1, \nu_2) = \frac{P_F(f, \nu_1, \nu_2)}{4\pi^2 \tau_0{}^2 f^2 + (1 + 4\pi^2 \lambda_0{}^2 \nu^2)^2}, \qquad (7)$$

where $P_F(f, \nu_1, \nu_2)$ is the spectrum of the forcing noise. If $P_F(f, \nu_1, \nu_2)$ is constant, the above spectrum for the random process is a well-behaved smooth function with a maximum at the origin and tapering off in all directions both in f and $|\nu|$. The input term $F(t, x, y)$ has an infinite variance and thus it is unrealistic in the limit of the infinite frequency and wavenumber domain. The divergence at the origin is inherent as long as white noise is assumed both in the two-dimensional space and in time, the difficulty being the spectrum $S(f, \nu_1, \nu_2)$ dies too slowly at infinity. Thus it may be said that the diffusion mechanism does not smooth the noise sufficiently and it is expected that the diffusion model may be only valid in the low frequencies–wavenumbers region in such a manner that the random process will have a finite variance.

The parameters of the space–time Markov process; that is, the inherent time scale τ_0 and inherent length scale λ_0, were estimated in our study from GATE data, to be 10.3 and 36.4 km for dataset 1 and 15.8 hours and 36.5 km for dataset 2. These values are defined as the autocorrelation time without spatial variation and space-correlation length without temporal variation. From our results the autocorrelation time for a zero wavenumber, which corresponds to virtually infinite wavelength, is approximately 13 hours and the space-correlation time for zero frequency, which corresponds to infinite time mean, is approximately 36 km. These values are in excellent agreement with those estimated by alternative methods (North and Nakamoto 1989).

In Figs. 3, 6 and 7 the spectrum and correlation functions of an AR(1) model were incorporated so that the observed values may be compared. Figure 8 shows the observed wavenumber spectra for the zero frequency case. From the figure it is seen that the deviation from an AR(1) process in the lower frequencies region is within the 95% confidence limit.

c. *Evaluation of mean square sampling error for GATE area*

The spectrum, obtained in our study from subsets of GATE Phase I records, is combined in this section with the design-dependent filter of the North and Nakamoto approach to estimate the mean square sampling error due to intermittent visits by a spaceborne sensor of the precipitation process. Two cases will be presented. The first uses the random diffusion spectrum with its parameters estimated from the observed spectrum; i.e., both diurnal and semidiurnal cycles are excluded. To simplify the calculations it was assumed that the diffusion model spectrum represents the observed rainfields up to frequencies of 3 cycles/day. Although the mechanism for producing the fourth-order power is not yet known, their contribution to the mean square error is much less than the extrapolated power of −2 and our estimations will then be on the conservative side. It was also assumed that the satellite will cover the entire GATE Phase I area (280 × 280 km) every 12 hours. For this case the following mean square sampling errors, ϵ_T, were obtained:

$$\epsilon_T = 0.044\sigma_A, \quad \text{(set 1)}, \qquad (8)$$

$$\epsilon_T = 0.033\sigma_A, \quad \text{(set 2)}. \qquad (9)$$

These values are close to that estimated by North and Nakamoto (1989) for a similar case. Laughlin (1981) used the same area (280 × 280 km) and found that the ratio of the standard deviation to the areal

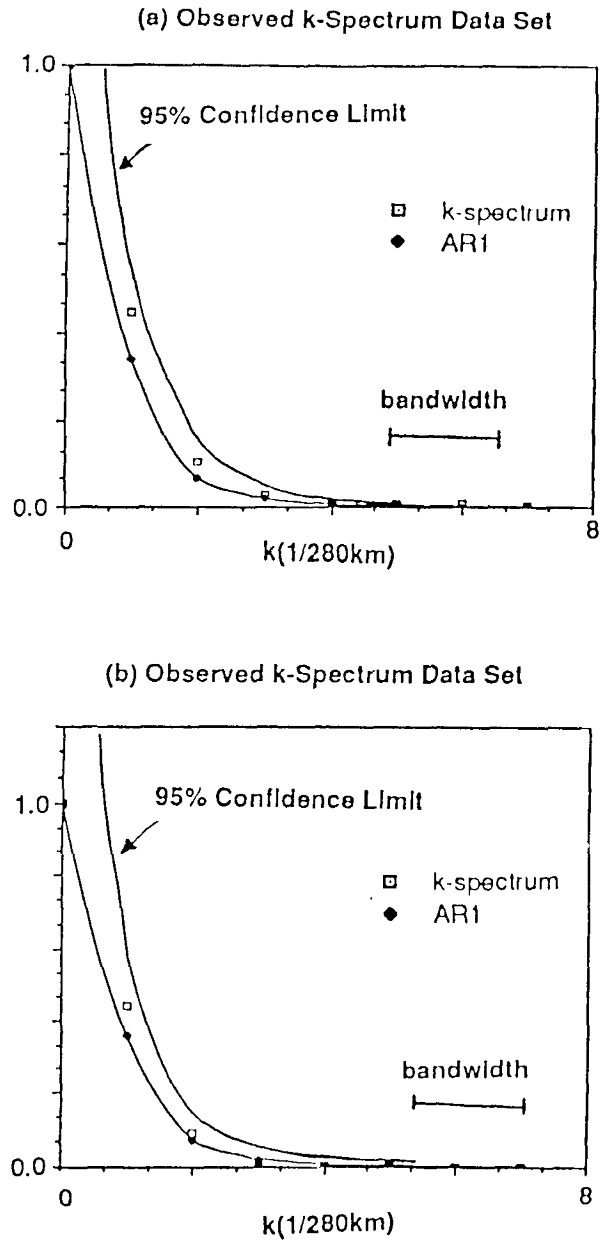

FIG. 8. Wavenumber spectra for zero-frequency case for (a) datasets and (b) 2. The AR(1) continuum, its associated 95% confidence limit, and bandwidth are added.

average was 1.3. To facilitate comparisons the previous results were scaled to represent a percentage of the areal mean rainrate and the sampling errors were estimated as 5.8% and 4.3% for sets 1 and 2, respectively.

Since the diurnal and semidiurnal cycles are noticeable in GATE Phase I records, the second case used the observed spectrum instead of the random diffusion spectrum, all other assumptions remained constant. Since the sampling time is 12 h; i.e., $\Delta t = 0.5$ days, the semidiurnal cycle may play an important role in the estimate of the sampling error. Since the observed spectra for both sets decay rapidly for frequencies larger than 1 cycle/day, only the first term of the sum may be taken. The ratio between the powers of the diffusion model and the observed spectrum, at a sampling frequency of 12 hours, was 1.9 and 1.72 for datasets 1 and 2, respectively. The sampling errors were then estimated as

$$\epsilon = 1.9\epsilon_T = 0.083\sigma_A, \quad \text{(set 1)} \qquad (10)$$

$$\epsilon = 1.72\epsilon_T = 0.056\sigma_A, \quad \text{(set 2)}. \qquad (11)$$

Scaling the above results the errors may be estimated to be 10.8% and 7.3% for datasets 1 and 2, respectively. It must be emphasized, however, that these results are for a sampling time of exactly 12 hours. The true diurnal and semidiurnal cycles powers should be very sharp, but due to the short length of GATE records, they appeared in the estimated spectrum as very broad bands. Hence if the missions visit the area at sampling times smaller than 12 hours, say 11.75 hours, the sampling error should be much smaller than the one calculated here and closer to the previous estimates.

4. Final comments

In this paper the frequency–wavenumber spectrum was evaluated from GATE Phase I records. Due to missing observations only a portion of the records were used to estimate the spectrum and its associated space–time correlation function. Although no other evaluation of this spectrum exists in the literature the results of our study were compared with previous studies that used the entire set and the agreement was significative. We believe that the results will add to the understanding of the space–time structure of tropical oceanic rainfall.

It was found that the observed spectrum, excluding the diurnal and semidiurnal cycles, can be reasonably represented by a space–time AR(1) process in the low frequency–low wavenumber region, with the inherent time scale to be on the order of 10–15 hours and the length scale to be approximately 36 km. Different slopes for the frequency spectrum in two different frequency regions were found. For frequencies lower than 1 cycle/day the slope was approximately −2 while it drops sharply for frequencies higher than 2 cycles/day. Further it was found that in the high frequency region the spectrum is independent of wavelength, suggesting that there may exist no length scale in the high frequencies.

The mean square sampling errors for estimates of areal and temporal averages of the rainfall process were estimated in our study using the North and Nakamoto (1989) approach. For areas the size of the GATE Phase I experiment (280 × 280 km) and a sampling period of 12 hours, the sampling errors were found to be around 3%–4%. This value is similar to those estimated by North and Nakamoto (1989). By including the semidiurnal cycle for sampling times of exactly 12 hours, the sampling error increases to 7%–10%. Hence, a sun-synchronous satellite might be expected to have nearly twice the sampling errors as one that precesses through the local hours. It is for this reason that the TRMM satellite will sample with a period slightly smaller than 12 hours (Simpson et al. 1988) and in this case the errors will be smaller (as discussed earlier).

If the derived spectrum is to be used to evaluate sampling errors of future satellite missions it must account for the fact that the spaceborne sensor will cover only partially the area under study (grid box) during each visit. The effect of this on the sampling errors should then be analyzed. To approach this problem Bell et al. (1989) and Shin and North (1988) took calculated satellite orbits to determine the fraction of the grid box covered in each pass and showed that for realistic orbits with partial area coverage the above sampling errors are roughly doubled.

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