A Test for the Period of 18 Cycles per Year in Rainfall Data

GLENN W. BRIER,
U. S. Weather Bureau, Washington, D. C.
RALPH SHAPIRO
Air Force Cambridge Research Laboratories, Bedford, Mass.
AND NORMAN J. MACDONALD[1]
Regis College, Weston, Mass.

ABSTRACT

A number of different analyses are used to examine 63 years of daily United States rainfall data to determine whether a suggested periodicity of 18 cycles per year exists. Although a slight tendency was found for such a period to persist in phase during the 63 years, the amplitude is extremely small. On the basis of the several tests performed we conclude that there is no reality to a periodicity at or near 18 cycles per year.

1. Introduction

In commenting on the results of the non-parametric statistical test that Brier (1961) applied to three sets of rainfall data, Hannan (1962) suggested that the association found between the series was due entirely to a periodicity in the data at or near 18 cycles per year. He based his conclusions on an analysis of the data published by Brier (1961) along with the daily means of the rainfall series that were supplied to him. He stated that a thorough examination of the spectrum for the original data of actual daily values would be needed to finally establish whether the effect found by Brier was due to this cycle. In reply, Brier (1962) presented the results of a spectral analysis of the daily values of the United States rainfall index for the period 1 January 1950 to 31 December 1961. A peak in the spectra did show up at approximately 18 cycles per year, but there were a number of other peaks just about as high. Daily values of the United States rainfall index have now been extended to cover the period 1 January 1900 to 31 December 1962 in connection with another study (1963). It thus becomes possible to examine some of the questions raised by Hannan through the analysis of a long series of homogeneous data. This paper reports the results of a study of the suggested periodicity of 18 cycles per year.

2. Data and analyses

The data used in this study are the same 63 years of daily United States precipitation data which were used by us in a study of calendaricities (1963). A number of different analyses were performed on these data to resolve the question of the existence of a peak in the rainfall spectrum around a frequency of 18 cycles per year. Power spectrum analysis of the raw data was used to determine whether any obvious peaks exist in the data. Harmonic analyses were performed to obtain phase information and also to determine whether concentrations of power exist in such narrow frequency intervals as to be undetected by the power spectrum analysis. In addition, a superposed epoch analysis was performed. This analysis, which is also capable of detecting concentrations of energy in the spectrum, has some advantages as compared with harmonic analysis.

Figs. 1 through 6 show the results of the power spectrum analyses of the 63 years of data. Because of limitations of the computer program, the data were divided into six periods—each containing 4000 data points (approximately 11 years). Table 1 shows the dates for the beginning and ending of each of these six periods. Some overlapping occurs between periods 5 and 6 in order to maintain 4000 data points in each period. In each of the figures, the ordinate is the fraction of the total variance of the time series contributed by each

TABLE 1. Dates of beginning and ending of six time periods (4000 days each) used in the power spectra of Figs. 1–6. Period 1 corresponds with Fig. 1, etc.

Period	Dates
1	1 Jan. 1900–14 Dec. 1910
2	15 Dec. 1910–26 Nov. 1921
3	27 Nov. 1921– 8 Nov. 1932
4	9 Nov. 1932–22 Oct. 1943
5	23 Oct. 1943– 4 Oct. 1954
6	19 Jan. 1952–31 Dec. 1962

[1] Now at the Air Force Cambridge Research Laboratories, Bedford, Mass.

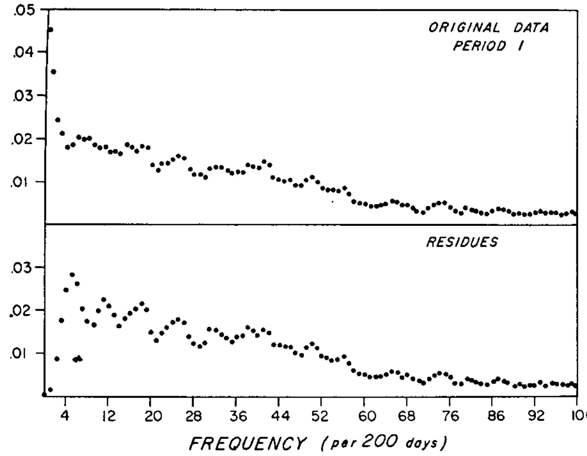

FIG. 1. Spectrum of rainfall data for period 1—1 January 1900–14 December 1910.

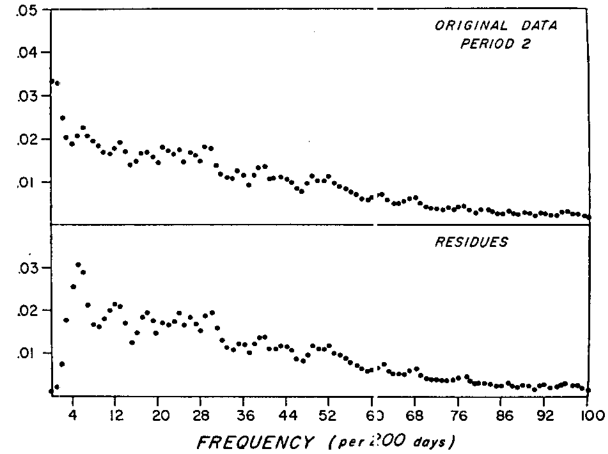

FIG. 2. Spectrum of rainfall data for period 2—15 December 1910–26 November 1921.

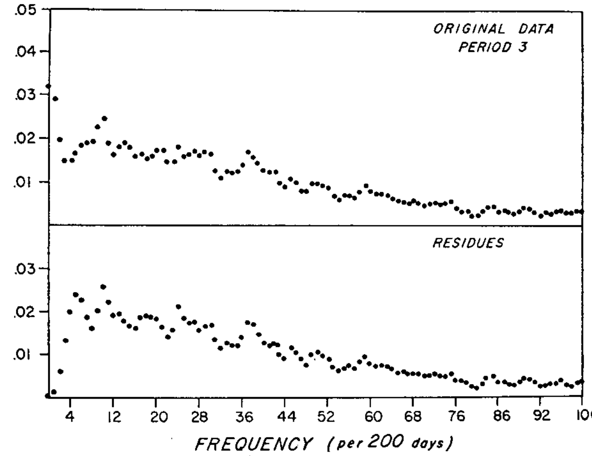

FIG. 3. Spectrum of rainfall data for period 3—27 November 1921–8 November 1932.

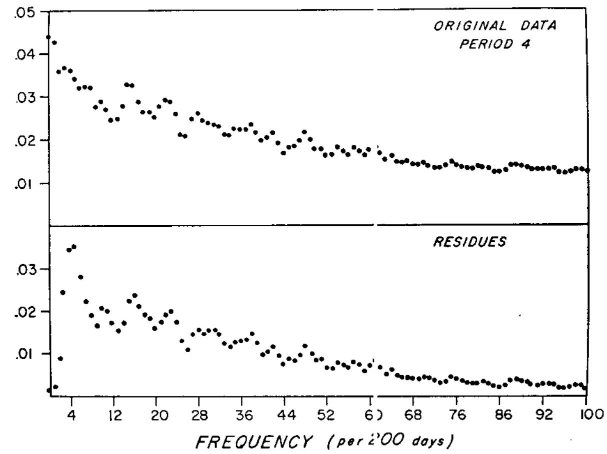

FIG. 4. Spectrum of rainfall data for period 4—9 November 1932–22 October 1943.

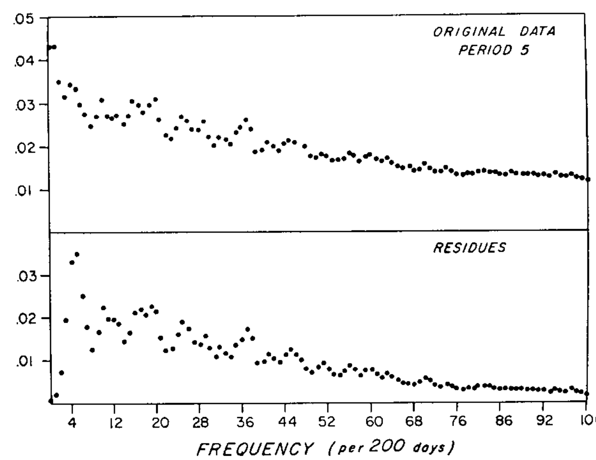

FIG. 5. Spectrum of rainfall data for period 5—23 October 1943–4 October 1953.

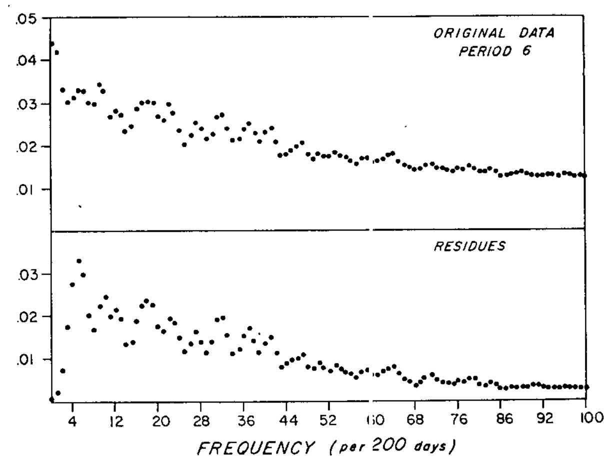

FIG. 6. Spectrum of rainfall data for period 6—19 January 1952–31 December 1952.

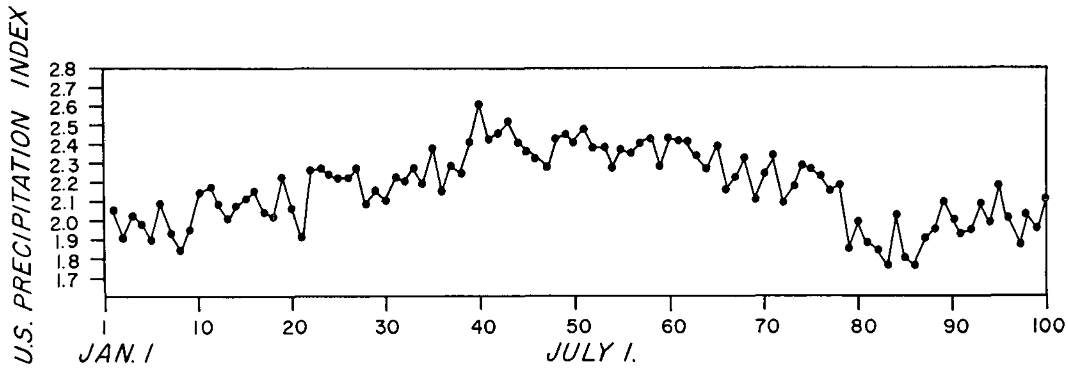

Fig. 7. Transformed precipitation index summarized according to the annual period divided into 100 classes for 1 January 1900 to 14 December 1910.

frequency. The abscissa is the frequency in cycles per 200 days on a linear scale. Therefore, a period of 20 days corresponds to the frequency equal to 10 in these figures. The upper curve in each figure shows the spectrum of the unfiltered, unnormalized, raw daily data. The lower curve in each figure shows the spectrum of the departures of the daily precipitation from a 61-day running mean. The principal effect of this filtering is to eliminate most of the power contained in the first few frequencies. In addition, however, it enhances the power contained in frequency 5 by almost 50 per cent. Since all of the spectra of the original data are decidedly "red" (decreasing power with increasing frequency), the spectra of the filtered data all show a manufactured peak at frequency 5.

The spectra of the raw data (upper curves in Figs. 1–6) all show a large amount of power at the low frequencies —a reflection of the annual period in the data. However, they show no obvious and consistent peaks around a period of 20 days (frequency 10). Although there is a large peak at frequency 10 in Fig. 3, the other figures do not show similar behavior, and in view of comparable peaks at other frequencies (e.g., frequencies 15–16 in Fig. 4), the peak in Fig. 3 must be attributed to sampling fluctuations. One must therefore conclude from this analysis that if a significant oscillation exists with a period around 20 days, either its amplitude is small or its band-width is extremely narrow. In any event, the fraction of the variance contained within it must be extremely small.

In order to meet the requirements for the spectral analysis program described above, the United States rainfall series was divided up into 6 periods. This same division was maintained in the harmonic analyses described here with the exception that the last (6th) period was adjusted to being immediately after the end of the 5th period.

The frequency distribution of the 23,010 values of the original rainfall index was found to be considerably skewed. Therefore the daily rainfall index was normalized by the cube-root transformation which produced values that fit very closely to a normal distribution. Since the basic period considered here was the annual one of 365.2422 days, each day from 1 January 1900 to 31 December 1962 was given an identification number to locate its position in respect to this cycle. 1 January 1900 was given an index of 0.00000, 2 January was given an index of $1/365.2422=0.00274$, 3 January 1900 was given the index 0.00548, etc. The precipitation data were then summarized in 100 classes, the first class contained all those days with index number 0.00000 to 0.00999, the second class contained all those days with index number 0.01000 to 0.01999, etc. This was done separately for each of the six periods of time.

The results for period 1 are shown graphically in Fig. 7. A large annual term is noted, in fact, a harmonic analysis of the means of these 100 classes indicated that over 67 per cent of the total sum of squares was accounted for by the annual term (frequency 1). Frequencies 1, 2, 3 and 4 together account for 75.9 per cent of the total sum of squares, leaving 24.1 per cent to be accounted for by the remaining 46 harmonics. Similar results were found for each of the other 5 periods, with the first four harmonics always accounting for at least 50 per cent of the sum of squares. Fig. 8 shows the result

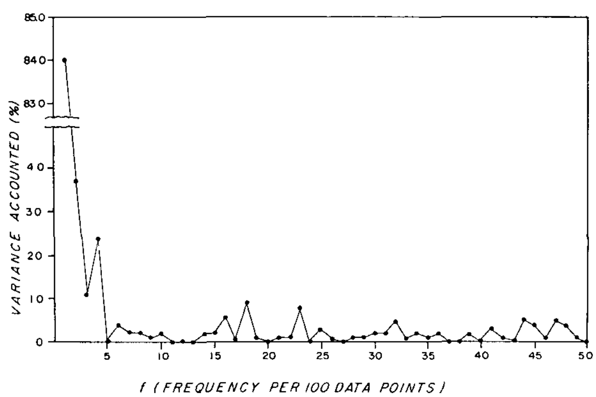

Fig. 8. Average variance contributed by each harmonic (frequency from 1 to 50 cycles per year).

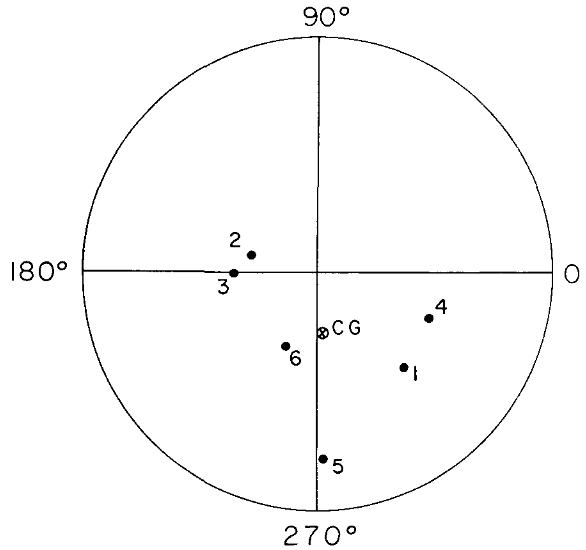

FIG. 9. Polar plot showing for each of the six time periods the amplitude and phase of the harmonic with a frequency of 18 cycles per year.

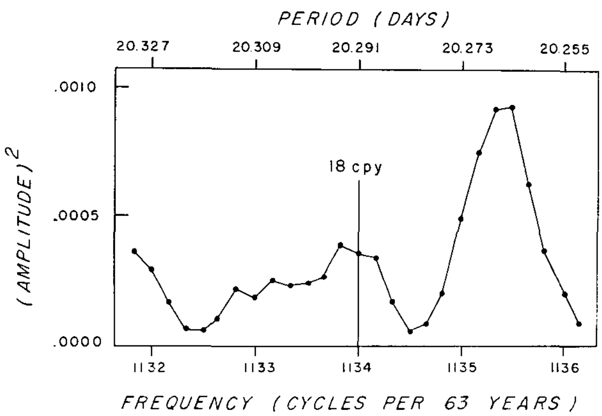

FIG. 10. Periodogram showing relative amplitude of harmonic waves with periods at or near 18 cycles per year.

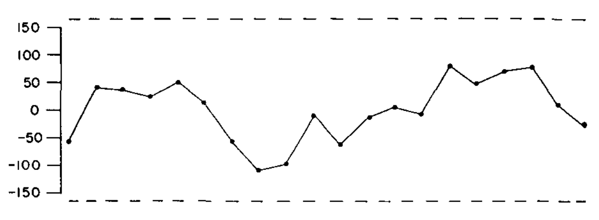

FIG. 11. Superposed epoch analysis of the 63 years of rainfall data based upon a frequency of 18 cycles per year. The ordinate scale is arbitrary and the abscissa represents one complete period of 365.24 divided by 18 days.

of the harmonic analysis of the 100 class means based on the entire 63 years of record. The first 4 frequencies account for 91.2 per cent of the variance and about 10 per cent of the remaining variance is accounted for by a peak corresponding to 18 cycles per year (cpy).

The next step was to determine if the phase of the period with 18 cpy was consistent during the six different periods. The amplitude and phase of the fitted harmonic for each of the six periods of time were determined. The results are shown in the polar plot of Fig. 9. The consistency of the phase from one time period to another can be tested by determining whether the center of gravity of the six points departs significantly from the origin. This result does not reach the 5 per cent level of significance, but it is of interest to point out that in Fig. 9 all phases are between 165° and 336°, none of them falling in the other semicircle of the dial.

The next question examined was whether the amplitude of the wave with a frequency of 18 cpy was significantly larger than those close to it. During the 63 years there were 23,010 daily observations and thus a wave with a frequency of 18 cpy corresponds to a frequency of $18\times63=1134$ cycles in 23,010 days. The amplitudes of other waves with frequency varying from about 1132 to 1136 cycles in 63 years were then determined. The results shown in Fig. 10 show no significant peak at 1134 cycles (18 cpy). In fact, a higher and sharper peak is found at about 1135.5 cycles per 63 years.

Superposed epoch analysis which is based upon fixed frequencies in a particular time interval is capable of revealing concentrations of power in narrow frequency bands. In addition to being simpler to perform than harmonic analysis, the superposed epoch analysis makes no assumption about the shape of the oscillations in the time series. Therefore, as an additional analysis tool, a superposed epoch analysis was performed on the raw precipitation data based upon a frequency of 18 cycles per 365.24 days. That is, 20 column-mean values were obtained. In each column, the individual data points are separated from each other, on the average, by multiples of 365.24/18 days. The results of this analysis are shown in Fig. 11 which shows the departures of the column means from the population mean. The dashed horizontal lines indicate approximately the 5 per cent significance level for these departures as determined by Student's t-test. If a significant oscillation with a frequency around 18 cycles per year (with a band-width of at least a few tenths of a day) is present in the United States precipitation data, one should expect much if not most of the variance of the curve in Fig. 11 to be contained in wave number one. It is obvious from the figure that this is not the case. Furthermore, none of the departures in the figure reach the 5 per cent significance level. Therefore, the bandwidth of any such oscillation, if it exists, must be less than a few tenths of a day and the total variance contained with it so small as to be masked by the noise fluctuations.

Finally, the first 10 years of data were examined to resolve what might appear to be an inconsistency between the results of the spectral analysis and the harmonic analysis. For example, the power spectrum for period 1 gave no evidence of a peak around 18 cpy while the harmonic analysis of the 100 class means indicated that this harmonic accounted for 2 per cent of the total variance of the 100 class means (8 per cent of the variance remaining beyond that accounted for by the first 4 frequencies). This is not significant when comparing it with the 14 per cent necessary by considering it as being selected as the largest out of a set of variances (see Cochran, 1941). Nevertheless, it is still higher than the variance accounted for by other frequencies such as 16, 17, 19 or 20 cpy. A possible explanation of this might be that the 18 cpy wave was more consistent or constant in phase than others. This was examined by determining the amplitude and phase of the 18 cpy wave separately for each of first 10 years in the same manner as that illustrated in Fig. 9. The waves with frequency 16, 17, 19 and 20 cpy were treated in the same way. The analysis showed a tendency for persistence of phase only for the 18 cpy wave but this was not statistically significant ($p > 0.20$). On the average, the amplitudes of the individual annual waves for 17 cpy were slightly greater, but the lack of persistence in phase for this harmonic during the 10 years resulted in a smaller net 10-year mean amplitude than the one for 18 cpy.

3. Discussion

The results from the harmonic analysis of the 63-year mean daily rainfall give some slight support to Hannan's suggestion that the rainfall data used by Brier (1961a) contain a component with a frequency of 18 cpy. However, a more detailed analysis of the original daily values shows that the amplitude of this component is extremely small and of doubtful statistical significance. Its slightly enhanced amplitude appears to be the result of a slight but not significant tendency for its phase to lock in with the annual cycle. The spectrum and superposed epoch analyses show no evidence of a peak around 18 cpy and periodogram and harmonic analyses show other periods with equal or greater amplitude. In view of these findings and the lack of any physical basis for such a periodicity, it is concluded that there is no reality to the period with a frequency of 18 cycles per year.

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