Computations of Transverse Circulation in a Steady State, Symmetric Hurricane[1]

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ABSTRACT

Radial and vertical circulations in a steady state, circular symmetric hurricane are computed from a given tangential wind field for various values of exchange and drag coefficients.

1. Introduction

It is generally agreed that of the three wind components in a hurricane the tangential is best known, the radial is measurable but contains large errors, and the vertical can only be deduced from the others. This work is an attempt to describe the transverse (i.e., radial and vertical) circulation in a hurricane from a known distribution of tangential velocity.

For the purpose of this calculation the hurricane is assumed to be in a steady state, and to be circular symmetric. The basic equations employed in the computation are essentially the same as those used by Krishnamurti (1961) for the same purpose. However the method of computation is different. In the present study the problem was programmed for solution on the IBM 7094 computer, whereas Krishnamurti employed the more laborious method of characteristics. More significantly, the present model includes a frictional boundary layer, and therefore produces a more realistic distribution of radial flow.

2. The basic equations

A cylindrical coordinate system is employed with pressure, p, as the vertical coordinate, azimuth angle, θ, in the counterclockwise direction, and r, the radial coordinate increasing outward from the hurricane center which will be considered as the origin.

The flow is restricted under the following assumptions:

a) the motion is in steady state;
b) all variables are axially symmetric.

A real hurricane may be in a steady state for only a short time, and may never be symmetric. However, it is probable that a major part of the hurricane's dynamics can be accounted for by the steady, symmetric approximation. In any event it appears reasonable to investigate the steady, symmetric vortex first before attempting experiments with the more complex, unsteady asymmetric case.

The assumption of steady state eliminates the local derivatives and reduces the equations to diagnostic form. The assumption about symmetry reduces the problem to two dimensions. Applying the assumptions to the general equations, the following horizontal equations of balanced motion are obtained:

$$u\left(\frac{\partial v}{\partial r}+\frac{v}{r}+f\right)+\omega\frac{\partial v}{\partial p}=\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)\right]+\frac{\partial}{\partial p}\left[\kappa\left(\frac{\partial v}{\partial p}\right)\right], \quad (1)$$

$$u\frac{\partial u}{\partial r}-v\left(\frac{v}{r}+f\right)+\omega\frac{\partial u}{\partial p}+g\frac{\partial z}{\partial r}=\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right]+\frac{\partial}{\partial p}\left[\kappa\left(\frac{\partial u}{\partial p}\right)\right]. \quad (2)$$

The third equation of motion is represented by the hydrostatic approximation

$$0=-g\frac{\partial z}{\partial p}-\frac{RT}{p} \quad (3)$$

and the equation of continuity reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}(ur)+\frac{\partial \omega}{\partial p}=0, \quad (4)$$

where v is the tangential velocity, positive in the direction (counterclockwise) of increasing θ, u is the radial velocity, positive outward, ω is the "vertical velocity," dp/dt, positive in the direction (downward) of increasing pressure, z is the geopotential height of an isobaric

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surface, g, is assumed to be 9.8 m sec^{-2}, f is the Coriolis parameter, T is the temperature, and R the gas constant. The horizontal and vertical kinematic eddy viscosity coefficient are ν and κ, respectively. In the p-coordinate system the "vertical" exchange coefficient, κ, has the dimension of pressure squared divided by time, while ν has the dimension of length squared divided by time. The relation between κ and the exchange coefficient, k, in the z-coordinate system is $\kappa=\rho^2g^2k$, where k has the same dimension as ν.

The first law of thermodynamics in the form

$$H=C_p\left[u\frac{\partial T}{\partial r}+\omega\frac{\partial T}{\partial p}\right]-\frac{RT}{p}\omega, \tag{5}$$

where C_p is the specific heat of constant pressure, and may be used to evaluate the diabatic heating rate, H.

3. The stream function and momentum

The absolute angular momentum, M, per unit mass about the vertical axis at the center of the hurricane is given by

$$M=vr+fr^2/2. \tag{6}$$

Then

$$\frac{1}{r}\frac{\partial M}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}(vr)+f=\eta, \tag{7}$$

where η is the absolute vorticity. Also

$$\frac{1}{r}\frac{\partial M}{\partial p}=\frac{1}{r}\frac{\partial}{\partial p}(vr)=\frac{\partial v}{\partial p}. \tag{8}$$

A stream function, ψ, which satisfies the continuity equation (4) is defined by:

$$ur=\frac{\partial\psi}{\partial p};\quad \omega r=-\frac{\partial\psi}{\partial r}. \tag{9}$$

Substituting (7), (8) and (9) into (1) we obtain

$$\frac{\partial M}{\partial r}\frac{\partial\psi}{\partial p}-\frac{\partial M}{\partial p}\frac{\partial\psi}{\partial r}=\frac{\partial}{\partial r}\left[\nu r^3\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right]+\frac{\partial}{\partial p}\left[\kappa r^3\frac{\partial}{\partial p}\left(\frac{v}{r}\right)\right]. \tag{10}$$

Since, for any prescribed field of v, (10) is a first order linear differential equation in ψ, one boundary condition is required for each of the independent variables r and p. The stream function on the axis of the model ($r=0$) is assumed to be zero as a boundary condition which is consistent with the symmetry assumption. For the top of the friction layer, boundary values of the stream function along r are either specified or computed. In one method used here, vertical velocities for the top of the friction layer are adopted from earlier studies and used to compute boundary values of the stream function from (9). In a second method, a frictional drag equation is used to compute the stream function at the top of the friction layer from a specified drag coefficient.

If the exchange coefficients and the tangential velocity field are known, (10) can now be solved for the stream function ψ. After evaluation of the stream function, radial and vertical velocities can be computed from (9). The geopotential height of the pressure surfaces can then be evaluated from (2) if the heights are known at some outer distance r_0 from the center. Using Eq. (3), the temperature distribution can be computed. Finally, (5) may be used to evaluate the diabatic heating distribution.

4. Model equations for the friction layer

Since (10) requires boundary values of the stream function along r for the bottom level of the main layer before computations can be performed, a method of evaluating it is required.

The vertical velocity at the lower boundary of the upper layer can either be specified as data, or computed. In this section the method of computing the stream function at the bottom of the upper layer (top of the friction layer) is described.

The following assumptions will be adopted for the friction layer:

(a) $\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)\right]\ll\frac{\partial}{\partial p}\left[\kappa\left(\frac{\partial v}{\partial p}\right)\right]$

(b) $\omega\frac{\partial v}{\partial p}$ is negligible.

Thus, from (1)

$$u\eta=\frac{\partial}{\partial p}\left(\kappa\frac{\partial r}{\partial p}\right)=-g\frac{\partial\tau}{\partial p}, \tag{11}$$

where τ is the stress associated with vertical shear of the tangential wind. Eq. (11) is integrated between the pressure p_s at the sea surface, and the pressure p_0, at the top of the friction layer where τ is assumed to vanish. From (11)

$$\int_{p_s}^{p_0}u\eta dp=g\tau_s, \tag{12}$$

where τ_s is the tangential surface wind stress.

The surface stress may be estimated from the semi-empirical formula

$$\tau_s=\rho C_D v_s|v_s|, \tag{13}$$

where C_D is a drag coefficient.

Replacing u by $\partial\psi/r\partial p$, (12) may be integrated to obtain

$$\bar{\eta}(\psi_0-\psi_s)=rg\rho C_D v_s|v_s|, \tag{14}$$

where $\bar{\eta}$ is the mean vorticity in the friction layer, ψ_0 is the stream function at P_0, and ψ_s is the stream function at the sea surface.

The boundary condition $\psi=0$ will be imposed at $r=0$, and it will be assumed that $\omega_s=0$.[3] Thus, from (9) it follows that $\psi_s=0$, and

$$\psi_0=\frac{rg\rho C_D v_s|v_s|}{\bar{\eta}}. \tag{15}$$

A similar equation has been used by Ooyama,[4] for computations of hurricane development.

In the strict sense of the derivation, the values of v_s and C_D to be used should be determined at the surface or in the surface boundary layer. However, since the value for v_s is hard to determine in that manner, values corresponding to the lowest level of the upper layer will be used. Similarly, $\bar{\eta}$ will be approximated by η_0, the value of η at the above-mentioned level.

The alternative method of evaluating ψ_0 is to specify the vertical velocity at level p_0. Then using (9) for ω_0, ψ_0 can be computed.

5. The model and boundary conditions

Essentially this is a two-layer model, with a lower layer (the friction layer) governed by (15), and an upper layer extending upward from the level p_0. Computations are carried out up to the 100-mb level, which is usually assumed to be near the top of the hurricane (Riehl, 1954). The top of the friction layer, p_0, will be assumed to be a fixed isobaric surface corresponding to 900 mb. This means that the height of the friction layer above the sea surface is assumed to decrease inward toward the hurricane center.

The values of the horizontal and vertical exchange coefficients will be assumed constant for each set of computations. Although this might not be the best assumption, this is the simplest approach in the absence of knowledge of the true variation. The drag coefficient will also be taken as a constant in (15).

6. Computational procedure

All the computations are done using a finite difference approximation for the differential equation.

The main grid is identified as (i,j), i positive to the right along radial distance and j positive upward along decreasing pressure p. An intermediate grid is identified as (k,l) where $k=i+\frac{1}{2}$ and $l=j+\frac{1}{2}$ grid distance. This is used to provide maximum resolution by permitting derivatives to be approximated by difference over one grid length rather than two. The tangential velocity data are given in the form of a table corresponding to the (i,j) grid.

A finite difference equation corresponding to Eq. (10) is

$$\bar{M}_r{}^p\bar{\psi}_p{}^r-\bar{M}_p{}^r\bar{\psi}_r{}^p=C(r,p), \tag{16}$$

where $\bar{M}_r{}^p$ implies averaging along p and taking the derivative along r. Other symbols have similar meaning.

The term $C(r,p)$ in Eq. (16) is made up of two parts, the first containing the horizontal exchange coefficient and the second the vertical exchange coefficient. This is computed between main grid points (at intermediate points), to be consistent with the averaging finite difference scheme that is used.

Since the stream function is known on the axis (boundary condition) and at the bottom of the main layer (either given or computed), the stream function at $r=\Delta r$ and $p=\Delta p$ can be computed from

$$\begin{aligned}\psi_{i,j}=[(\bar{M}_r{}^p)_{k,l}\Delta r(\psi_{i,j-1}+\psi_{i-1,j-1}-\psi_{i-1,j})\\ -(\bar{M}_p{}^r)_{k,l}\Delta p(\psi_{i,j-1}-\psi_{i-1,j}-\psi_{i-1,j-1})\\ -2C_{k,l}\Delta r\Delta p]\div[(\bar{M}_r{}^p)_{k,l}\Delta r+(\bar{M}_p{}^r)_{k,l}\Delta p].\end{aligned} \tag{17}$$

Starting at $r=0$, $p=P_0$(i.e., i, $j=1$, 1), computation can proceed either along r or along p-negative. Computations of other parameters can be done as previously outlined.

7. Input values

For the input, tangential velocities for hurricane Cleo 1958 (Krishnamurti, 1961) were used. Tangential velocities for this hurricane were given at grid points at 15 km apart to a distance 150 km from the center, and at 50-mb intervals from 900 mb to 100 mb. Tangential velocities at the center of the hurricane are assumed to be zero, which is consistent with the coordinate system being used. The values of tangential velocities used in the computations are shown in Table 1. (Tangential velocities for Carla 1961[5] were also used as input values for separate experiments, but are not reproduced here.)

Vertical velocities for 900 mb were also supplied as boundary conditions in cases where the equation for the friction layer was not used. These velocities were computed by Krishnamurti (1961) for Cleo 1958.

The horizontal and vertical exchange coefficients are two important physical quantities that have to be supplied in order to carry out the computations. As a starting point a value of 10^8 cm^2 sec^{-1} was assumed[6] for

[3] This latter assumption is not strictly valid. In the steady symmetric hurricane, $\omega_s=\frac{dp_s}{dt}=\left(u\frac{\partial p}{\partial r}\right)_s$, a term that arises because of trans-isobaric flow.

[4] Ooyama, Katsuyuki, 1963: A dynamic model for the study of tropical cyclone development. Presented at the 43rd Annual Meeting of the American Meteorological Society in New York, January 1963, 26 pp.

[5] Ho, Te-chun, 1963: An investigation of hurricane Carla 1961. S. M. Thesis, Department of Meteorology and Oceanography, New York University, 39 pp. (unpublished).

[6] Cohen, Lester A., and Jerome Spar, 1963: Eddy stress in hurricane Donna (1960) over Long Island. Presented at the Third Technical Conference on Hurricanes and Tropical Meteorology, Mexico City, June 1963, 8 pp. (to be published).

TABLE 1. Tangential velocities (in knots), Cleo 1958.

Pressure (mb) \ Radius (km)	0*	15	30	45	60	75	90	105	120	135	150
100	0	3.0	5.0	7.5	9.8	11.6	14.0	12.0	9.7	7.5	6.0
150	0	4.8	7.5	10.0	15.0	22.0	25.0	24.0	20.0	12.0	9.5
200	0	6.0	9.5	14.0	22.0	29.5	32.0	30.4	27.4	21.0	15.0
250	0	7.5	12.2	20.0	27.4	34.4	37.5	36.0	31.8	27.0	22.0
300	0	9.2	15.2	24.0	32.0	40.0	42.0	40.0	36.6	31.8	27.6
350	0	10.0	18.0	29.5	40.0	44.6	45.5	43.8	41.4	37.6	32.0
400	0	10.4	20.4	37.0	46.0	49.5	49.4	47.5	45.0	41.8	38.0
450	0	13.0	25.5	44.0	52.5	53.6	52.5	51.0	48.5	45.0	41.6
500	0	14.8	30.0	51.0	56.4	56.0	55.7	53.6	51.0	48.5	44.8
550	0	15.0	38.0	55.2	59.0	57.5	56.5	54.9	52.5	50.8	48.0
600	0	17.0	42.0	57.5	60.2	58.5	57.2	55.8	53.8	52.0	50.0
650	0	19.0	47.0	60.0	61.2	59.5	58.0	56.4	54.2	53.0	51.0
700	0	19.6	50.0	61.0	62.0	60.0	58.4	57.0	54.8	53.4	52.0
750	0	19.6	52.0	62.0	63.0	60.5	59.0	56.8	55.0	53.6	52.6
800	0	19.8	52.0	63.0	64.0	61.0	59.0	56.4	55.0	53.6	53.0
850	0	20.0	51.0	64.0	64.0	60.5	59.0	55.8	54.2	53.4	53.0
900	0	20.0	50.0	65.0	64.0	60.0	57.6	55.0	54.0	53.0	52.8

* Tangential velocities at the axis assumed zero from symmetry consideration.

ν and 10^5 cm^2 sec^{-1} for k, where $k=\kappa/\rho^2 g^2$ is the vertical exchange coefficient in the x, y, z coordinate system. For dimensional reasons it is more convenient at this point to employ k rather than κ. However, in the numerical computations κ was kept constant and not k. The ratio ν/k was carried in the experiments in order to investigate if the results might be dependent on the ratio. Several ratios were tried for the Cleo data.

Another important input value is the drag coefficient C_D. This is necessary to solve the equation governing the friction layer. A value of 10^{-3} was first used. This is comparable to the value of 2.7×10^{-3} computed by Miller[7] from momentum balance considerations. A value slightly less than Miller's was used at the start because the values of v_0 substituted in (15) are for higher level (900 millibars in this case) than that used by Miller. Other values of C_D were also tried.

8. Experimental computations of the transverse circulations

Inasmuch as one of the main purposes of this paper is to determine reasonable values of the friction coefficients, several sets of experiments were done on the same input data. Horizontal exchange coefficient, vertical exchange coefficient, and drag coefficient were varied systematically based on the present limited knowledge about them for different sets of experiments. Over a hundred experiments were performed for the tangential velocity fields for Cleo and Carla. Results of the experiments were analyzed and the relevant ones to illustrate the pertinent results are presented here.

The most important results of these studies are the computed flow patterns as shown by the stream function fields, from which the radial velocities and vertical velocities can be evaluated. These fields of stream function depend on four sets of given conditions, namely (1) tangential velocity, (2) drag coefficient, (3) horizontal and vertical exchange coefficients, and (4) boundary conditions and modelling approximations used. Tangential velocities for two hurricanes have been used. However, only results of computations for hurricane Cleo 1958 are presented here inasmuch as there was better data-resolution in Cleo, and no significant difference appeared in the Carla computations.

Hurricane Cleo 1958 was a small size storm compared to Carla. The maximum winds were found about 25 n mi from the center, the eye had a diameter of about 15 n mi, and hurricane winds were observed up to 300 mb and 100 n mi from the center.

The conditions in the friction layer are critical for the resulting flow pattern. Several major characteristics of the transverse circulations are changed appreciably by slight variations of the drag coefficient. Computed vertical velocity at the 900-mb level as a function of radius for three different drag coefficients were compared with the values computed by Krishnamurti (1961). It was observed that the value of 2×10^{-3} for C_D is the one that most closely approximates Krishnamurti's results. Although the magnitudes are almost the same, the maximum vertical velocities do not coincide in radial distance. The difference is to be expected. It may be recalled here that Krishnamurti's model does not include a friction layer; thus the 900-mb computed vertical velocities in his results are due only to horizontal and vertical exchange coefficients. Also the 900-mb level is internal in his model, while in this

[7] Miller, Banner I., 1962: On the momentum and energy balance of hurricane Helene (1958). National Hurricane Research Project Report No. 53, U. S. Weather Bureau, 19 pp.

paper it is the lower boundary for the main layer. A value of 5×10^{-3} for C_D was also tried; however, the 900-mb level vertical velocity obtained was far too large. It was also observed that possibly a variation of C_D with radius and wind speed will produce a smoother flow, but it is difficult to determine what variations will give the best results.

The friction layer in this paper means the layer from the surface to the bottom of the main layer (900 mb). (The friction layer is indicated in the figures by the dashed line.) The inflow layer refers to the layer from the ground to the level where radial velocity is zero. The depth of the friction layer decreases toward the center because the isobaric surface slopes upward with positive r, while the inflow layer has an irregular upper boundary and includes the friction layer.

The depth of the inflow layer was investigated for different conditions. The depth of the inflow layer has a variation with radius similar to that found by Rosenthal.[8] The computed inflow layer decreases in general toward the center of the hurricane, which is opposite to that obtained by Estoque (1962). Although it can be seen that the depth of the inflow layer in general decreases with increasing C_D, other things being equal, it is obvious that C_D is not the only factor. The internal friction in the fluid associated with the exchange coefficients affects the flow considerably. Regions of inflow and outflow, and regions of upward and downward motions are different for any change in any of the boundary conditions. Regions of outflow for some sets of different extreme conditions were also investigated. with the exchange coefficients kept constant for different experiments, the depth of the inflow layer decreases as C_D increases. This means, of course, that if C_D is big, all the inward mass transport is occurring in a low shallow layer while most of the upper layer exhibits outflow. The inflow layer and its radial variation computed here is similar to that obtained by Rosenthal,[9] but is about 100 mb deeper. In view of the fact that Rosenthal used a linear model with no horizontal exchange coefficient, the agreement is quite satisfactory.

Figs. 1 to 4 show flow patterns computed for different friction parameters and boundary conditions for hurricane Cleo 1958. The field of stream function is shown in units of 10^4 (n mi)^2 mb hr^{-1}. Except for extreme values of the coefficients, the well-known features of hurricanes are reproduced. Such features as the eye, the eyewall, the low-level radial inflow, and the high-level outflow are clearly illustrated.

Fig. 1 was computed using $C_D=0$, simulating a perfectly smooth surface. This condition gives rise to very strong and deep inflow in the main layer, as the friction layer now causes none of the inflow. Since an equal amount of outflow is required to satisfy the continuity equation, a very strong outflow and upward motion at the 100-mb level has to occur. This also shows that the hurricane should be strong even above the 100-mb level. However, the case of $C_D=0$ is believed not to occur in nature, and such a circulation is just hypothetical.

Fig. 2 is the cross section of the flow obtained by using the 900-mb level vertical velocity computed by Krishnamurti (1961) as the lower boundary condition instead of C_D. The computed transverse circulation is similar to Krishnamurti's as expected. However, the maximum radial velocity occurs at a lower level.

Fig. 3 is another extreme case assuming $\nu=0$, $k=0$, and $C_D=10^{-3}$. This simulates the condition that there is no internal friction, and the circulation is attributed only to frictional drag on the surface. As expected the flow above the friction layer is mainly upward and out. The conditions imply also that the inflow is confined mainly to the friction layer. It may be noted that the region of maximum upward vertical velocity is shifted away from the center of the hurricane compared with the results computed including internal friction. This can be remedied by increasing the drag coefficient toward the center of the hurricane. However, as previously mentioned, the kind of variation of C_D that occurs in nature is not exactly known. Comparison of the stream functions in Figs. 1 and 3 shows that Fig. 3 is smoother. Internal "friction" (turbulent mixing) should produce smoother profiles. The fact that it does the reverse indicates that the irregularity is probably computational. This is due to the fact that in the computation of Fig. 1, second derivatives of tangential velocities are involved.

Results for different combinations of ν, k, and C_D were investigated. The lowest non-zero value used for C_D was 8×10^{-4}. Even this small value gave results that were very different from those for $C_D=0$ (Fig. 1). A non-zero value of C_D produces a more realistic profile than that shown in Fig. 1, and illustrates the point that frictional drag cannot be neglected.

A commonly used value for C_D in theoretical studies is 10^{-3}. This value was used in the computations of Fig. 4. In these computations important characteristics of the hurricane are reproduced, e.g., sinking motion inside the eyewall, inflow layer below and outflow layer aloft. The values of ν and k used for the computation of Fig. 4 were 10^8 and 5×10^5 cm^2 sec^{-1}, respectively. Computations (not shown) were also carried out with different values of ν and k for C_D equal to 10^{-3}. The general result of these computations was an increase in the transverse circulation with increasing values of the exchange coefficients.

9. Final computations of the transverse circulations

After several experiments using the tangential velocity data of Cleo 1958 and Carla 1961 were analyzed,[10]

[8] Rosenthal, Stanley L., 1962: A theoretical analysis of the field of motion in the hurricane boundary layer. National Hurricane Research Project Report No. 56, U. S. Weather Bureau, 12 pp.

[9] Rosenthal, *op. cit.*

[10] Ho, *op. cit.*

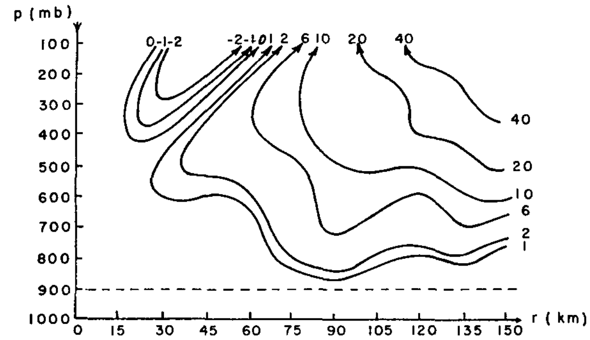

FIG. 1. Stream function (unit: 10^4 (n mi)^2 mb hr^{-1}). $C_D=0$; $\nu=3\times10^8$ c.g.s.; $k=2\times10^5$ c.g.s.; Cleo 1958.

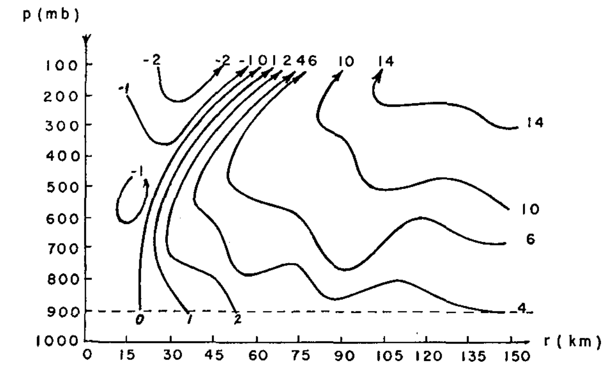

FIG. 2. Stream function (unit: 10^4 (n mi)^2 mb hr^{-1}). ω given at 900 mb; $\nu=10^9$ c.g.s.; $k=5\times10^5$ c.g.s.; Cleo 1958.

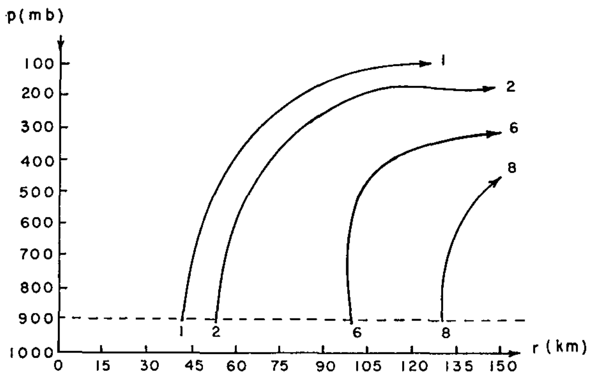

FIG. 3. Stream function (unit: 10^4 (n mi)^2 mb hr^{-1}). $C_D=2\times10^{-3}$; $\nu=0$; $k=0$; Cleo 1958.

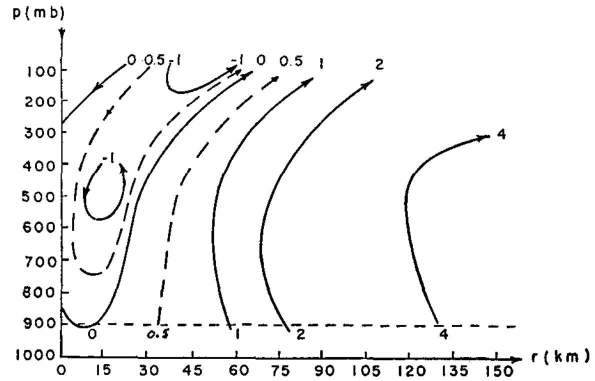

FIG. 4. Stream function (unit: 10^4 (n mi)^2 mb hr^{-1}). $C_D=10^{-3}$; $\nu=10^8$ c.g.s.; $k=5\times10^5$ c.g.s.; Cleo 1958.

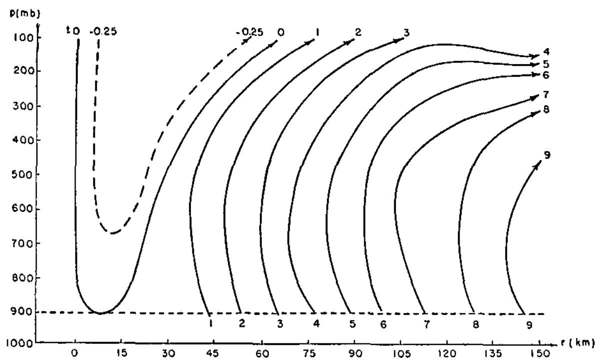

FIG. 5. Stream function, Cleo 1958 (unit: 10^4 (n mi)^2 mb hr^{-1}).

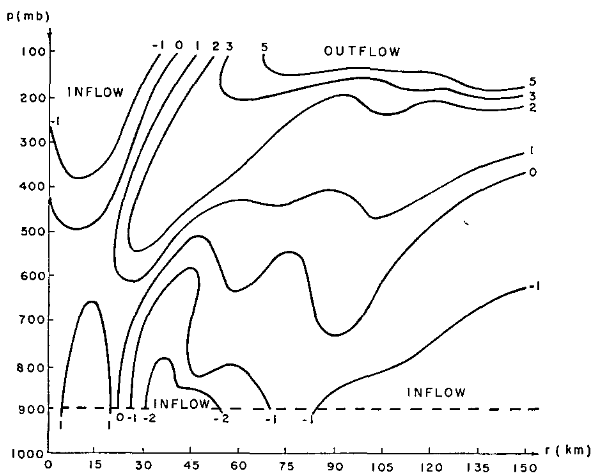

FIG. 6. Radial velocity, Cleo 1958 (unit: knot).

a final set of computations was carried out on Cleo 1958 data using what appeared to be the most reasonable values of the friction coefficients. These were $C_D=2\times10^{-3}$, $\nu=3\times10^8$ cm^2 sec^{-1}, and $k=2\times10^5$ cm^2 sec^{-1}. This combination of boundary conditions seems to give the best values compared to earlier studies.

The field of stream function is presented in Fig. 5. The eyewall is represented by the zero line of stream function that divides the regions of upward and downward motion. The transverse circulations represented by the streamlines have already been discussed in the previous section.

The computed radial velocity distribution (Fig. 6) may be compared with the result of Krishnamurti (1961). The inflow layer is shallower in this result, the maximum radial velocity occurring in the friction layer. Although not shown in Fig. 6, the average inflow in the friction layer (from the surface to 900 mb, i.e., below the

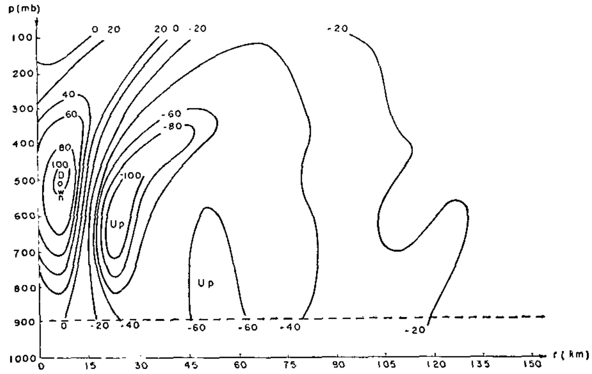

FIG. 7. Vertical velocity, Cleo 1958 (unit: mb hr^{-1}).

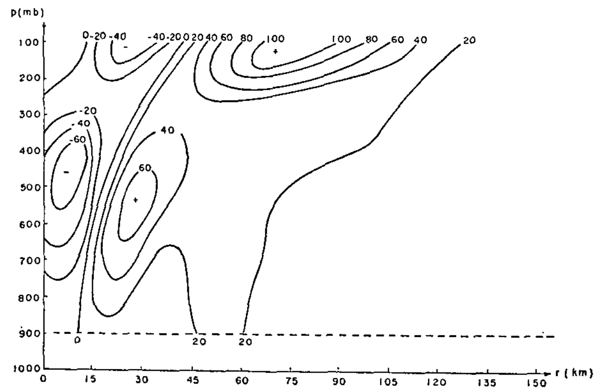

FIG. 8. Diabatic heating, Cleo 1958 (unit: 10^{-5} calorie $(\text{gram-sec})^{-1}$).

dashed line in the figures) increases outward from 4 knots at 45 km to 12 knots at 135 km from the center. (It may be recalled here that no friction layer is included in Krishnamurti's model.) There is a small inflow layer on top near the center which is accompanied by the sinking motion in the eye. The field differs from that of Estoque (1962), in that the thickness of the inflow layer here does not increase toward the center as computed by Estoque.

The profile of vertical velocity for Cleo 1958 (Fig. 7) exhibits all the important characteristics of the hurricane (Riehl, 1954). The zero line of vertical velocity may be considered as the inner boundary of the eyewall. A strong upward motion is computed at the eyewall and sinking motion in the eye. It may be seen also that the eyewall slopes outward with height. The upward motion at 150 km is an order of magnitude lower than that of the eyewall. The region of maximum upward motion here lies inside the region of maximum tangential wind speed, which does not agree with some evidence found by Lavoie (mentioned by Estoque, 1962).

After the computations of u and ω, everything in Eq. (2) is known except the geopotential height, z. This can, in principle, now be computed from (2) if z is given on the outer boundary as a function of pressure. When this is done in the computational program using all terms, the resulting z-field is very irregular. This is due to the fact that the terms in (2) are computed separately, and contain errors which are magnified when derivatives are taken. The z-field is then computed by adding the individual terms. This method leads to large computational errors, which indicates that the z-field can probably not be computed more accurately than from the gradient wind equation, which depends only on the given tangential wind field.

The field of diabatic heating (Fig. 8) is obtained using Eq. (5). The actual magnitude of these results may not be too representative due to the fact that computational errors may be large in this calculation. However, the general positive and negative regions are fairly smooth. On the whole it seems that heating (positive values) is associated with upward motion, and vice versa, with maximum heating and cooling regions of maximum vertical velocity. The heating is undoubtedly caused by release of latent in the ascending air. There is also a negative region (cooling) inside the eye associated with downward motion. The cooling in the eye, which partially offsets the adiabatic warming of the subsiding air there, is probably due to evaporation of cloud water from the interior of the eyewall. There is a maximum positive area aloft in Fig. 8 which is not easily explained, except perhaps as an error due to uncertainty in the static stability in that region.

10. Summary and conclusions

It has been shown, following Krishnamurti (1961), that the transverse circulation in a hurricane can be determined from a given tangential velocity field provided that the exchange coefficient and drag coefficient are known. The main problem encountered in the studies is the determination of the correct values of the exchange and drag coefficients.

The inflow in the friction layer depends on the drag coefficient for a given tangential velocity. In general, the depth of the inflow layer may be reduced by increasing the drag coefficient.

The magnitude of the radial and vertical velocities vary with changes in the three coefficients: drag, horizontal exchange, and vertical exchange coefficients. The transverse circulation is found to be more sensitive to changes in the lateral exchange coefficient than to changes in the vertical exchange coefficient for a given tangential velocity distribution. Keeping the exchange coefficients fixed, and changing only the drag coefficient, produces appreciable changes in the magnitude of the transverse circulation, showing that the computed circulation is strongly dependent on surface friction.

Although the computations are limited to the 100-mb level, this level is not considered to be the lid of the hurricane in this study. The vertical motion at the 100-mb level was obtained explicitly from the computations. It is possible that the computed vertical velocities at the 100-mb level are partly due to the assumption that ν and κ are constants.

The computed maximum downward motion in the eye has nearly the same magnitude as the computed upward motion in the eyewall. For the actual hurricane the upward motion may be considerably larger; however, the results considered here are for an average, steady-state symmetric case.

The field of diabatic heating should be used qualitatively only because computational errors may be large in this particular result.

It is important to note that the computed radial velocity fields in this study do not show the strong mid-tropospheric inflow obtained by Krishnamurti (1961). The inclusion of the friction layer results in a more realistic transverse circulation.

Within the framework of this model the transverse circulation is the result of the exchange processes within the hurricane. There is some evidence that the drag coefficient increases toward the center of the hurricane, but the form of variation is not yet known. After consideration of several factors, it may be concluded that the following values seem to be the most appropriate for the hurricane: $C_D = 2 \times 10^{-3}$; $\nu = 3 \times 10^8$ cm^2 sec^{-1}; $k = 2 \times 10^5$ cm^2 sec^{-1}. It may be emphasized here that only C_D is used in the friction layer, while above the friction layer only ν and k must be specified to carry out the computations.

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