Comments on "Examination of a Wind Profile Proposed by Swinbank"[1]

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Barad's conclusion (*J. appl. Meteor.*, **2**, 747–754, 1963), that the wind profile equation proposed by Swinbank (1960) is not verified by data of Project Prairie Grass appears to be stronger than warranted by the test applied. It is true that the equation is shown not to be as universally excellent as Swinbank had hoped. This Barad does by applying a test suggested by Swinbank to Halstead's data (see Barad, 1958). However, this test is not the most critical one, and it makes terrific demands on fine points of the accuracy and suitability of the data.

Between them, Swinbank and Barad do not give a satisfactory discussion of the "Swinbank profile equation" that follows the "Swinbank hypothesis" (and some additional assumptions). The relation between the ideal conditions of Swinbank's model and good data from sites that *approach* the ideal conditions is not discussed by either. I do not have a pro-Swinbank counterattack; my objection is that Barad's study follows too closely the methods of analysis suggested by Swinbank, and thereby gives only a partial examination.

The Swinbank profile equation may be written more clearly if one of the following functions is used:

$$P(t)=1+\frac{t}{2!}+\frac{t^2}{3!}+\cdots=t^{-1}(e^t-1), \tag{1}$$

$$B(t)=\left\{1-\frac{t}{2!}+\frac{t^2}{3!}-\frac{t^3}{4!}+\cdots-\cdots\right\}^{-1}=t(1-e^{-t})^{-1}. \tag{2}$$

A brief table of these functions and their natural logarithms is given in Table 1.

Note that $P(t)=[B(-t)]^{-1}$; $1=P(0)=B(0)$; and $0.5=dP/dt(0)=dB/dt(0)$.

The Swinbank profile equation can be written as:

$$u=\frac{u_*}{k}\ln\left\{\frac{z-d}{z_0}\frac{P(\gamma(z-d))}{P(\gamma z_0)}\right\}, \tag{3}$$

and its slope as:

$$\frac{du}{dz}=\frac{u_*}{k(z-d)}B(\gamma(z-d)$$
$$=u_*\{k(z-d)P(-\gamma(z-d))\}^{-1}. \tag{4}$$

Here u_* is friction velocity, k is von Karman's constant, $k(z-d)$ is adiabatic mixing length, $P(-\gamma(z-d))$ is the ratio of mixing length to adiabatic mixing length, and surface roughness z_0 is the value of $(z-d)$ at which wind speed extrapolates to zero. The reciprocal of the Monin-Obukhov length is

$$\gamma=L^{-1}=-kgH/c_p\rho\bar{T}u_*^3.$$

Signs are chosen so that $H>0$ and $\gamma<0$ for heat flux upward. In most applications, $P(\gamma z_0)$ may be taken as unity; and $(z-d)$ may often be approximated by z or $z+z_0$.

At high elevations, (large $|\gamma z|$), Swinbank's equation approaches:

$$u=\frac{u_*}{k}\ln\left(\frac{1}{-\gamma z_0}\right) \qquad \text{Daytime}$$

$$u+\frac{u_*}{k}\left\{\ln\left(\frac{1}{\gamma z_0}\right)+\gamma z\right\} \qquad \text{Nighttime.}$$

It is noted that the function $P(t)$ appears also in the Deacon wind profiles:

$$u+\frac{u_*}{k}\ln\left(\frac{z-d}{z_0}\right)\cdot P\left[(1-\beta)\ln\left(\frac{z-d}{z_0}\right)\right]. \tag{5}$$

Eq (3) is the idealized equation, assuming that friction velocity, u_*, heat flux, H, and γ are constant with height in the surface layer. For actual profiles, the equation remains an explicit function of height z if particular values of u_* and γ are specified, rather than allowing $u_*(z)$ and $\gamma(z)$ on the right-hand side. The limiting values as $(z-d)\rightarrow z_0$ are convenient. Values so fixed will be indicated by subscript zero, e.g., u_{*0}.

The "wind profile in thermally stratified flow" (this is the title of Swinbank's paper) is then given by:

$$u=\frac{u_*}{k}\ln\left\{\frac{z-d}{z_0}\frac{P[\gamma_0(z-d)]}{P(\gamma_0 z_0)}\right\}+E(z), \tag{6}$$

where the deviation term $E(z)$ goes to zero as $(z-d)\rightarrow z_0$. Measured profiles would include also an error term. Taking (6) as a statement of Swinbank's equation for real situations, what is the meaning of its validity or non-validity? Surely its validity requires at least that $E(z)$ be small in a height interval $z_0<z-d<h$; where u_{*0} and γ_0 are not merely "profile parameters," but have their basic physical meaning:

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TABLE 1. The functions defined by Eqs. (1) and (2), and their natural logarithms.

t	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
$P(t)$	0.245	0.316	0.432	0.632	1	1.718	3.19	6.36	13.40	29.5	67.0	156.5	372.5
$B(t)$	0.0746	0.1571	0.314	0.582	1	1.580	2.31	3.16	4.09	5.01	6.01	7.00	8.00
$\ln P(t)$	-1.404	-1.148	-0.839	-0.459	0	0.541	1.160	1.85	2.59	3.38	4.20	5.05	5.91
$\ln B(t)$	-2.60	-1.85	-1.160	-0.541	0	0.459	0.840	1.152	1.408	1.612	1.794	1.948	2.08

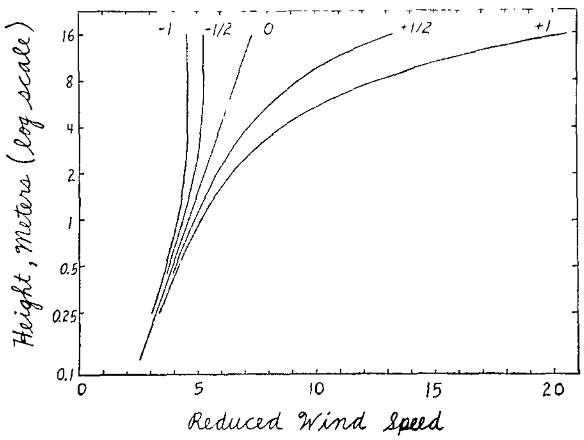

FIG. 1. Reduced wind speed profiles according to Swinbank's equation. Reduced wind speed is ku/u_*. Drawn for $z_0=0.01$ meter, $d=0$, and $\gamma_0=0, \pm0.5$ and ±1.0 meter^{-1}.

$$u_{*0}=\lim_{z-d\to z_0}\sqrt{\tau/\rho}$$

$$\gamma_0=\lim_{z-d\to z_0}\frac{-kgH}{c_p\rho\quad u_*^3}.$$

That is, the ideal equation should account for most (not all) of the systematic features of a good set of measured profiles, where the tying-together parameters in it have the true micrometeorological significance attributed to them. The exact function $P(\gamma_0(z-d))$ is the offering of Swinbank, and its rightness is the matter at issue.

Some reduced wind profiles (hypothetical) are sketched in Fig. 1. On the x-axis is ku/u_{*0} and on the y-axis is $z-d$, on a logarithmic scale. Swinbank profiles are drawn for $z_0=0.01$ meter and $\gamma_0=0$, ±0.5, and ±1.0 meter^{-1}. Barad's conclusions may be paraphrased and extended as follows: Consider the Swinbank profile for $\gamma_0=+1$ meter^{-1}, and grant that for small heights and until it has a moderate deviation from the neutral ($\gamma_0=0$) profile, it corresponds to a real profile with $\gamma_0=C$. Whether this value of γ_0 is close to or far from Swinbank's value is not considered. But that profile, whatever its γ_0, deviates less from the neutral profile with increasing height than the Swinbank equation says it should. And this difference is believed largely due to inexact accounting for buoyancy effects in the Swinbank equation.

It should be said that the Swinbank profile equation is simple, clever, elegant and convenient. It does not have any mathematical breakdown points. Its reputed freedom from "arbitrary" constants seems to me entirely imaginary, however. If Eq (3) is generalized to:

$$u=\frac{u_*}{k}\ln\left\{\frac{z-d}{z_0}\frac{P(a\gamma(z-d))}{P(a\gamma z_0)}\right\}, \tag{6}$$

Swinbank has "unarbitrarily" taken the hypothesis equivalent to assuming $a=1$. But Barad's analysis not only does not hold the Swinbank profile equation to this particular value $a=1$, but does not hold it to any single value in the set of profiles examined. Only the constancy of $(a\gamma)$ within individual profiles is required. Thus, Barad's $\gamma=1/L$ is treated strictly as a shape parameter defined by:

$$Q_z=\frac{u_z-u_{z/4}}{u_{4z}-u_z}=\frac{\ln4+\ln P(z)-\ln P(z/4)}{\ln4+\ln P(4z)-\ln P(z)}. \tag{7}$$

It is found not to be constant with height. This "nonverification" of Swinbank's profile equation is, I think, really a disproving of a method of analysis proposed by Swinbank. The fact that a Richardson number term should not account for all systematic features of the profiles is ignored. Barad seems to anticipate the following futile chain of argument:

Mr. Tambo. So γ is not constant with height? Well, the same analysis gives three values of z_0 and u_* with height also. If u_* is constant with height, and γ not constant with height; then heat flux is not constant with height, and the conditions of the equation do not hold. If u_* is not constant with height, then shear stress is not constant with height, and the conditions of the equation do not hold. In either case, the profile equation is not tested by suitable data, and is not disproved.

Mr. Bones. Then the equation has nothing to do with micrometeorological data, and should be forgotten.

This pseudo-dilemma comes from trying to account for all systematic features of wind profiles by adding only a Richardson number term to the logarithmic profile equation. And the method of analysis does not use all the micrometeorological information available.

More decisive results would come from plotting the mass of reduced wind profiles, after Businger (1959), as:

$$\frac{u}{V_{*0}}-\ln\left(\frac{z-d}{z_0}\right) \quad \text{against} \quad (z-d)g\left[\frac{K_H}{K_M}\frac{d\theta}{du}\Big/\bar{T}V_*\right]_0,$$

where $V_*=u_*/k$ and $\theta=T+gc_p^{-1}z$.

In contrast to Barad's present analysis, which allows z_0, V_*, and $(K_H/K_M\cdot d\theta/dz)$ to fall where they will; that procedure requires optimum estimates of these three parameters. According to Swinbank's equation, if the constant-flux assumption is valid, the points from all the profiles should show ordinary statistical scattering about the single, beautiful curve,

$$y=\ln P(t).$$

To the extent that constant flux, steady conditions, etc. assumptions are not met, systematic deviations at increasing heights are to be expected.

REFERENCES

Barad, Morton L., 1958: Project Prairie Grass, A field program in diffusion. *Geophysics Research Paper No. 59*, Vols. I and II, Air Force Cambridge Research Laboratories, AFCRL-TR-58-235 (ASTIA Document No. AF-152572).

——, 1963: Examination of a wind profile proposed by Swinbank. *J. appl. Meteor.*, **2**, 747–754.

Businger, J. A., 1959: A generalization of the mixing length concept. *J. Meteor.*, **16**, 516–523.

Swinbank, W. C., 1960: Wind profile in thermally stratified flow. *Nature*, **186**, 463–464.