On the Estimation of Climatological Z–R Relationships

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ABSTRACT

A statistical framework for climatological Z–R parameter estimation is developed and simulation experiments are conducted to examine sampling properties of the estimators. Both parametric and nonparametric models are considered. For parametric models, it is shown that Z–R parameters can be estimated by maximum likelihood, a procedure with optimal large sample properties. A general nonparametric framework for climatological Z–R estimation is also developed. Nonparametric procedures are attractive because of their flexibility in dealing with certain types of measurement errors common to radar data. Simulation experiments show that even under favorable assumptions on error characteristics of radar and raingages, large datasets are required to obtain accurate Z–R parameter estimates. Another important conclusion is that estimation results are generally quite sensitive to radar and raingage measurement thresholds. For fixed sample size, the simulation results can be used to provide quantitative assessments of the accuracy of Z–R model parameter estimates. These results are particularly useful for error analysis of precipitation products that are derived using climatological Z–R relations. One example is the large-area rainfall estimates derived using the height–area rainfall threshold (HART) technique.

1. Introduction

Determination of parameters for the power-law model, relating radar-measured reflectivity factor and rainfall rate on the ground, has been a longstanding problem. The problem will assume additional importance with deployment of the Next Generation Weather Radar (NEXRAD) system in the United States and through national and international activities related to rainfall climatology (Wilkerson 1988; Simpson et al. 1988). Radar rainfall estimation from reflectivity factor observations is a complex problem requiring numerous quality control and processing steps (see Hudlow et al. 1984; Joss and Waldvogel 1990; Austin 1987; Smith and Krajewski 1991). Nonetheless, accurate specification of Z–R parameters provides the fundamental building blocks for constructing high-quality radar rainfall estimates.

The traditional methods of Z–R estimation make use of either drop-size distribution data (see Hudlow et al. 1979) or concurrent measurements from radar and raingages (Zawadzki 1975; Wilson and Brandes 1979; Smith 1990). Drop-size data have been collected largely for intensive research programs and are not likely to meet requirements for calibration of operational radar systems. Furthermore, drop-size data collected at the ground are not necessarily representative of radar sampling. Use of Z–R relations derived from drop-size data typically require calibration steps utilizing raingage data for removal of systematic bias (see Hudlow et al. 1979). Numerous procedures have been proposed for estimating parameters of the Z–R relationship from concurrent observations of radar reflectivity factor and gauge accumulations. The most appropriate, perhaps, is nonlinear least-squares regression as discussed by Smith et al. (1975). The main drawback of these methods is the need to develop a large dataset of concurrent radar and gauge observations (more will be said later on what is meant by "large").

In response to the need for accurate Z–R relationships, a climatological method of estimating parameters of the Z–R relationship, which utilizes nonsynchronous radar and raingage observations, has gained popularity. The concept was originally suggested by Miller (1972) and has been followed by the work of Calheiros and Zawadzki (1987), Atlas et al. (1990), Smith et al. (1989) and Rosenfeld et al. (1990). In general terms, the method establishes Z–R relations by relating reflectivity and rainfall-rate values corresponding to the same probability of exceedance. These values can be obtained not only from the concurrent pairs, but from all the available historical radar and raingage data as well. The approach could be particularly attractive if used in conjunction with methods such as the area–

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time integral (ATI) (Doneaud et al. 1984) and the height–area rainfall threshold (HART) technique (Atlas et al. 1990) to estimate areally averaged rainfall.

The purpose of this paper is to consider several aspects of the climatological Z–R relationship. Some of these aspects concern the statistical framework of the problem; others are more general. We present a statistical framework for parameter estimation (section 2) followed by analysis of sampling properties of climatological Z–R estimators. In section 3, we describe simulation experiments, designed and performed, to illustrate various estimation issues. In particular, data requirements and error analysis for climatological Z–R procedures are examined. Conclusions and recommendations for future research are given in section 4.

2. Framework for Z–R parameter estimation

A fundamental feature of climatological Z–R relationships is that they can be developed from nonsynchronous radar reflectivity observations (i.e., observations that are not taken at the same time and place). The central topic of this section is estimation of Z–R parameters from nonsynchronous datasets. At the end of the section it is shown that, in certain circumstances, synchronous datasets are required for estimating Z–R parameters.

Let us assume that reflectivity factor Z and rainfall rate R are related by a power-law model of the form:

$$Z = aR^b, \tag{1}$$

where a and b are unknown constants. Equation (1) implies that the Z–R model holds without error. The implications of model and measurement error are examined later in this section and in section 4.

The observations available for estimating a and b are assumed to consist of random samples $Z_1, \ldots, Z_m$ of reflectivity and $R_1, \ldots, R_n$ of a rainfall rate. It is further assumed that the samples of rainfall rate and reflectivity are mutually independent. The interpretation is that m reflectivity factor values are obtained from radar and that n rainfall-rate observations are obtained from raingages. In statistical terms, the problem is to estimate the unknown parameters a and b from the samples of reflectivity and rainfall rate. Two major classes of procedures can be distinguished: parametric and nonparametric. The estimation problems are formulated below for both classes.

a. Parametric Model

In a parametric model, the probability law is specified by a vector of parameters. Maximum likelihood procedures are particularly attractive for parametric models because of their optimal large sample properties (see Bickel and Doksum 1977). We assume that the distribution of rainfall rate,

$$F_R(r) = P(R \leq r), \tag{2}$$

admits a density function,

$$f_R(r) = \frac{dF_R(r)}{dr}, \tag{3}$$

that is specified by a vector of parameters $\tilde{\theta}$. It follows from Eq. (1) that

$$F_Z(z) = F_R(\alpha z^\beta), \tag{4}$$

where $\beta = b^{-1}$ and $\alpha = a^{-\beta}$. This implies that

$$f_Z(z) = f_R(\alpha z^\beta)\alpha\beta z^{\beta-1}. \tag{5}$$

The log-likelihood function for the sample of reflectivity and rain-rate observations is the natural logarithm of the joint density of the observations, expressed as a function of model parameters and given by:

$$\mathbf{L}(\theta \mid R_1, \ldots, R_n, Z_1, \ldots, Z_m) = \sum_{i=1}^{n} \ln f_R(R_i) + \sum_{i=1}^{m} \ln f_R(\alpha Z_i^\beta) + m \ln(\alpha\beta) + (\beta - 1) \sum_{i=1}^{m} \ln(Z_i), \tag{6}$$

where $\theta = (\alpha, \beta, \tilde{\theta})$ (henceforth, conditioning on the observations in the likelihood function will be omitted.). The parameters $\tilde{\theta}$ include the unknown parameters of the density function $f_R(R)$. Maximum-likelihood estimators are solutions, if they exist, to the system of equations:

$$\nabla \mathbf{L}(\theta) = 0. \tag{7}$$

The estimation problem is illustrated below under the assumption that rainfall rate follows an exponential distribution. Use of the exponential distribution for Z–R models has an antecedent in the work of Seed and Austin (1990). The density function of an exponential distribution is given by:

$$f_R(r) = \begin{cases} 0 & r \leq 0 \\ \lambda e^{-\lambda r} & r > 0, \end{cases} \tag{8}$$

where λ is the sole unknown parameter of the rainfall-rate distribution. For this model the mean rainfall rate is λ^{-1}. The likelihood function, obtained from Eq. (6), is given by:

$$\mathbf{L}(\alpha, \beta, \lambda) = (m + n)\ln\lambda - \lambda\left(\sum_{i=1}^{n} R_i + \alpha \sum_{i=1}^{m} Z_i^\beta\right) + m(\ln\alpha + \ln\beta) + (\beta - 1)\sum_{i=1}^{m} \ln Z_i. \tag{9}$$

Solutions for Eq. (7), with the likelihood function given by (9), can be obtained numerically.

An attractive feature of maximum-likelihood procedures is that the accuracy of estimates can be assessed using large sample properties of the estimators, particularly asymptotic normality (see Bickel and Doksum

1977). If the Z–R model of Eq. (1) holds and parametric model assumptions are valid for all observations of both samples, the Z–R relations should be obtained from maximum-likelihood estimation via Eq. (7).

b. Nonparametric model

Parametric assumptions may not be valid over the entire range of radar and raingage samples. For radar reflectivity data, representativeness of small and large observations is often brought into question. Nonparametric methods provide parameter estimators that are more robust to certain types of errors that may contaminate reflectivity and rain-rate observations. The climatological Z–R procedure of Calheiros and Zawadzki (1987) is, in spirit, a nonparametric procedure and serves as the basis for the following development.

A key feature of nonparametric procedures is their reliance on order statistics; that is, on the ordered values of the sample. The order statistics of the rain-rate sample are denoted by

$$R_{(1)} \leqslant \cdots \leqslant R_{(n)}. \tag{10}$$

Similarly, the order statistics of the reflectivity sample are denoted:

$$Z_{(1)} \leqslant \cdots \leqslant Z_{(m)}. \tag{11}$$

The quantile function of the reflectivity distribution is defined by

$$Q_z(p) = F_Z^{-1}(p). \tag{12}$$

In words, the quantile $Q_z(p)$ provides the reflectivity value that is exceeded with probability $1 - p$. Similarly, $Q_R(p)$ will denote the quantile function of the rainfall-rate distribution. Nonparametric procedures for estimating climatological Z–R relationships can be based on the observation that

$$Q_Z(p) = aQ_R(p)^b, \tag{13}$$

which follows directly from Eq. (1). Climatological Z–R relations can be obtained by choosing a and b to minimize an expression of the form

$$H(a, b) = \int_0^1 h\{\ln[Q_Z(p)] - \ln[aQ_R(p)^b]\}\, w(p)dp, \tag{14}$$

where h is a nonnegative error function and w is a nonnegative weighting function. If, for example,

$$h(x) = x^2 \tag{15}$$

and $w(p) = 1$ for all p, Eq. (14) will provide estimators that minimize the integrated squared difference between $\ln[Q_Z(p)]$ and $\ln[aQ_R(p)^b]$. If, on the other hand

$$h(x) = |x| \tag{16}$$

and

$$w(p) = \delta(p - 1/2), \tag{17}$$

where $w(p)$ is the Dirac delta function that puts all weight on the probability $1/2$, then Eq. (14) provides estimators that minimize the absolute difference between the median value of $\ln(Z)$ and the median value of $\ln(aR^b)$.

Calheiros and Zawadzki (1987) obtain Z–R relations by relating $Q_R(p_i)$ and $Q_Z(p_i)$ for k values of exceedance probability, $p_1, \ldots, p_k$. Paralleling their development, Z–R parameters can be chosen to minimize the sum of squared differences:

$$H(a, b) = \sum_{i=1}^{k} \{\ln[Q_Z(p_i)] - \ln[aQ_R(p_i)^b]\}^2. \tag{18}$$

This is a special case of Eq. (14) with $h(x)$ given by Eq. (15) and

$$w(p) = \sum_{i=1}^{k} \delta(p - p_i). \tag{19}$$

The flexibility afforded by Eq. (14) is convenient in dealing with samples that are more likely contaminated by errors in one part of the distribution than in others. For example, radar data at low rainfall rates are notoriously suspect (see Zawadzki 1982, 1984; Doviak and Zrnić 1984).

To implement estimators based on Eq. (14), the quantile functions must be replaced by estimators obtained from samples of reflectivity and rainfall rate. Sample quantile estimators are of the form

$$\hat{Q}_R(p) = R_{(i)} \quad \text{if} \quad \frac{i}{n+1} \leqslant p < \frac{i+1}{n+1}. \tag{20}$$

More sophisticated quantile estimators based on order statistics are discussed in Serfling (1980).

It can be shown (Cramer 1946) that the probability density function of the mth element in an ordered random sample of size N is given by

$$f_{R,m,N}(R) = \frac{N!}{(m-1)!(N-m)!} p(R)^{m-1} \times [1 - p(R)]^{N-m} f_R(R), \tag{21}$$

where $f_R(R)$ is the probability density function of the random variable R and $p(R)$ is the probability of exceedance. In some cases, this result can be used to evaluate sampling properties of quantile estimators, since it holds that $m = Np(R)$. Even for parametric models, however, Eq. (21) will often be too complicated to provide useful sampling properties of quantile estimators. An attractive feature of quantile estimators, including the sample quantile estimator of Eq. (20), is asymptotic normality (Serfling 1980) represented as follows:

$$N^{1/2}\{[\hat{Q}(p_i), \ldots, \hat{Q}(p_k)] - [Q(p_i), \ldots, Q(p_k)]\} \xrightarrow{\mathcal{D}} N(\mathbf{0}, \boldsymbol{\Sigma}), \quad (22)$$

where $N(\mathbf{0}, \boldsymbol{\Sigma})$ denotes a multidimensional normal distribution with the zero mean vector and a $k \times k$ covariance matrix $\boldsymbol{\Sigma}$. This result will provide a guide for the simulation experiments carried out in the following section.

Nonparametric procedures have two major advantages: parametric distributional assumptions are not required, and estimators can be derived that are more robust to common sources of error in radar and raingage samples. A disadvantage of nonparametric procedures, compared to parametric models using maximum-likelihood estimates, is that it is more difficult to quantitatively asssess the accuracy of the estimators.

c. *Effects of sampling and measurement errors*

To conclude this section, we consider the case in which model error is explicitly incorporated. A simple form of the Z–R relationship with model error is given by

$$Z = aR^b\epsilon, \quad (23)$$

where ϵ is a positive random variable representing model error. We consider only the parametric case and, in particular, the case in which all of the processes, including the error process, have a lognormal distribution. Parameters of the processes are as follows:

$$E(\ln\epsilon) = 0, \quad (24)$$

$$\mathrm{var}(\ln\epsilon) = \sigma_\epsilon{}^2, \quad (25)$$

$$E(\ln Z) = \mu_Z, \quad (26)$$

$$\mathrm{var}(\ln Z) = \sigma_Z{}^2. \quad (27)$$

The parameters (mean and variance) of the rainfall-rate distribution are determined from the above equations and are given by

$$\ln R \sim \mathrm{LN}[\mu_Z b^{-1} - \ln a, b^{-2}(\sigma_Z{}^2 + \sigma_\epsilon{}^2)], \quad (28)$$

where LN denotes a lognormal distribution. Denoting the sample mean and variance of the natural logarithms of rainfall rate and reflectivity by $\hat{\mu}_R$, $\hat{\sigma}_R{}^2$, $\hat{\mu}_Z$, $\hat{\sigma}_Z{}^2$, respectively, it follows that moment estimators for a and b must satisfy

$$\hat{\mu}_R = \hat{\mu}_Z b^{-1} - \ln a, \quad (29)$$

$$\hat{\sigma}_R{}^2 = b^{-2}(\hat{\sigma}_Z{}^2 + \sigma_\epsilon{}^2). \quad (30)$$

If $\sigma_\epsilon{}^2$ is unknown, there are two equations and three unknown parameters. It follows that the parameters a and b cannot be estimated from nonsynchronous samples unless the variance parameter of the error process is known.

If the variance parameter of the error process is unknown, we must have synchronous observations of radar reflectivity and rainfall rate:

$$(R_1, Z_1), \ldots, (R_n, Z_n), \quad (31)$$

where the (R, Z) pairs are observed at the same location and time. In this case, the problem of estimating a and b is a standard regression problem, which will be discussed in more detail in section 3.

Now, the effects of both the sampling and measurement errors on the results of climatological Z–R estimation will be discussed further. The climatological Z–R estimation can be performed as either a linear or nonlinear regression of quantiles. The linear case corresponds to Eq. (18) where a straight line is fit to the k pairs of the logarithms of the quantiles. For the nonlinear case, the minimized function could be

$$H(a, b) = \sum_{i=1}^{k} [Q_Z(p_i) - aQ_R(p_i)^b]^2. \quad (32)$$

As a result of the earlier discussion, it should be clear that both members of quantile pairs $[Q_Z(p_i), Q_R(p_i)]$ are subject to sampling error in addition to their respective measurement errors. Let us assume, for the moment, that the measurements of both raingage rainfall R_i and radar reflectivity factor Z_i are error free. Therefore, only the sampling error will be considered.

Now, the objective is to estimate the parameters a and b of the Z–R relationship from k pairs of the quantile quantities $[Q_Z(p_i), Q_R(p_i)]$ for $i = 1, \ldots, k$ defined by Eq. (12).

We have

$$Q_R(p_i) = Q_R^*(p_i) + \epsilon_i, \quad (33)$$

$$Q_Z(p_i) = Q_Z^*(p_i) + \delta_i, \quad (34)$$

where ϵ_i and δ_i are random errors associated with $Q_R(p_i)$ and $Q_Z(p_i)$, respectively. The asterisk denotes the true value of the variables of interest. Since Z and R are related by a nonlinear function, the estimation problem can be approached as nonlinear regression. Development of an appropriate model is difficult, however, especially in cases where both the predictor and the predictand are corrupted by errors. To illustrate this point, let's assume a hypothetical situation, and suppose that a linear relationship between R and Z is sought. Then, one could postulate the following model,

$$Q_R^*(p_i) = \beta_0 + \beta_1 Q_Z^*(p_i). \quad (35)$$

Substituting Eqs. (33) and (34) into Eq. (35) we obtain

$$Q_R(p_i) = \beta_0 + \beta_1 Q_Z(p_i) + \epsilon_i^*, \quad (36)$$

where

$$\epsilon_i^* = (\epsilon_i - \beta_1\delta_i). \quad (37)$$

It can be shown (Draper and Smith 1981) that the expected value of the estimator of the coefficient β_1 is approximately

$$E(\hat{\beta}_1) = \beta_1(1 + \rho r)(1 + 2\rho r + r^2)^{-1}, \quad (38)$$

where ρ is the correlation coefficient between the true random variable $Q_Z^*(p_i)$ and its error δ, and r is the standard deviation ratio of the error δ and the variable $Q_Z^*(p_i)$. Therefore, the estimator $\hat{\beta}_1$ is biased.

Sampling error is also a source of bias in the parameters if Z–R estimation is performed as a linear regression resulting from Eq. (18). The occurrence of this bias is demonstrated in the next section. In the case of Z–R estimation using a nonlinear regression, the effects of sampling and measurement errors can be investigated only numerically via simulation.

3. Monte Carlo simulation experiment

In order to test the discussed estimation of climatological Z–R relationship the following Monte Carlo simulation experiments were designed and performed. In the first scenario of the simulation we assumed a perfectly known Z–R relationship. Two sets of parameters were considered. The first was the familiar Marshall–Palmer relationship for which $a = 200$ and $b = 1.6$, and the second was the relationship used in GATE (see Hudlow and Patterson 1979) with $a = 230$ and $b = 1.25$. Independent samples of varied size were generated from the distribution of R and Z [see Eqs. (1) and (8)]. The sample sizes generated were 100, 1000, and 10 000. For each sample, nine quantiles corresponding to the probabilities 0.031, 0.063, 0.125, 0.250, 0.500, 0.750, 0.875, 0.938, and 0.967 were calculated using the method of Tukey (1977). Then, both a nonlinear and a linear regression were performed based on the model given by Eqs. (32) and (18), respectively. The process was repeated for 250 independent realizations, and statistics of the parameter estimators were calculated. Figure 1a shows the so-called box plot of the results obtained for the Marshall–Palmer Z–R relationship using a nonlinear regression. In the box plot, the top and bottom horizontal lines corre-

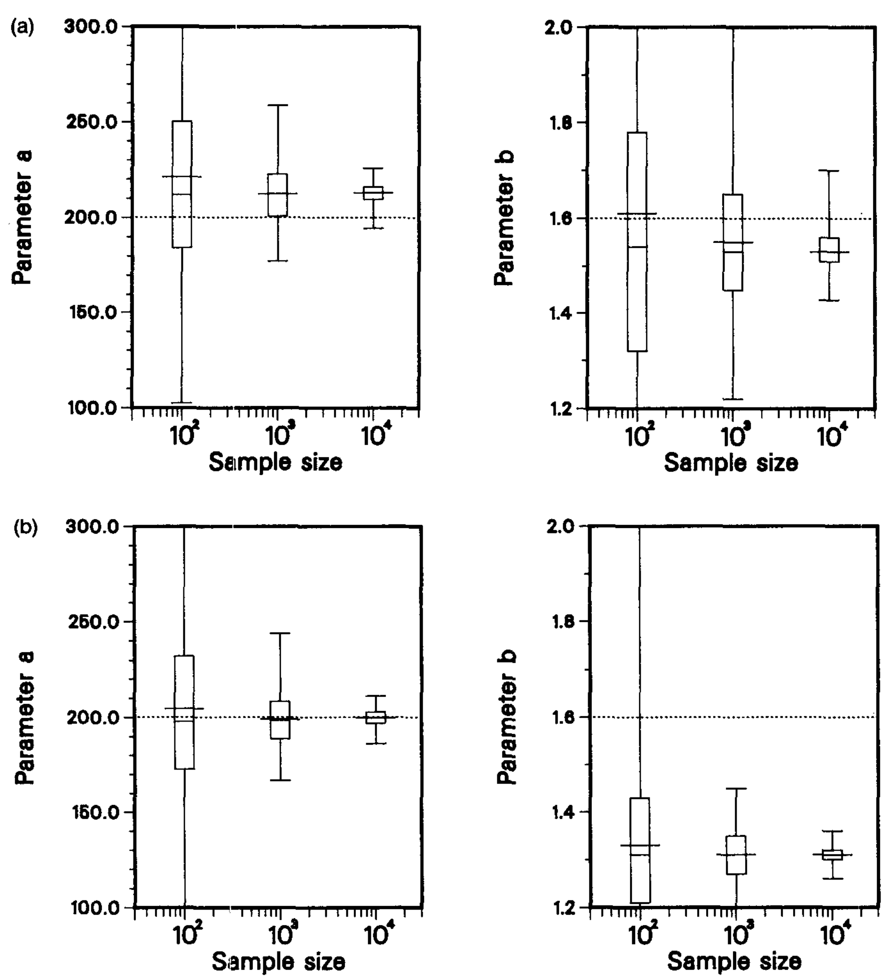

FIG. 1. (a) Results of a simulation experiment of Z–R parameter estimation. The true parameters are marked with the dashed lines. Rainfall was assumed to be exponentially distributed with the mean rate of 2 mm h^{-1}. Parameters were estimated using nonlinear regression. (b) Same as (a) but the parameters were estimated using linear regression.

spond to the maximum and the minimum values obtained in the Monte Carlo sample; the top and bottom limits of the box correspond to the 75 and 25 percentiles, respectively; the middle line is the median and the horizontal line extending across the box is the mean. The most important feature of the plot is that a large sample is required to obtain good estimates. Even for 10 000 independent observations the spread in both parameters is quite large. The results for the GATE Z–R relationship look qualitatively the same. Figure 1b shows the analogous results obtained using the linear regression method. The striking result is a significant bias in the estimates of parameter b. This is an expected result in view of the discussion in section 2.

In the second scenario, the measurement errors in both R and Z were introduced. For R the standard deviation of the additive error used was 10% of the true value. This is a conservative estimate of raingage accuracy (Larson and Peck 1978) since the generated error accounts for the fact that most raingages do not measure rainfall rate but only the accumulated rainfall. For the radar reflectivity, random errors with standard deviation of 1–2 dBZ were used. The lower end of this error range is in line with good-quality radar accuracy estimates used by Sachidananda and Zrnić (1987) and Chandrasekar and Bringi (1987), among others. The upper end of the range is not uncommon for operational radar installations. The results are plotted in Fig. 2a and Fig. 3a for nonlinear estimation and in Fig. 2b and Fig. 3b for linear estimation, respectively. The effect of the added measurement errors is an increase in the spread of the estimates.

The third investigated scenario simulated a real data situation even more closely. In this case, thresholds were introduced in both raingage rainfall and radar reflectivity. The most important observation of these

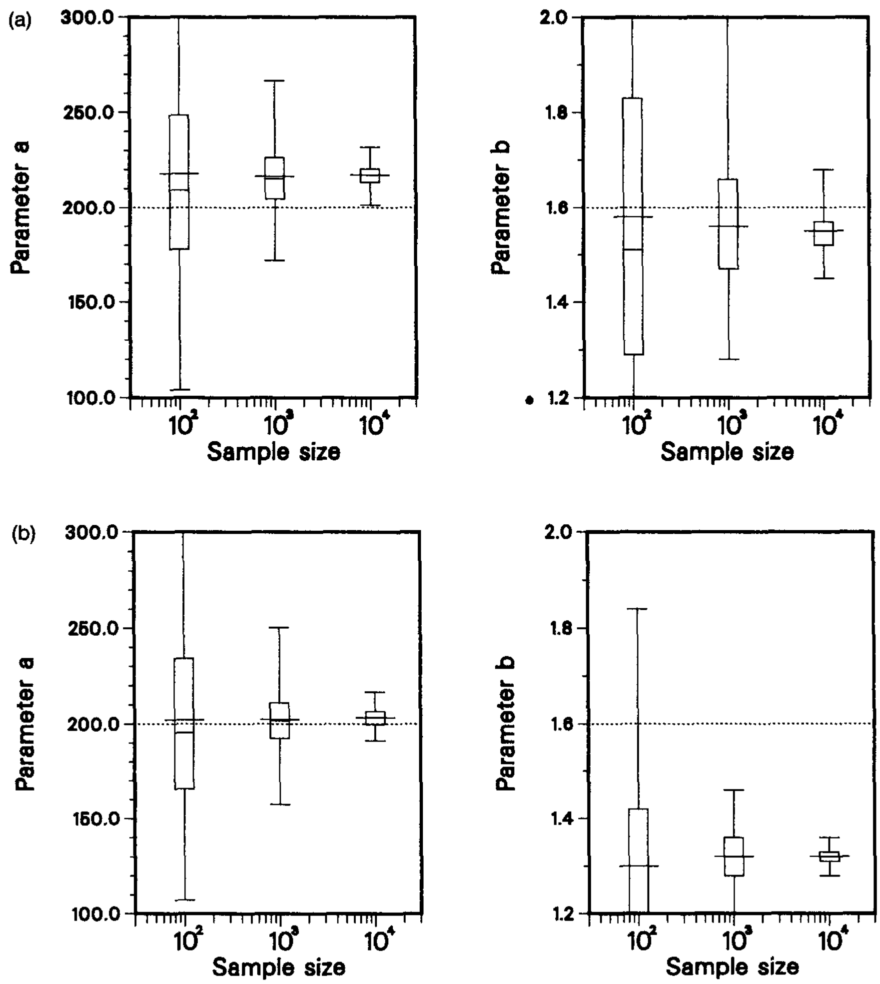

FIG. 2. (a) Same as Fig. 1a, but rainfall measurement error was 10%, and reflectivity measurement error was 1 dBZ. Parameters were estimated using nonlinear regression. (b) Same as (a) but the parameters were estimated using linear regression.

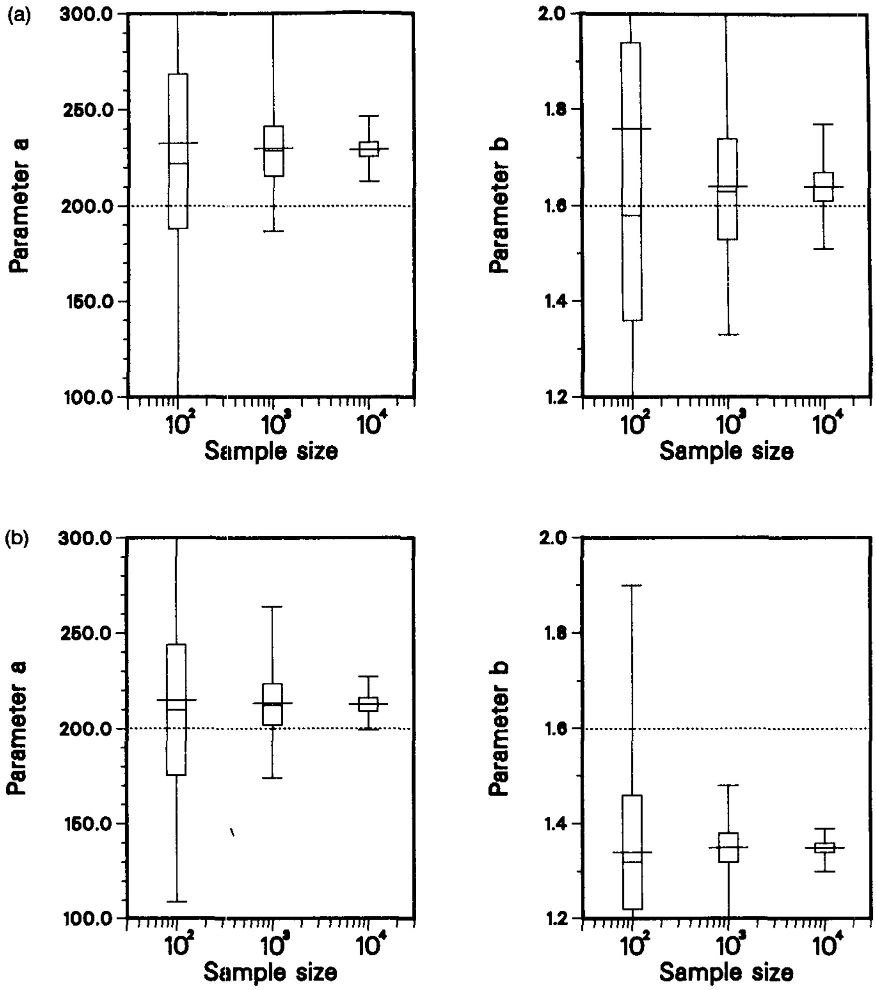

FIG. 3. (a) Same as Fig. 2a, but reflectivity measurement error was 2 dBZ. Parameters were estimated using nonlinear regression. (b) Same as (a) but the parameters were estimated using linear regression.

experiments was the high sensitivity of the results with respect to the selection of the thresholds. When the thresholds are equivalent (i.e., related through the true Z–R relationship) their introduction improves the results of the linear regression. However, if the thresholds are selected arbitrarily (especially the rainfall-rate threshold, since the radar reflectivity can be determined by the sensitivity of the radar system) this immediately results in biased estimates. Figures 4 and 5 illustrate this point.

To contrast the above results, the corresponding statistics, obtained by using concurrent pairs of Z and R and the nonlinear regression, are presented in Fig. 6. In this case, of course, assumption of one-to-one correspondence between Z and R leads to a trivial estimation problem unless the generated values of Z and R represent error-corrupted measurements. As evident from the plots, the spread of the results is significantly less than for the nonsynchronous observations. Small biases are byproducts of measurement error and the thresholds used.

4. Conclusions and recommendations

A statistical framework for climatological Z–R estimation was discussed. Both parametric and nonparametric methods were described. The requirements for use of parametric models are constraining for Z–R parameter estimation problems. In the parametric case, however, the statistical sampling properties of the Z–R parameter estimators can be investigated analytically using standard results for maximum likelihood estimators.

For the nonparametric models, sampling properties of the estimators were investigated via a numerical simulation experiment. The experiment was conducted

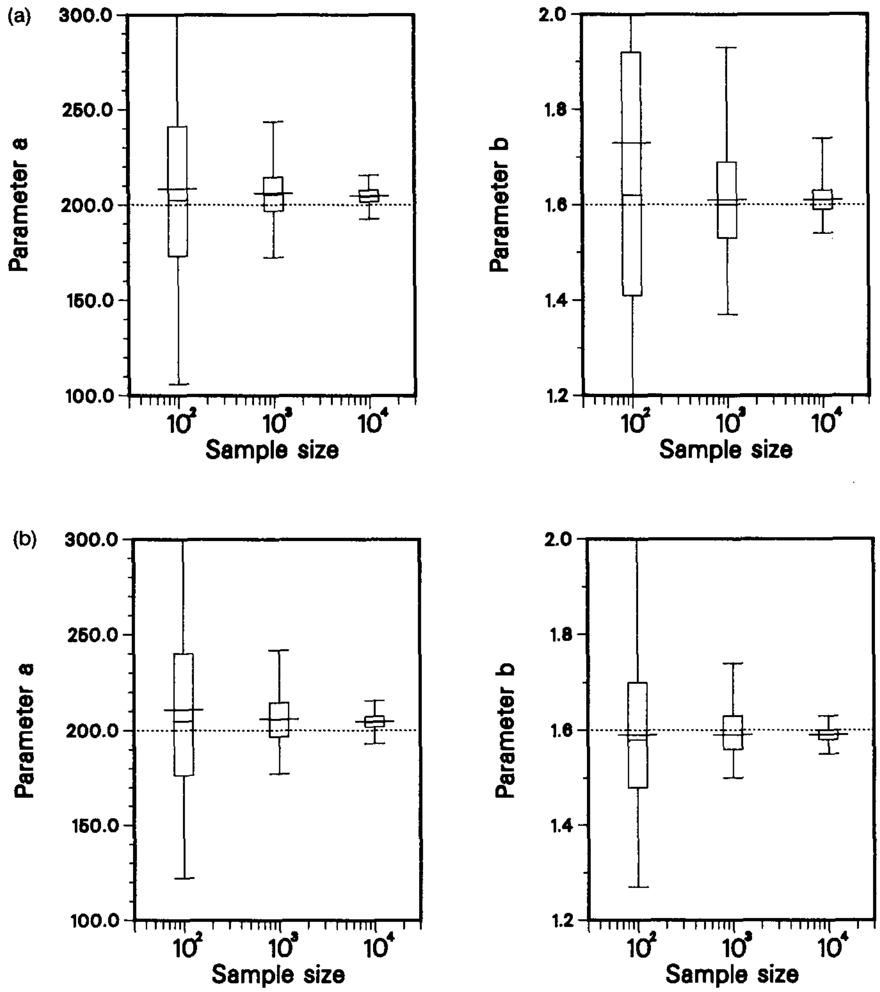

FIG. 4. (a) Same as Fig. 2a, but rainfall threshold was 0.15 mm h^{-1}, reflectivity threshold was 10 dBZ. This is the case of corresponding thresholds. Parameters were estimated using nonlinear regression. (b) Same as (a) but the parameters were estimated using linear regression.

using a favorable assumption that a power law holds between rainfall rate and radar reflectivity. In the experiment, an exponential distribution was assumed for the true rainfall rate. However, this assumption has no effect on the qualitative conclusions of this work. The most important conclusions are: 1) large (over 10 000) samples of statistically independent observations are required to obtain valid (with small variance) estimates of Z–R parameters, 2) linear regression leads to biased estimators, and 3) the results of estimation are sensitive to the selection of thresholds typically applied to both raingage rainfall data and radar reflectivity data.

The presented results are statistically valid since care was taken in selecting the number of realizations needed to obtain stable statistics. This number of independent realizations was determined experimentally to be 250. This many realizations were used in all the simulations.

It is interesting to note the high correlation coefficient associated with the linear regression. In all cases it was found to be higher than 0.98. This is in agreement with the observation made by Rosenfeld et al. (1990). However, it is attributed to the usually high correlation among any ordered sample statistics.

All the presented simulations are for the case where observations constituting the samples were independent. It is well known that rainfall and radar reflectivity are autocorrelated, both in time and space. In order to use the presented results as guidelines for development of rainfall estimation system, it is necessary to account for these correlations. The following speculative argument can be used to help put the investigated sample sizes in proper perspective: For each hour of GATE-like rainfall [spatial correlation distance of about 20 km, fractional coverage of 0.08, (Bell 1987), and 1 h for temporal correlation distance, (Laughlin 1981)],

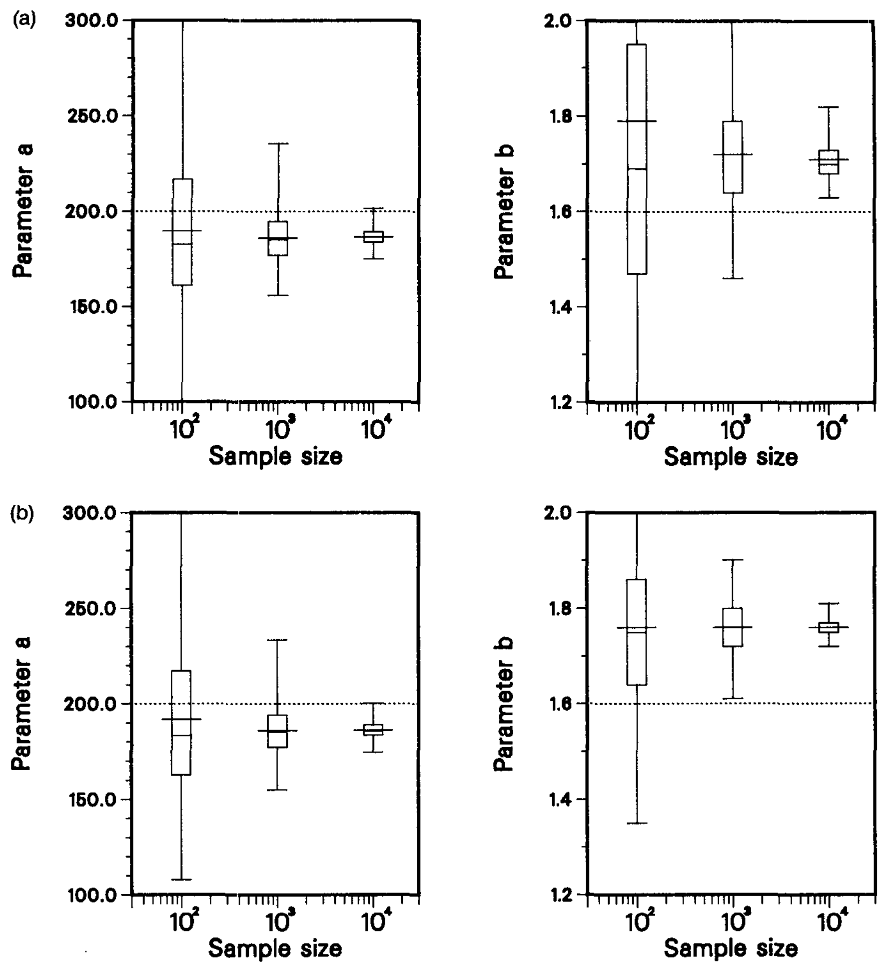

FIG. 5. (a) Same as Fig. 4a but rainfall threshold was 0.20 mm h^{-1}. This is the case where the thresholds do not correspond to each other. Parameters were estimated using nonlinear regression. (b) Same as (a), but the parameters were estimated using linear regression.

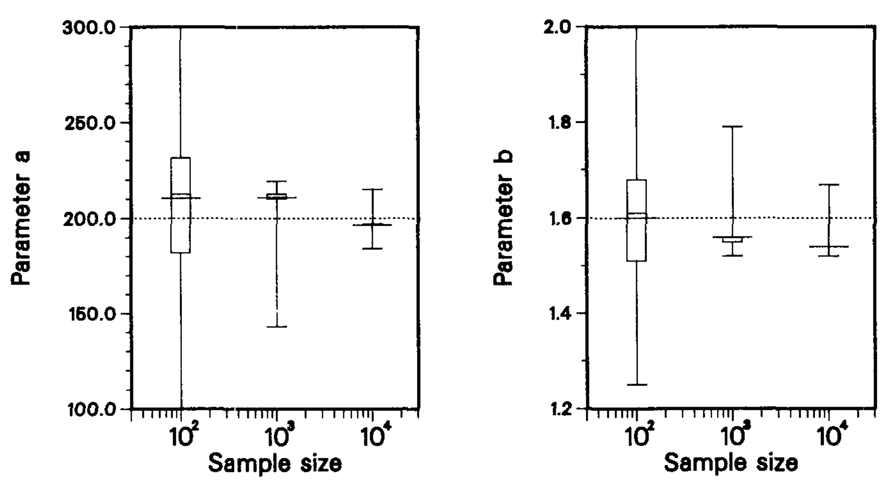

FIG. 6. Results of a simulation experiment of Z–R parameter estimation from synchronous observations. The true parameters are marked with the dashed lines. Rainfall was assumed to be exponentially distributed with the mean rate of 2 mm h^{-1}. Rainfall measurement error was 10%; reflectivity measurement error was 1 dBZ. Rainfall threshold was 0.15 mm h^{-1}; reflectivity threshold was 10 dBZ. Parameters were estimated using nonlinear regression.

a radar with 200-km range provides approximately eight independent 4×4 km^2 observations. As far as raingage observations are concerned, the corresponding number, for the same rainfall regime, is one observation per gauge per hour. The presence of correlated data in the sample increases the sample size requirements needed to achieve the same accuracy as for a sample of independent data.

An important point in interpreting our results is the assumption that a power-law model holds over the entire range of rainfall-rate values. In reality, it almost certainly does not—a simple argument is the extraordinary complexity of the rainfall scattering problem [see Kiriaki and Krajewski (1990) for analytical solutions of scattering by ellipsoidal raindrops in Raleigh regime]. Therefore, the presented results are probably optimistic. To show the quantitative aspects of Z–R estimation under physically realistic conditions requires a more complex simulation setup. Work on development of such a scenario is underway by the authors.

The results presented in this work could assist in error analysis of radar rainfall estimation products used in hydrology and meteorology. An example is the large-area rainfall estimation using the HART technique (Atlas et al. 1990).

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