Multiple Disdrometer Observations of Rainfall

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ABSTRACT

Raindrop sampling instruments, such as disdrometers and optical array probes, have been used by researchers to observe raindrop-size distributions. Estimates of raindrop-size distribution (RSD) can be made from these instruments, which can subsequently be used to study several derived parameters such as liquid-water content, rainfall rate, and radar reflectivity. These instruments have limited sampling volume, which affects the estimates of raindrop-size distribution and the derived integral parameters. Although fluctuations in the derived parameters obtained from the same disdrometer samples are correlated, estimates from independent disdrometers are not. This paper addresses the issues involved in comparing data from disdrometers. Data from four disdrometers sampling the same rain volume are analyzed to study the measurement fluctuations in a single disdrometer with time and between disdrometers at the same time interval. Theoretical analysis and data show that the correlation between derived parameters of sampled RSD helps in observing the mean feature between the parameters with much less scatter. It is also shown that the scatter between the derived parameters of a single disdrometer, or between different disdrometers, is a function of the correlation between the estimates.

1. Introduction

Raindrop-size distributions (RSD) can be estimated by using surface instruments like the disdrometer (Joss and Waldvogel 1967) or using probes mounted on instrumented aircraft (Knollenberg 1981). Joss and Gori (1978) have conducted studies combining RSD estimates from many disdrometers in order to observe the influence on mean shape with averaging. Observations of single disdrometer or multiple disdrometer measurements have also been used to study the natural variability of RSD (Gori and Joss 1980; Ulbrich 1983). Comparison of data from different RSD sampling instruments is controlled by several factors, including spatial, temporal, and measurement fluctuations. In this paper we address the issues involved in comparing data between different disdrometers. We consider the mean feature common to the disdrometers as well as the difference between the measurements. We extend this principle of analysis to comparison of derived parameters such as reflectivity Z and rainfall R between disdrometers, in order to interpret the variability between measurements.

The paper is organized as follows. Section 2 describes the RSD model and computation of integral parameters from the RSD estimate. Section 3 provides the theoretical basis for analyzing multiple disdrometer observations. Data collected using four disdrometers sampling the same rainfall are presented in section 4. Section 5 summarizes the results of this paper.

2. Raindrop-size distribution

The space–time variability of raindrop-size distribution (RSD) is typically due to a variety of physical processes, for example, evaporation, collision–coalescence, collisional breakup, inhomogeneities produced by turbulent fluctuations, drop sorting, etc. The RSDs are important in determining characteristics of rain medium such as reflectivity Z, liquid water content W, and rainfall rate R. Ulbrich (1983) showed that a gamma RSD can describe many of the natural variabilities in the RSD. The gamma form of the RSD can be written as

$$N(D) = N_0 D^m \exp(-\Lambda D), \qquad (1)$$

where $N(D)$ is the number of raindrops per unit volume per unit size interval (D to $D + \Delta D$). This distribution can be written in terms of the total number of drops per unit volume (N_T) as

$$N(D) = \frac{N_T}{\Gamma(\alpha)\beta^\alpha} D^{\alpha-1} \exp(-D/\beta), \qquad (2)$$

where $\alpha > 0$, $\beta > 0$, $D \geqslant 0$. Note that $m = \alpha - 1$,

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$\Lambda = \beta^{-1}$, and $\Gamma(\ )$ indicates gamma function. The median volume diameter D_0 is defined such that all drops with diameter $\leqslant D_0$ contribute to one-half of the total liquid water content. This property can be expressed in terms of RSD as

$$\int_0^{D_0} D^3 N(D) dD = \int_{D_0}^{\infty} D^3 N(D) dD. \quad (3)$$

Ulbrich (1983) has shown that $\Lambda D_0 \approx 3.67 + m$. Reflectivity Z, rain intensity R, and other characteristics of the rain medium can be expressed as moments or integrals over the RSD,

$$Z = \int_0^{\infty} D^6 N(D) dD \quad (\mathrm{mm}^6\ \mathrm{m}^{-3}) \quad (4)$$

$$R = \frac{\Pi}{6} \int_0^{\infty} D^3 N(D) v(D) dD \quad (\mathrm{mm\ h}^{-1}), \quad (5)$$

where $v(D)$ is the raindrop fall speed. Here $v(D)$ can be approximated in still air as $v(D) = CD^{0.67}$, with $C = 17.67\ \mathrm{m\ s}^{-1}\ \mathrm{cm}^{-0.67}$ (Atlas and Ulbrich 1977). In general, an integral parameter F of the RSD can be expressed as

$$F = \int_0^{\infty} f(D) N(D) dD, \quad (6)$$

where $f(D)$ describes the drop-size dependent property contributing to parameter F. When $f(D)$ is of the power-law form then F can be shown to be a moment of the RSD.

Raindrop-size distributions are sampled using surface instruments like the disdrometer (Joss and Gori 1978) or using 2D-PMS probes mounted on instrumented aircraft (Knollenberg 1981). Estimates of integral parameters of the RSD such as Z and R are computed from the observed raindrop samples as follows:

$$\hat{Z} = \frac{1}{V} \sum_{i=1}^{n} D_i^{\,6} \quad (\mathrm{mm}^6\ \mathrm{m}^{-3}) \quad (6a)$$

$$\hat{R} = \frac{7.12 \times 10^{-3}}{V} \sum_{i=1}^{n} D_i^{\,3.67} \quad (\mathrm{mm\ h}^{-1}), \quad (6b)$$

where V is the sampling volume.

The preceding can be written in a general form for parameter F, such as

$$\hat{F} = \frac{1}{V} \sum_{i=1}^{n} f(D_i). \quad (6c)$$

The general estimates for the moments of the RSD can be written as

$$\hat{P}_\alpha = \frac{C_\alpha}{V} \sum_{i=1}^{n} D_i^{\,\alpha}, \quad (6d)$$

where $P_\alpha = C_\alpha \int D^\alpha N(D) dD$.

Gertzman and Atlas (1977) and Chandrasekar and Bringi (1988) have shown that the fractional standard deviation (FSD) of these estimates is given by

$$\mathrm{FSD}(\hat{P}_\alpha) = \left(\frac{\Lambda^{m+1}}{N_0 V}\right)^{1/2} \frac{\Gamma^{1/2}(m + 2\alpha + 1)}{\Gamma(m + \alpha + 1)}. \quad (7)$$

Chandrasekar and Bringi (1987) have shown that two different moments of the RSD estimated from the same raindrop samples are correlated. For example, the correlation between reflectivity and rainfall rate estimated from the same disdrometer is fairly high, having a typical value of 0.8. This correlation helps in the observation of mean relationships between derived parameters like Z and R from the RSD, reducing the scatter due to measurement errors.

3. Multiple disdrometer observations

Spatial sampling of RSD can be done by several disdrometers located at various regions or by moving the same instrument through different regions, as in the case of PMS probes mounted on research aircrafts. The observed variabilities in the RSD or other integrated parameters in these measurements are composed of three parts, namely, 1) spatial variability, 2) temporal variability, and 3) measurement fluctuations. In the case of aircraft-mounted probes data are collected with the aircraft in motion, and, hence, all three of the previous factors are observed together. However, in the case of multiple disdrometers variability can be observed between spatially distributed instruments at a given time, which will have fluctuations due to 1) and 3). If there is more than one disdrometer at the same location, then the variability between the instruments at a given time gives the measurement fluctuations, which are not physical variabilities. The extent of measurement fluctuations could depend on the mean value of the physical parameter measured; for example, the standard deviation in the estimate of rain rate will vary with mean rainfall rate.

Let $\hat{R}_j$, $\hat{R}_k$ be the rainfall-rate estimates obtained from independent identical disdrometers j and k (located very close in space) sampling the same rainfall. The variance of the difference $(\hat{R}_j - \hat{R}_k)$ can be written as

$$\mathrm{var}(\hat{R}_j - \hat{R}_k) = 2\ \mathrm{var}\hat{R}. \quad (8)$$

This equation can be written in general for any parameter P as

$$\mathrm{var}(\hat{P}_j - \hat{P}_k) = 2\ \mathrm{var}\hat{P}.$$

Thus, more than one disdrometer located at the same place can be used to measure the standard error in the estimates, directly from measurements.

Conventional techniques to obtain Z–R relationships from disdrometers involve computing $\hat{Z}$ and $\hat{R}$ from the same disdrometer and fitting a curve through starts tracking R more closely. However, if two different disdrometers are used to compute $\hat{R}_1$ and $\hat{R}_2$, the term $\mathrm{var}(\epsilon_1 - \epsilon_3)$ cannot be neglected. Thus, the preceding analysis shows that the data from the same disdrometer can be used to obtain mean relationship between parameters, but the variability between them is both physical and statistical in nature and cannot be attributed to one of them easily.

4. Data analysis

The data used in this analysis are obtained from four Joss-RD-69 (Joss and Waldvogel 1967) disdrometers placed on a square grid 1 m apart. The data from the four disdrometers were collected at Locarno Monti, Switzerland, sampling widespread rain over 120 contiguous samples, 1 min long. Figures 1a–c show time series of $\hat{R}$, $\hat{Z}$, and $\hat{Z}_{DR}$ obtained from disdrometer measurements, where the computation of Z_{DR} was made using equilibrium axis ratio relationship. The time scale shown is in minutes. It can be seen from Fig. 1a that the dataset includes light to moderate rainfall. Table 1 shows the results of standard deviation in the estimates of R, Z, and Z_{DR} calculated at S band. These standard deviations are calculated comparing estimates from the four disdrometers, considering two at a time, using the method described in section 3. The standard deviation results of Table 1 indicate that Z and Z_{DR} can be estimated to typical accuracies of 1 and 0.1 dB, respectively; however, the accuracy of rain rate changes with mean rain rate varying between 0.2 and 1.2 mm h^{-1}. These experimental results agree well with theoretical accuracies reported earlier in text by Gertzman and Atlas (1977) and Chandrasekar and Bringi (1988).

Disdrometers typically collect raindrop data over a fixed time interval (typically 1 min), and integral parameters obtained from such measurements are plotted as a function of time as shown in Fig. 1. The time series of these measurements have fluctuations due to inherent time variability of the parameters as well as due to measurement. To quantify these variabilities, temporal standard deviation of an estimated (say $\hat{R}$) single disdrometer is defined

$$\mathrm{var}\hat{R}(\Delta t) = \mathrm{var}(\hat{R}_t - \hat{R}_{t+\Delta t}). \quad (17)$$

The true time fluctuation variance can be obtained by

TABLE 1. Measurement standard deviation of $\hat{Z}$, $\hat{R}$, and $\hat{Z}_{DR}$ stratified with reflectivity.

Range of Z	$\sigma(\hat{Z})$ (dB)	$\sigma(\hat{Z}_{DR})$ (dB)	$\sigma(\hat{R})$ (mm h^{-1})
20–25	1.06	0.07	0.15
25–30	1.00	0.1	0.18
30–35	0.75	0.1	0.34
35–40	0.65	0.13	0.48
40–45	0.85	0.22	1.35

TABLE 2. Temporal standard deviation of $\hat{Z}$ and $\hat{R}$ given as a function of time lag Δt.

Δt (min)	$\sigma[\hat{Z}(\Delta t)]$ (dB)	$\sigma[\hat{R}(\Delta t)]$ (mm h^{-1})
2	3.7	1.9
3	4.1	2.2
4	4.3	2.5
5	5.2	2.4

subtracting twice the variance of $\hat{R}$. Table 2 shows the temporal standard deviation of $\hat{R}$ and $\hat{Z}$ for $\Delta t = 2$, 3, 4, and 5 min. In Table 2 it can be seen that as Δt increases the temporal standard deviation increases, which represents true variability in the measurement not due to instrument. When $\Delta t = 2$ min the temporal standard deviation is twice that of the standard deviation due to measurement error; hence, any considered variability in $\hat{R}$, observed at time intervals Δt of 2 min or more, is physically significant.

Figure 2a shows a scatterplot of specific attenuation at X band divided by Z calculated for S band versus Z_{DR} at S band, where all three estimates are obtained from the same disdrometer. Figure 2b shows similar results except that A, Z, and Z_{DR} are computed from three different disdrometers at the same time instant. The correlation between the triplet of measurements A/Z and Z_{DR} is responsible for small scatter in Fig. 2a, comparing well with similar results reported by Aydin et al. (1986). However, such correlation is absent in Fig. 2b and, hence, a wider scatter is seen. Thus, the results of Fig. 2a show that RSD measurements from a single disdrometer can be used to infer mean relationship between integral parameters with good accuracy.

Figure 3a shows a scatterplot of $\hat{R}$ obtained using (6b) versus $\hat{R}(Z, Z_{DR})$ where $\hat{R}(Z, Z_{DR})$ is obtained from $\hat{Z}$ and $\hat{Z}_{DR}$, and the following expression (Chandrasekar and Bringi 1988)

$$R(Z, Z_{DR}) = 1.9 \times 10^{-3} Z^{0.94} Z_{DR}^{-1.014} \quad (\mathrm{mm\ h^{-1}}), \quad (18)$$

where Z is in standard units (mm^6 m^{-3}) and Z_{DR} is in units of decibels (dB). The results of Fig. 3a were obtained with data from the same disdrometer. Figure 3b shows similar results except the estimates are obtained from three different disdrometers. Comparing Figs. 3a and 3b it can clearly be seen that the mean feature, that is, the ability of $R(Z, Z_{DR})$ to estimate R, is shown well with very little scatter about the 1:1 line. However, the scatter in Fig. 3b is significantly larger than that of Fig. 3a even though the mean feature exists. The results of Fig. 3 again bring out the point that RSD measurements from a single disdrometer can be used successfully to evaluate the mean relationships; however, the scatter about the line is a strong function of the correlation between the integral parameters as discussed in sections 3 and 4. The scatter about the integral pa-

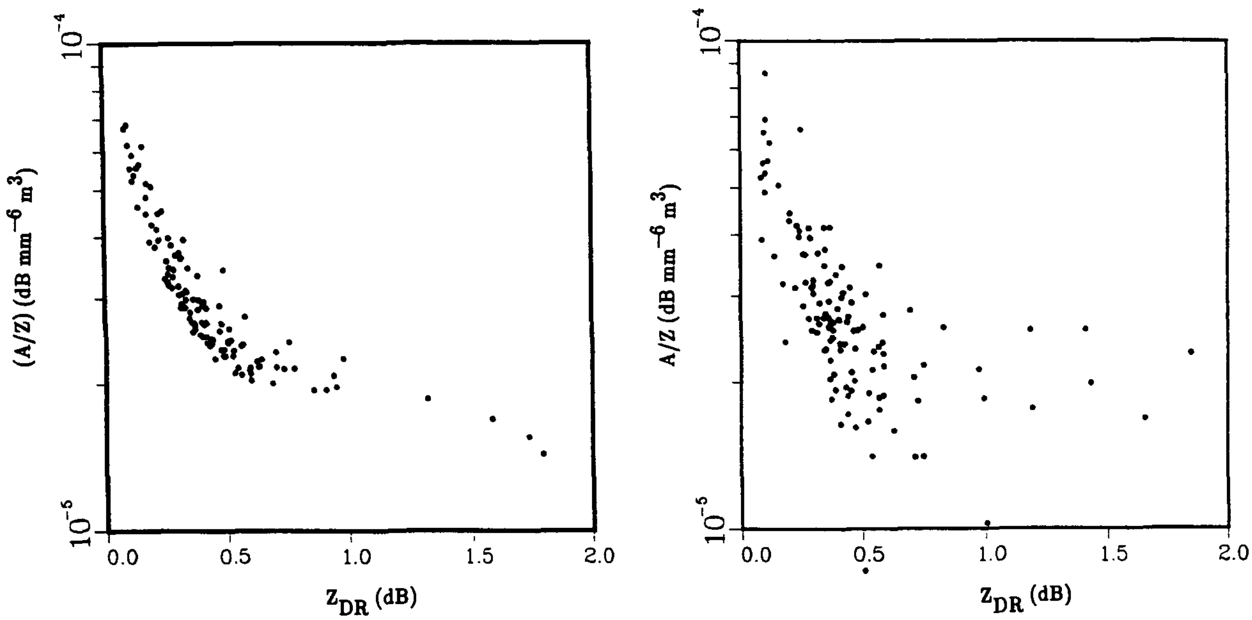

FIG. 2. (a) Scatterplot of specific attenuation estimates at X band divided by $\hat{Z}$ at S band versus $\hat{Z}_{DR}$ at S band. All three estimates are obtained from the same disdrometer. Note the small scatter between the triplet of measurements. (b) Same as (a), except that the three estimates $\hat{A}$, $\hat{Z}$, and $\hat{Z}_{DR}$ are obtained from three different disdrometers sampling the same rainfall. Note that in comparison with (a), (b) shows larger scatter.

rameter estimates cannot be taken as a direct measure of the scatter in the technique. However, the scatter about the mean features observed from the single disdrometer observations gives a general idea about the scatter in the mean value of parameters.

5. Conclusions

We have analyzed the fluctuations in the estimates of derived parameters like R and Z from raindrop sampling devices. These fluctuations are composed of

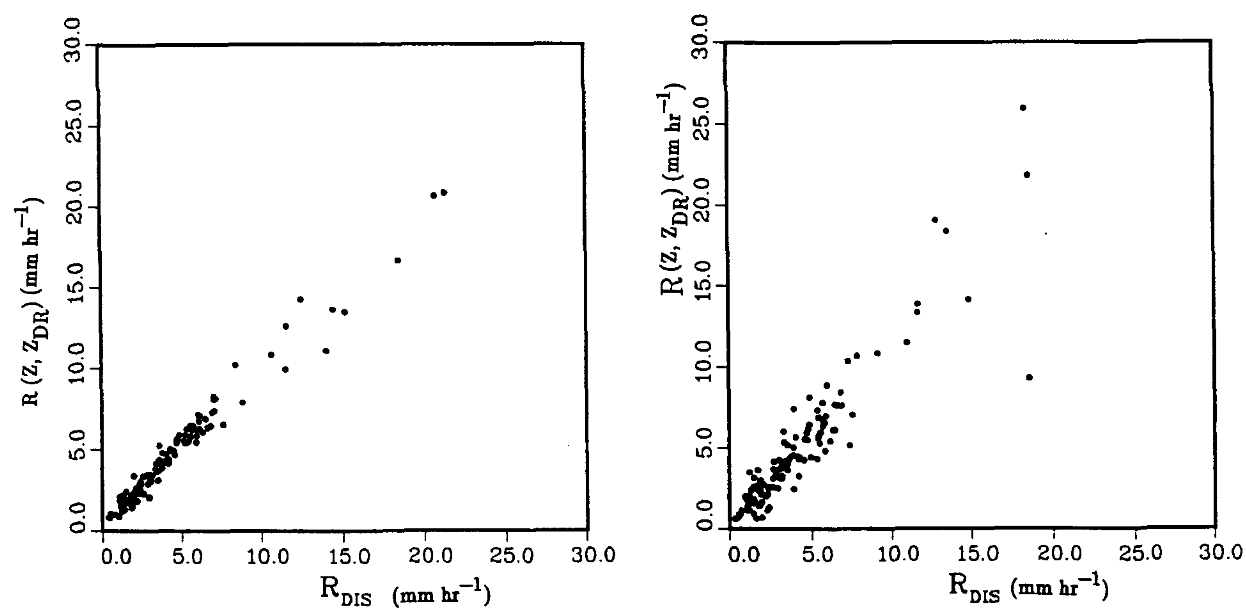

FIG. 3 (a) Scatterplot of rainfall estimates, $\hat{R}$ calculated as a direct integral of the measured RSD versus the rainfall estimates calculated indirectly from the estimates $\hat{Z}$ and $\hat{Z}_{DR}$, $\hat{R}(Z, Z_{DR})$. The two estimates for rainfall are obtained from the same disdrometer. (b) Same as (a), except that the two estimates of rainfall $\hat{R}$ and $\hat{R}(Z, Z_{DR})$ are calculated from three different disdrometers. Note the wider scatter compared to (a).

three parts; namely, 1) spatial variability, 2) temporal variability, and 3) measurement fluctuation. We have utilized the data from four disdrometers sampling the same rainfall to study the characteristics of fluctuations in the estimates of integral parameters. We have estimated the standard error in the measurements of reflectivity, differential reflectivity, and rainfall rate using data from four disdrometers and show that they agree with the theoretical results of Gertzman and Atlas (1977) and Chandrasekar and Bringi (1988). We have demonstrated using multiple disdrometer data that different integral parameter estimates, such as R and Z, are correlated if obtained from the same disdrometer. However, this correlation is much less when the estimates are obtained from different disdrometers. We have demonstrated using theoretical analysis and observations that this correlation between integral parameter estimates enables us to observe the mean relationship between them without much scatter in observations. It is also shown that the scatter between the integral parameter estimates is controlled by the amount of correlation between the estimates, and, hence, care should be exercised in comparing scatter between two sets of intercomparisons.

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