The Accuracy of the Area-Threshold Method: A Model-based Simulation Study

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ABSTRACT

A Monte Carlo simulation study is conducted to investigate the performance of the area-threshold method of estimating mean areal rainfall. The study uses a stochastic space–time model of rainfall as the true rainfall-field generator. Simple schemes of simulating radar observations of the simulated rainfall fields are employed. These schemes address both random and systematic components of the radar rainfall-estimation process. The results of the area-threshold method are compared to the results based on conventional averaging of radar-estimated point rainfall observations. The results demonstrate that when the exponent parameter in the *Z–R* relationship has small uncertainty (about ±10%), the conventional method works better than the area-threshold method. When the errors are higher (±20%), the area-threshold method with optimum threshold in the 5–10 mm h^{-1} range performs best. For even higher errors in the *Z–R* relationship, the area-threshold method with a low threshold provides the best performance.

1. Introduction

Estimation of areally averaged rainfall rate at scales on the order of 100 000 km^2 is an important element of climatic studies. Recently, an estimation method based on measurement of the area with rainfall rates above a threshold has received considerable attention. Several authors, including Doneaud et al. (1981; 1984), Chiu (1988), Lopez et al. (1989), Atlas et al. (1990a), Kedem et al. (1990), Rosenfeld et al. (1990), and Braud et al. (1991) reported high correlation between the fractional area above a fixed rainfall-rate threshold and the areally averaged rainfall rate in radar and raingage datasets. This forms the basis for the techniques of estimating space–time accumulated rainfall known as the area–time integral (ATI) method and the height–area threshold (HART) method (Rosenfeld et al. 1990). These methods may have important implications for climatological rainfall estimation from satellites and ground-based radars (Simpson et al. 1988).

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Space–time rainfall integrals could be estimated by measuring fractional areal coverage and integrating over time. In this paper, the aforementioned approach to estimating space–time integrals of rainfall will be referred to as the *area-threshold method.*

In order to implement the area-threshold method with radar-based observations, a conversion of the radar-measured quantity—radar reflectivity—into rainfall rate is required. To use the method with satellite-based passive microwave observations, a conversion of the brightness temperature into rainfall rate is involved (Chiu and Kedem 1990). In this paper, only radar-based rainfall estimation is addressed. The considerations are further restricted to situations in which the only radar-measured parameter is radar reflectivity. Therefore, multiparameter radar rainfall-estimation methods are excluded from the analysis.

The conversion of radar reflectivity into rainfall rate appears in two steps. For application of the area-threshold method, the first step is calibration of the relationships between the fractional areal coverage above a selected threshold and mean areal rainfall. If estimation of the areal rainfall is based on radar observations, then it is obvious that a conversion of radar reflectivity into rainfall rate is necessary. The second step is determination of fractional area covered by rainfall rate above the preselected threshold. Since the threshold is often specified in terms of rainfall rate, again a conversion of radar-measured reflectivity into rainfall rate is involved. Once the fractional area is determined, application of the method is completed by using the previously calibrated relationship to obtain an estimate of mean areal rainfall.

The potential effects of reflectivity–rainfall conversion on the accuracy of the area-threshold method have been recognized by Kedem et al. (1990) and Atlas et al. (1990b), among others. They express hope, however, that the conversion will have a minor effect (relative to its use in traditional radar-based estimation of rainfall). Rosenfeld et al. (1990a) state that ". . . we do not yet have an adequate independent means of validating the area-integral methods, all of which have thus far been based on radar observations."

The purpose of this paper is to present a simple simulation study that sheds some light on the accuracy of the area-threshold method. The study accounts for the "radar effects" as manifested by reflectivity measurement error and the conversion of radar reflectivity to rainfall rate. Other sources of radar rainfall-measurement error (see Zawadzki 1984 for discussion) are ignored here. The study does not attempt to offer a comprehensive analysis—it merely attempts to complement the real-data investigations performed hitherto with simulation results.

The study is based on simulations of rainfall from the stochastic space–time model developed by Bell (1987). The model was calibrated to mimic the statistical behavior of GATE [Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment] rainfall (Hudlow and Patterson 1979). It has been applied in several studies concerning the design of the Tropical Rainfall Measuring Mission (TRMM) planned for the late 1990s by the National Aeronautics and Space Administration (NASA) (Simpson et al. 1988; Bell et al. 1990).

The paper is organized in the following order. A brief description of the area-threshold method is given in section 2. Section 3 describes the simulation experiment performed to assess the accuracy of the method. The results of the experiment are presented in section 4. Conclusions and recommendations for future research conclude the paper.

2. Area-threshold method

Let the domain of interest be denoted as A. Let rainfall rate at location $\mathbf{u} = (x, y)$, $\mathbf{u} \in A$, be $R(\mathbf{u})$. The mean rain rate R_A over the domain A can be expressed as

$$R_A = \frac{1}{|A|} \int_A R(\mathbf{u}) d\mathbf{u}, \qquad (1)$$

where $|A|$ is the total area of the domain A and $d\mathbf{u}$ is a differential area element. Define the fractional area of rainfall above threshold τ as

$$F(\tau) = \frac{1}{|A|} \int_A \mathbf{I}[R(\mathbf{u}) > \tau] d\mathbf{u}, \qquad (2)$$

where $\mathbf{I}(\ )$ is the indicator function

$$\mathbf{I}[R(\mathbf{u}) > \tau] = \begin{cases} 1, & \text{if } R(\mathbf{u}) \geqslant \tau \\ 0, & \text{otherwise.} \end{cases} \qquad (3)$$

According to the area-threshold method, the mean areal rainfall rate can be estimated from the following simple expression:

$$\hat{R} = S(\tau)\hat{F}(\tau) + I(\tau), \qquad (4)$$

where $S(\tau)$ and $I(\tau)$ are the slope and intercept computed during calibration of the method. The hat "ˆ" signifies an estimate.

A linear regression formulation of (4) for the relationship between fractional area and mean areal rainfall rate was used by Rosenfeld et al. (1990) on data from GATE, Texas, South Africa, and Darwin, Australia. They found very high correlations associated with relationship (4) exceeding .98.

Although for the purpose of this paper the mathematical description of (4) is sufficient, it is helpful to express the relationship between mean areal rainfall and fractional area above a threshold in terms of probability distribution of rainfall rate. Assuming homogeneity of the probability distribution of rainfall rate, the aforementioned quantities can be given in terms of the corresponding probability density function $f_r(R)$. According to Kedem et al. (1990), fractional area of rain rate exceeding threshold τ can be approximated by probability of rainfall rate exceeding the threshold $P(R > \tau)$

$$F(\tau) \rightarrow P(R > \tau) \qquad (5)$$

and

$$\hat{R} \rightarrow E\{R\} = \int_0^\infty R f_r(R) dR, \qquad (6)$$

where $E\{\ \}$ denotes expectation operator. The slope function $S(\tau)$ reduces to the following ratio of conditional quantities:

$$S(\tau) = \frac{E\{R | R > 0\}}{P(R > \tau | R > 0)}. \qquad (7)$$

The importance of this representation of the area-threshold method is in stressing the statistical character of the method. The formulation given by Kedem et al. (1990) ignores the role of intercept $I(\tau)$ in (4). More discussion on this issue will be given later.

Thus, if one knows the slope function $S(\tau)$ and the intercept $I(\tau)$, one can estimate the areally averaged rainfall rate simply by measuring the fractional coverage of the area $F(\tau)$ above the threshold τ. This is a very attractive concept in view of the fact that the TRMM satellite will be equipped with a radar capable of making such measurements over oceans.

The basic questions that need to be asked are: 1) what is the accuracy of the method? and 2) what are the most critical aspects of its application? As pointed out by Atlas et al. (1990), a comprehensive uncertainty analysis of the ATI and HART methods, which are variants of the area-threshold method, has not been performed. Recently, Kedem et al. (1990) formulated the general conditions under which the method should work well, and performed a limited sensitivity analysis with respect to the parameters of commonly used rainfall probability density functions.

There are at least two important estimation issues of direct relevance to the method. First, since radar does not measure rainfall, but rather rainfall reflectivity, it is necessary to convert the radar-measured reflectivity factor to rainfall rate. Such a conversion is usually accomplished using a Z–R relationship. The problem of Z–R parameter estimation was discussed in hydrometeorological literature many times with recent important developments reported by Atlas et al. (1990), Krajewski and Smith (1991), and Smith and Krajewski (1991). In the context of the area-threshold method, Rosenfeld et al. (1990) warn against erroneous specification of the threshold τ, which ". . . could result from either a systematic calibration error in the observing system or a systematic error in the Z–R relation, presuming that one is using a radar."

The second issue concerns the estimation of the probability density function of rainfall rate needed to estimate the slope function $S(\tau)$. This probability density function can be estimated either from raingage measurements of rainfall rate or from converted radar reflectivity measurements. In the first case, the estimation is subjected to all the common sampling errors experienced by typically sparse raingage networks; in the second case, it is subject to errors in the Z–R relationship parameters.

In this paper, we deal only with the first issue, that is, the conversion of radar-measured reflectivity into rainfall rate. An analytical approach to the problem seems difficult, for it would require an analytical form of both a space–time model of rainfall and a corresponding space–time model of radar rainfall-measurement error.

In cases where analytical solutions are difficult or impossible to obtain, a straightforward Monte Carlo simulation can offer a viable alternative. The utility of such simulations was demonstrated by Bell et al. (1990) in studies of TRMM sampling error; Goldhirsh (1988), who studied the performance of spaceborne radar rainfall measurements; and Krajewski and Smith (1991), who investigated the estimation aspects of climatological Z–R relationships.

The proposed simulation experiment is based on a statistical space–time model of rainfall. Over the past decade or so, many stochastic space–time models have been proposed in the literature. Some examples include the work of Waymire et al. (1984), Smith and Karr (1985), Rodriguez-Iturbe et al. (1986), Rodriguez-Iturbe and Eagleson (1987), Smith and Krajewski (1987), and Bell (1987). All of these models are attractive for our purposes since they can be easily adopted to a simulation framework. The most useful for the purposes of this paper, however, appears to be the Gaussian-type model described by Bell (1987). Bell's model was the basis for several sampling studies related to TRMM (see Bell et al. 1990). The model was calibrated to mimic the GATE (Hudlow and Paterson 1979) statistics. Since many studies of the area-threshold method, or its variants, are based on GATE data, the use of the Bell model seems further justified. Next, we describe a numerical simulation experiment that uses the Bell model as the true rainfall-field generator together with simulated radar observations of those fields.

3. Experimental setup

A problem with previous studies of the area-threshold technique, which have been based on observed data, is that the true rainfall field remains unknown. Therefore, the accuracy of the method cannot be easily established. It has been established without a doubt that there is a strong linear relation between the area over a threshold and the areawide rainfall. The degree of such linear dependency is a function of the selected threshold.

In Fig. 1, the relationship is shown for four thresholds (0, 5, 10, and 20 mm h^{-1}) based on 2000 rainfall fields generated with Bell's model. The model parameters were the same as in Bell et al. (1990). Rainfall was generated over a 400-km $\times$ 400-km domain with a resolution of 4-km $\times$ 4-km pixels. The nonlinearity present in the plot corresponding to the lowest threshold [also existing in the original GATE data; see Kedem et al. (1990)] will be ignored here, and we will consider the linear relationships only. These results can be explained by heuristic considerations according to which the intercept corresponding to the zero threshold should be zero, and it could increase as the threshold increases up to a value no greater than the threshold itself. Also, the slope increases with the threshold, as expected.

Figure 2 shows the threshold-dependent regression parameters, such as correlation, slope, intercept, and mean-square error. It is observed that the smallest mean square-root error thresholds occur for values between 9 and 10 mm h^{-1}. Table 1 gives a comparison of the regression parameters obtained for the Bell model and the original GATE data. Large similarities are evident.

The simulation experiment was designed to have three scenarios denoted I, II, and III. Within each scenario, two samples are used, each with 2000 generated

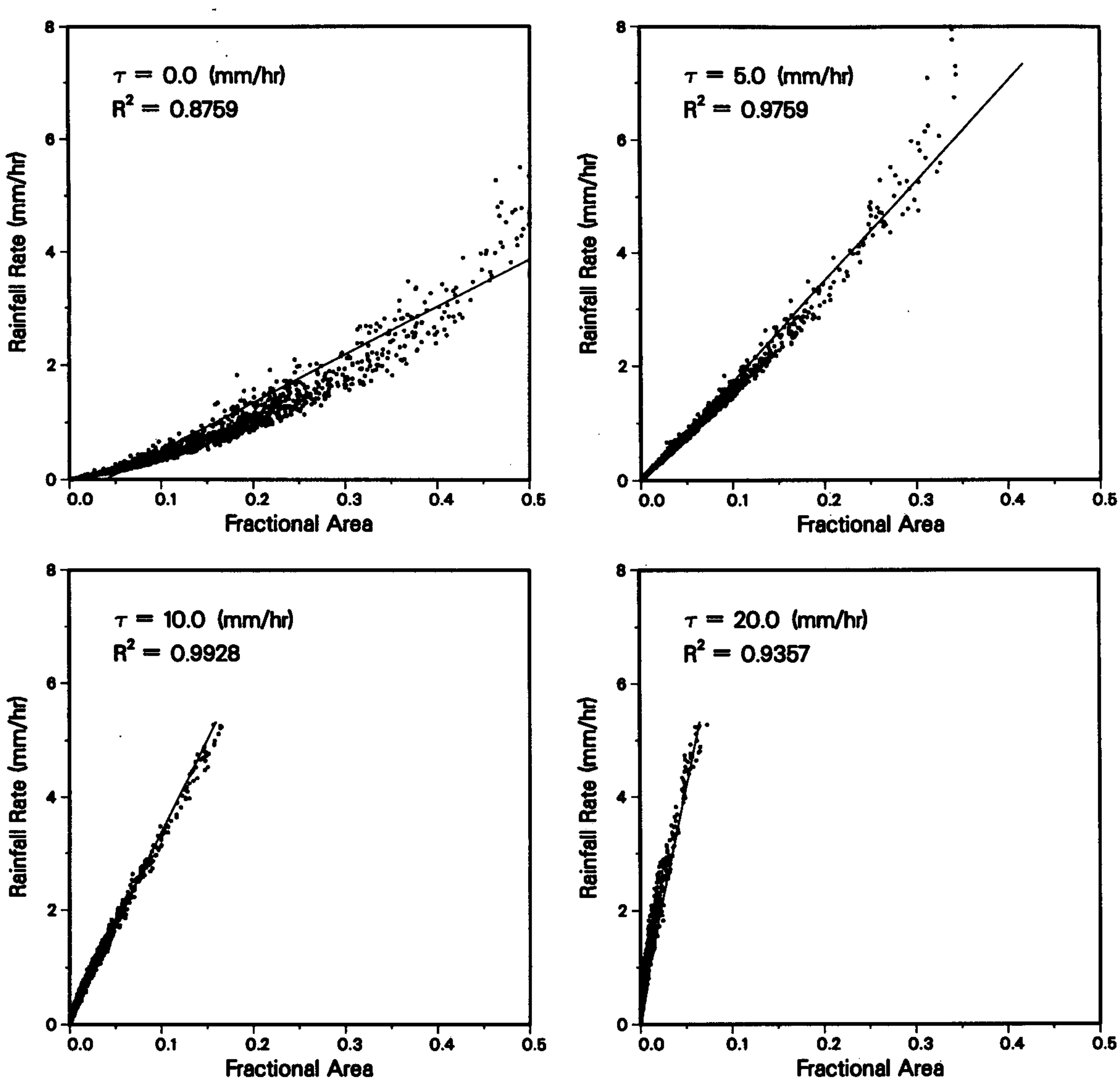

FIG. 1. Regressions between areal coverage above threshold and mean areal rainfall obtained from 2000 rainfall fields generated by Bell's model (calibration sample). The corresponding thresholds are, clockwise, 0, 5, 10, and 20 mm h^{-1}.

rainfall fields. One sample represents a calibration sample, that is, a sample of "historical" data used for development of the regression relationships, while the second sample, statistically independent, is called a validation sample and serves as the subject of the developed regressions. This process imitates the real-world situation where relationships (models) developed on one dataset are applied to an independent set for predictive purposes.

Scenario I does not involve simulated radar observations. It is concerned only with the errors due to the estimated regression relationships between areally averaged rainfall values and fractional area above a threshold. Linear regressions between the mean areal rainfall and the fractional area are developed for the calibration sample and applied to the fractional area from the validation sample. Then, errors are computed, and their distributional statistics are presented as box plots. The resultant errors are due both to the random error about the regression and to the sampling error of the regression.

Scenario II considers effects of the random error present in radar-measured reflectivities. Simulation of this effect is accomplished by first converting the model-generated rainfall rate R to radar reflectivity factor Z using an assumed Z–R relationship. In this way, we obtain the "true" radar reflectivity, which is then contaminated with a random Gaussian error of zero mean and standard deviation of 1 or 2 dBZ. The contaminated (erroneous) reflectivity field is converted back to rainfall rate. In scenario II, this conversion is accomplished using the original Z–R relationship. A cut-

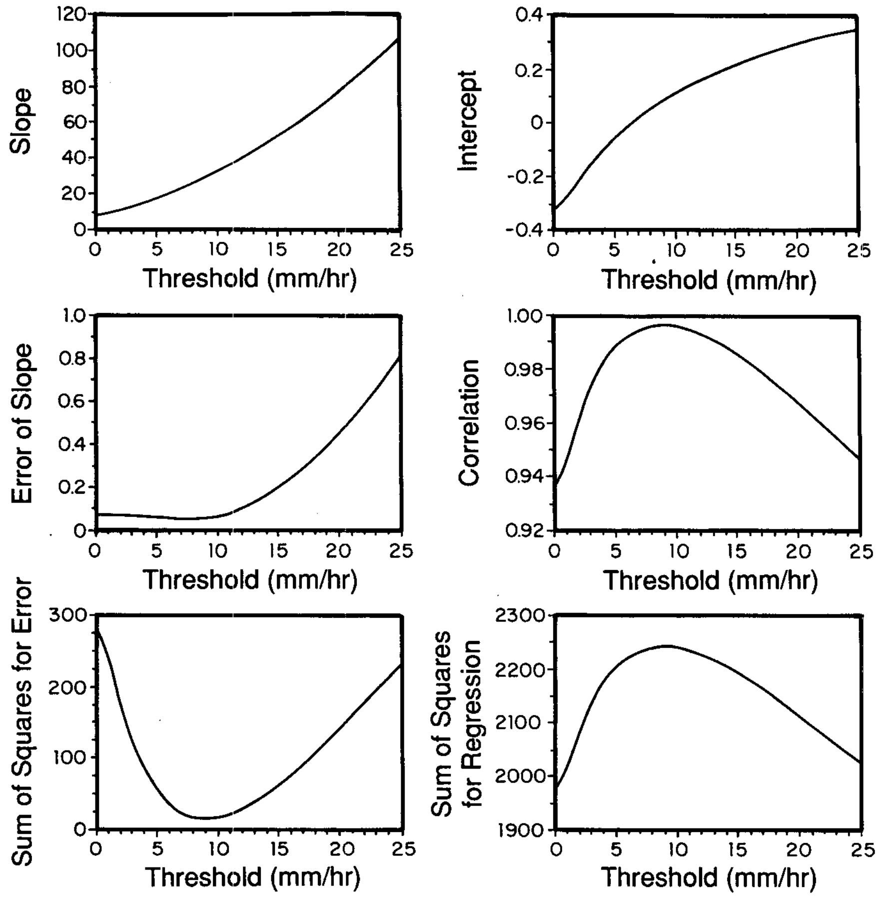

FIG. 2. Area-threshold-method regression statistics as function of threshold. Obtained from 2000 rainfall fields generated by Bell's model (calibration sample).

off of 20 dBZ is used to simulate radar sensitivity—there is zero rainfall for those locations where the simulated reflectivity is below the sensitivity threshold. This value of radar sensitivity threshold is rather conservative—modern radars, such as the WSR-88D used in the Next Generation Weather Radar (NEXRAD) system, have sensitivity of −8 dBZ at 50 km, which roughly corresponds to a rainfall rate of 0.01 mm h^{-1}. The use of a conservative radar sensitivity value will affect the performance of the area-threshold method for the low thresholds. The effects of radar beamwidth spreading and the sidelobes, which may lead under certain conditions to overestimation of the rain area, are ignored in this paper.

TABLE 1. Regression coefficients obtained from 2000 rainfall fields generated by Bell's model (calibration sample). GATE values are given for comparison (after Chiu 1988).

Dataset	Threshold (mm h^{-1})	Slope	Intercept	R^2
Bell model	0	8.93	−0.32	.875
	1	9.27	−0.28	.895
	5	17.73	−0.05	.975
	10	32.68	0.11	.992
	20	77.29	0.30	.935
GATE I	0	5.06	−0.13	.776
	1	7.21	−0.01	.886
	5	15.85	−0.05	.979
	10	33.27	0.12	.947
	20	93.14	0.16	.853
GATE II	0	4.94	−0.09	.706
	1	7.52	−0.07	.853
	5	18.09	−0.01	.970
	10	36.24	0.07	.960
	20	82.42	0.14	.874

Four sets of results were generated within the scope of scenario II. They are compiled in Table 2. The random error added to simulate radar-measured reflectivities corresponds to good-to-average quality radars and is consistent with values used by Sachidananda and Zrnić (1987) and Chandrasekar and Bringi (1987), among others.

Scenario III accounts both for the measurement error of reflectivities and the error of converting reflectivities to rainfall rates. This conversion is manifested as a systematic error leading to biased rainfall estimates in most cases (sometimes there may be a canceling effect of the erroneous parameters of Z–R relationship). In order to investigate this effect, the original conversion of model-generated rainfall rate into radar reflectivity using a "true" Z–R is followed by a conversion back to rainfall rate using an erroneous Z–R relationship. This is done after adding a random component to the generated Z. A radar sensitivity threshold of 20 dBZ is applied as in scenario II. The Z–R parameters' error domain is considered from −50% to 50% for the a parameter and from −30% to 50% for the b parameter. The original Z–R relationship used was the "standard" Marshall–Palmer relationship with $a = 200$ and $b = 1.6$.

For scenarios II and III, in addition to calculating errors based on the area-threshold method, errors were calculated for the "reflectivity method." By the reflectivity method, we mean the averaging of the simulated radar rainfall estimates, just as in the traditional radar-based rainfall-estimation approach.

4. Results

The following error definitions are used in this section. The Z–R parameter errors are defined as

$$p_{\text{relative error}} = 100\% \frac{(p_{\text{true}} - p_{\text{error}})}{p_{\text{true}}}, \qquad (8)$$

where p_{true} is the true value of the parameter and p_{error} is the erroneous value of the same parameter. The true areally averaged rainfall $\bar{R}$ is calculated from the values generated by the Bell model. The estimated rainfall is a function of the threshold used, and is estimated for each field using relationship (2), where $F(\tau)$ is the fractional coverage over rain rate greater than τ millimeters per hour, and $S(\tau)$ and $I(\tau)$ are the computed slope and intercept of the regression corresponding to threshold τ. The mean areal rainfall error (expressed in millimeters per hour) is simply

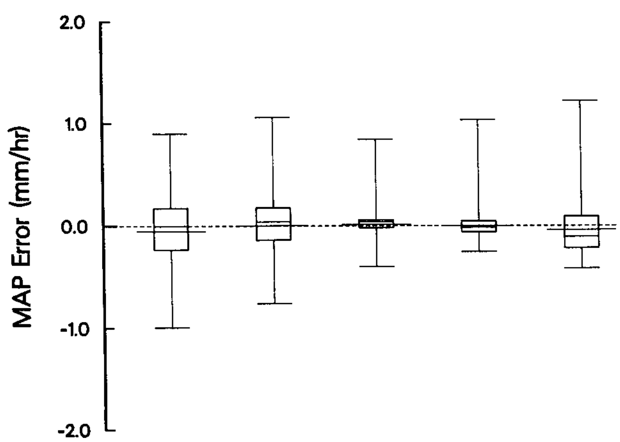

FIG. 3. Box plot of results calculated in scenario I for 2000 fields of the validation sample. The graphs correspond to, from left to right, 0, 1, 5, 10, and 20 mm h^{-1} thresholds.

TABLE 2. Compilation of cases investigated within scenario II.

Case	Description
1	Validation sample contains observations corrupted by 1-dBZ measurement noise. True regressions are used in the estimation process.
2	Validation sample contains observations corrupted by 2-dBZ measurement noise. True regressions are used in the estimation process.
3	Validation sample contains observations corrupted by 1-dBZ measurement noise. Regressions are obtained from error-contaminated (1-dBZ) observations in the calibration sample.
4	Validation sample contains observations corrupted by 2-dBZ measurement noise. Regressions are obtained from error-contaminated (2-dBZ) observations in the calibration sample.

$$\epsilon_{\text{MAP}} = \bar{R} - \hat{R}(\tau). \qquad (9)$$

The resultant values of ϵ_{MAP} are computed for all the fields (2000), and comprise a series of errors that are subsequently statistically analyzed.

a. Scenario I

The regressions presented in Table 1 are based on the calibration sample. When applied to the validation sample, the errors presented in Fig. 3 were obtained. In the box plots, the top and bottom horizontal lines correspond to the maximum and the minimum values obtained in the validation sample; the top and bottom limits of the box correspond to the 75th and 25th percentiles, respectively; the middle line is the median; and the horizontal line extending across the box is the mean.

The best results, as evidenced by the smallest spread of errors, correspond to the threshold somewhere between 5 and 10 mm h^{-1}. This agrees well with the results obtained in previous studies (see section 2). These results represent the performance limit of the area-threshold method. The only error is due to the regression error and the error about the regression. Far more interesting, from the practical point of view, are the results of the next two scenarios.

b. Scenario II

In this scenario, four cases are analyzed (Table 2). In addition to the errors calculated based on the threshold method, error statistics are calculated for the rainfall estimates obtained by simply averaging the simulated radar rainfall data. These results are referred to as the reflectivity-method estimates. In the first two cases, the true regressions (see scenario I) are applied to the validation sample observed with 1- and 2-dBZ random error, respectively. The second two cases represent a more realistic situation in which the regressions are calculated from the calibration sample observed with 1- and 2-dBZ random errors, and then applied to the validation sample with the corresponding errors. This way the initial regression is corrupted with errors, just as it would be in an actual application of the method. Figures 4 and 5 show the results for five thresholds, as well as for the reflectivity method (first box). The results for the reflectivity method are the same in both figures since they are unaffected by the regressions of the threshold method. It is evident that the reflectivity method is characterized by the smallest spread for both assumptions on the error magnitude. It is also evident that the systematic error (bias) introduced by the area-threshold method is negligible. Small differences between results corresponding to the true regression and the contaminated regressions indicate that the dominant error is due to sampling and regression errors of the area-threshold method.

The very good performance of the reflectivity method should be interpreted with caution. In the simulation, many errors faced in real-world applications of the method were neglected. These include range effects, partial beam filling, anomalous propagation, bright band, attenuation due to intervening rain, the ubiquitous problem of a detection threshold, etc.

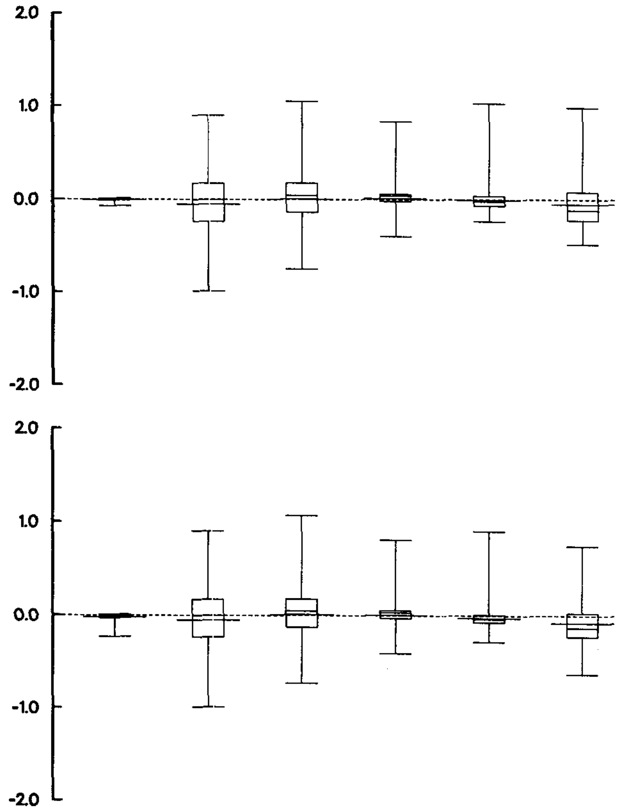

FIG. 4. Box plot of results calculated in scenario II for 2000 fields of the validation sample. The top and bottom plots have 1- and 2-dBZ errors, respectively. The boxes correspond to, from left to right, the reflectivity method and the 0, 1, 5, 10, and 20 mm h^{-1} thresholds. The 0 mm h^{-1} threshold corresponds to the simulated radar sensitivity threshold of 20 dBZ. The true regressions from the calibration sample were used.

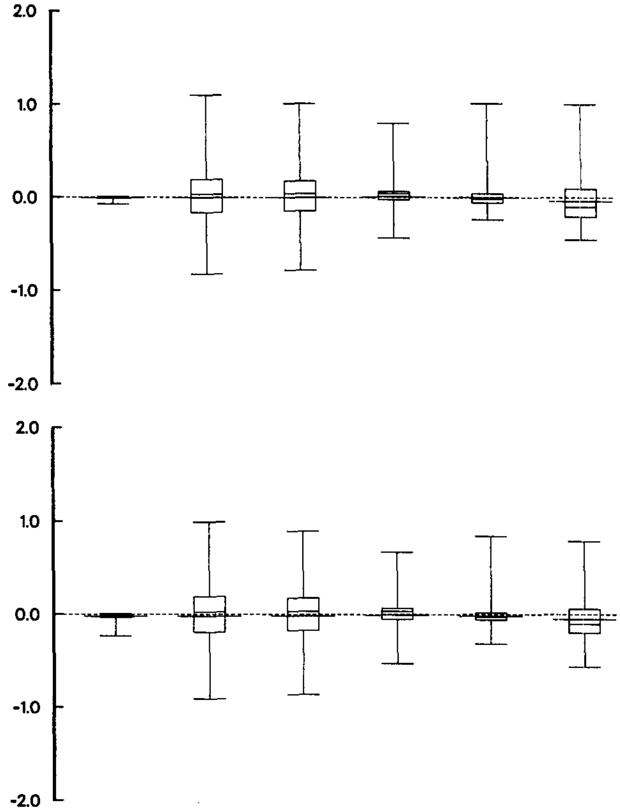

FIG. 5. Box plot of results calculated in scenario II for 2000 fields of the validation sample. The top and bottom plots have 1- and 2-dBZ errors, respectively. The boxes correspond to, from left to right, the reflectivity methods, 0, 1, 5, 10, and 20 mm h^{-1} thresholds. The 0 mm h^{-1} threshold corresponds to the simulated radar sensitivity threshold of 20 dBZ. The error-contaminated regressions from the calibration sample were used.

c. *Scenario III*

Scenario III combines the effects of random measurement error and systematic error (bias) due to the conversion of the radar reflectivity factor to rainfall rate. The performance of the area-threshold method is analyzed, given an erroneous Z–R relationship. Since both a and b coefficients of the Z–R relationship may be in error, the joint effect was studied. The statistical results are given for the two-dimensional space enclosing percent errors in parameter a from -50% to 50%, and in parameter b from -30% to 50%. The reason for restricting the lower bound of the error for parameter b to -30% (rather then -50% as for the parameter a) is that for the Marshall–Palmer parameters used in the study as the true parameters, underesti-

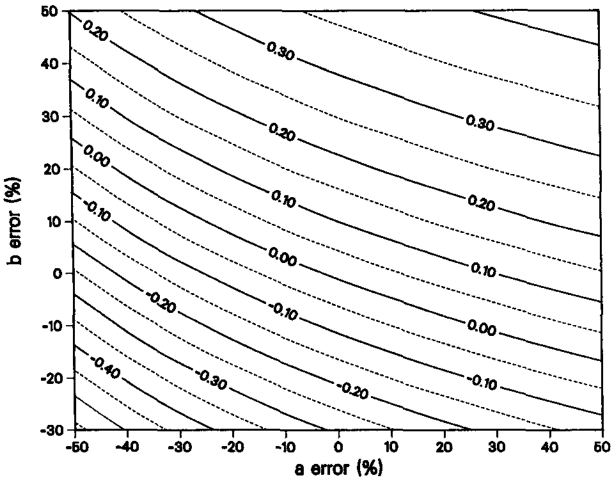

FIG. 6. Mean error (mm h^{-1}) corresponding to the threshold of 5 mm h^{-1}. True regressions were used.

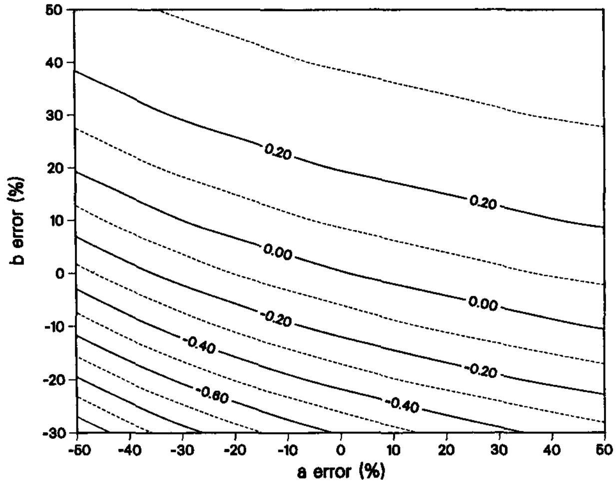

FIG. 8. Mean error (mm h^{-1}) corresponding to the threshold of 10 mm h^{-1}. True regressions were used.

mation errors of less than −30% would result in $b < 1.0$, which is an infeasible value to be acceptable [see Battan (1973) for a compilation of Z–R relationships and Smith and Krajewski (1992) for physical arguments].

Figure 6 presents the contour lines of the mean error in the areally averaged rainfall predicted using the area threshold method with the threshold of 5 mm h^{-1}. The mean error surface has a very regular behavior, with a slightly increasing gradient as the parameters are underestimated. Also, greater sensitivity with respect to parameter b can be observed. The error standard deviation (Fig. 7) displays a similar pattern. The errors (both in terms of the mean error and the error standard deviation) are increased for the threshold of 10 mm h^{-1} (Figs. 8 and 9, respectively). The patterns are the same as for a threshold of 5 mm h^{-1}, but the gradients are steeper.

The performance of the area-threshold method with both thresholds (5 and 10 mm h^{-1}) is compared with that of the reflectivity method. In the parameter error domain, where the Z–R parameters are overestimated, the reflectivity method performs better. In the corner of the domain, where both Z–R relationship parameters are severely underestimated, the area-threshold method with threshold of 5 mm h^{-1} is the best. This is demonstrated by comparison of Figs. 6–9 with Figs. 10 and 11.

Actually, the best strategy, as far as threshold selection is concerned under the conditions of large uncertainties in the Z–R parameters, is to use the area-threshold method with a low threshold. With a low

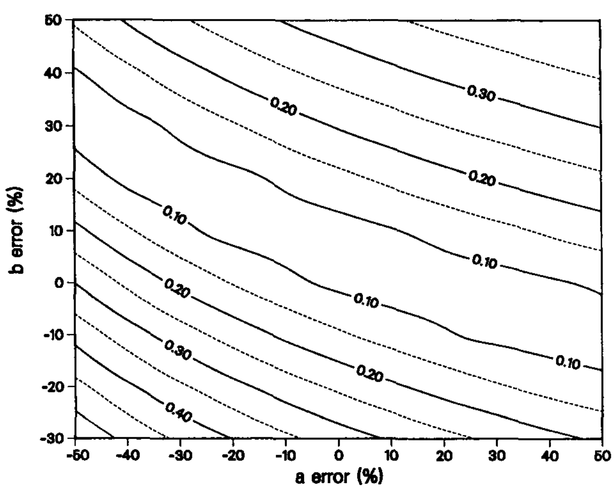

FIG. 7. Error standard deviation (mm h^{-1}) corresponding to the threshold of 5 mm h^{-1}. True regressions were used.

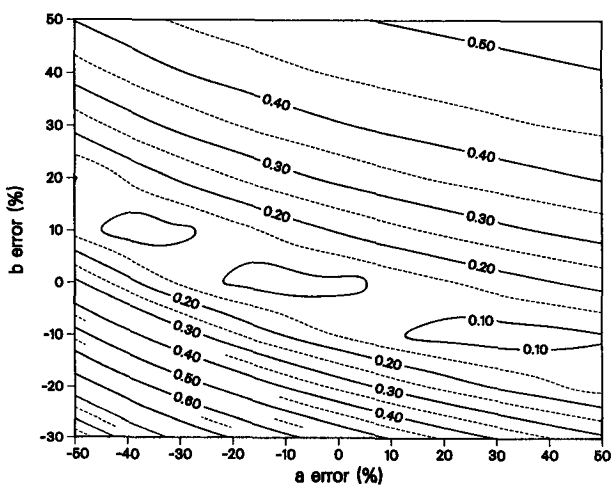

FIG. 9. Error standard deviation (mm h^{-1}) corresponding to the threshold of 10 mm h^{-1}. True regressions were used.

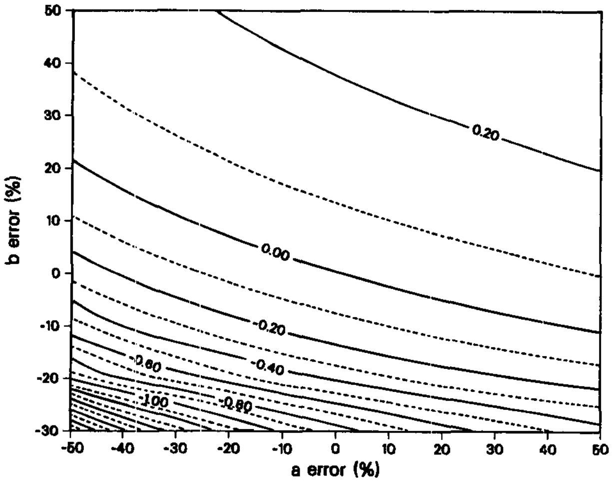

FIG. 10. Mean error (mm h^{-1}) corresponding to the reflectivity method. True regressions were used.

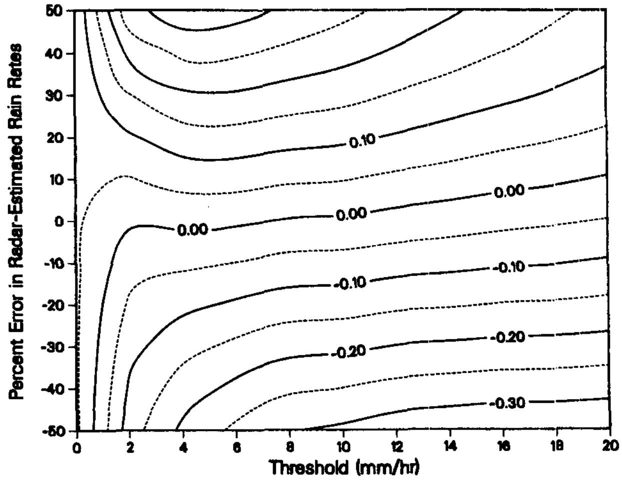

FIG. 12. Mean error (mm h^{-1}) corresponding to the area-threshold method. True regressions were used.

threshold, the radar is used to indicate rain–no-rain areas, and therefore, the method is not affected much by the reflectivity to rainfall conversion. Indeed, radar is considered to be a very good rainfall "detector" given that ground clutter and anomalous propagation errors are properly resolved. For the lowest threshold corresponding to the radar sensitivity level (or 0 mm h^{-1} threshold in scenario I), however, there is large scatter about the regression relationship between the fractional area and the mean areal rainfall. The results of the simulations performed in scenario III indicate that the threshold of 1 mm h^{-1} is a good compromise. It performed well for large portions of the Z–R parameter error domain. Contour plots of its performance are not shown since both surfaces of the mean error and the error standard deviation are very flat, and the values are comparable to the values of the lowest threshold.

In this study, the mean error corresponding to the lowest threshold was −0.057 mm h^{-1}, and the error standard deviation was 0.2 mm h^{-1}. The error standard deviation represents worse performance than the reflectivity method if the errors of the Z–R parameters are within ±10%. Actually, parameter overestimation is "safer" for the area-threshold method than parameter underestimation. These findings shed new light on the notion of optimal threshold, which is previously defined as the threshold that maximizes the correlation between the area sample mean rainfall and the sample fractional area above the threshold (see Kedem et al. 1990; Short et al. 1993; Kedem and Pavlopoulos 1991).

Another way of looking at the results is presented in Figs. 12 and 13. The contours of mean error and the error standard deviation are plotted as a function of threshold (from 0 to 20 mm h^{-1}), and the percent

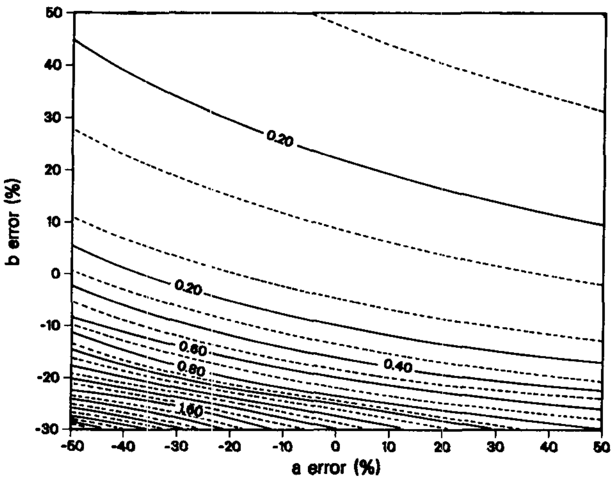

FIG. 11. Error standard deviation (mm h^{-1}) corresponding to the reflectivity method. True regressions were used.

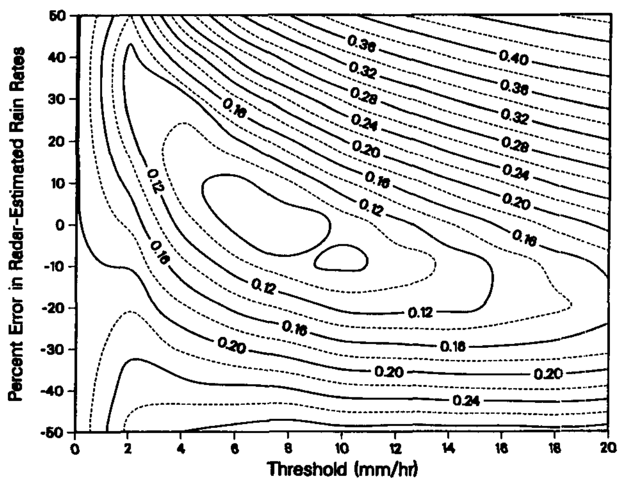

FIG. 13. Error standard deviation (mm h^{-1}) corresponding to the area-threshold method. True regressions were used.

error in estimation of point rainfall (whatever the cause) ranging from −50% to 50%. The standard deviation shows the domain of minimum error to extend over thresholds within a 4–12 mm h^{-1} range, but the unbiased performance is confined to a narrow strip near the zero-error line or the lowest threshold vicinity.

This discussion and these results assumed the use of true regression, that is, regression obtained from the calibration sample of the true rainfall fields generated by Bell's model. The regressions, however, also suffer from the systematic error due to *Z–R* conversion or other similar causes. Figures 14 and 15 show the values of the regression intercepts and slopes for the 5 mm h^{-1} threshold as estimated from rainfall data based on contaminated *Z–R* relationships. For comparison, the parameter values of the true regressions are given in Table 1. It is evident that these regressions are significantly in error, even for the error range of the *Z–R* parameters being within a reasonable ±20%. This fact combined with the previous discussion leads once again to the conclusion that the safest decision is to use the area-threshold method with a low threshold.

If the uncertainty associated with a particular *Z–R* relationship is high, as is likely the case for climatologically estimated *Z–R*'s (see Krajewski and Smith 1991), the selection of a low threshold would mitigate these errors since estimation of the rainy area is less dependent on *Z–R* conversion. By the same token, a spaceborne radar, such as the TRMM radar, even when subjected to a strong attenuation effect will "see" well the projection of the rainy area, since subcloud evaporation is unlikely to affect intensive rain. On the other hand, the measurement of areal extent of rainy area is affected by the sensitivity threshold of the radar system—the minimum detection threshold. Low-intensity rainfall may be cut off. Although the contribution of such rainfall to rainfall climatology is insignificant (see Atlas et al. 1990b), the effect may be minimized if care is taken in the area-threshold-method calibration by specifying an appropriate low-end threshold.

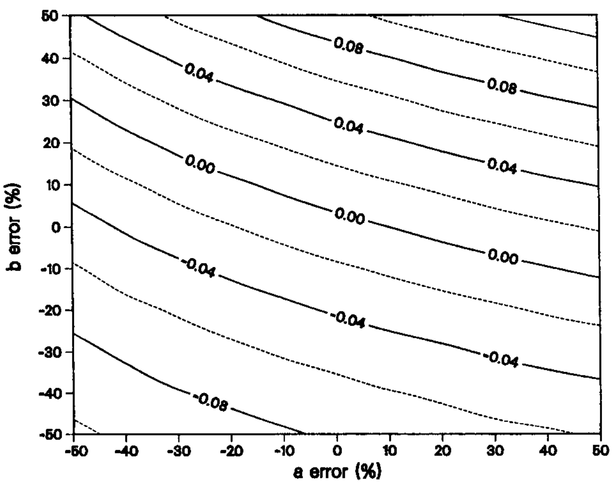

FIG. 14. Estimated regression intercepts from the calibration sample. Threshold is 5 mm h^{-1}.

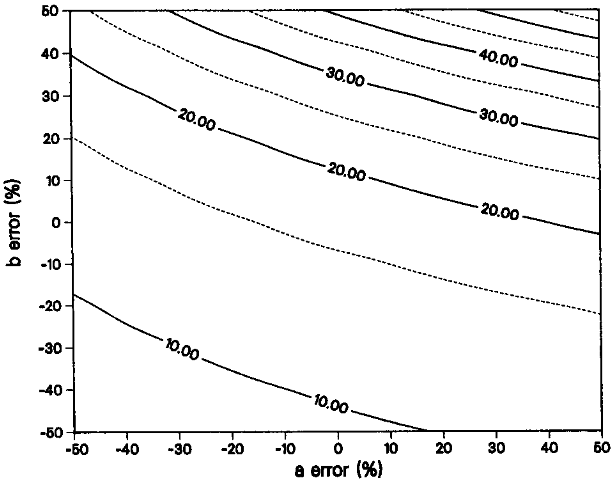

FIG. 15. Estimated regression slopes from the calibration sample. Threshold is 5 mm h^{-1}.

5. Conclusions and recommendations

Several conclusions are evident from this study. The performance of the area-threshold method depends critically on quality control and preprocessing of the radar data. An especially important problem appears to be anomalous propagation. Substantial areas of undetected anomalous-propagation echoes could severely impair the accuracy of the method. It should be emphasized that the effects simulated in scenario III represent more then just the *Z–R* errors. Similar performance of the area-threshold method would be evident if other systematic errors were considered. These include electronic miscalibration, strong signal attenuation by rainfall, and range effects under certain circumstances. Final judgement on the performance of the area-threshold method cannot be made without quantitative specification of the performance of other components of the radar rainfall-estimation system. These include signal processing, data quality control, and estimation of the *Z–R* relationship.

The simulation study presented in this paper is intended to be the first in a series of quantitative analyses of the areal rainfall-estimation methods. Future research will examine a number of issues including the use of multiple *Z–R* relationships (Atlas et al. 1990b). This study is based on a single *Z–R*. The results obtained are valid for the low limit of the applicability of the area-threshold method specified by Rosenfeld et al. (1990) as 10 000 km^2.

The results of this study demonstrate that when the exponent parameter in the *Z–R* relationship has small uncertainty (about ±10%), the conventional method works better than the area-threshold method. When the errors are higher (±20%), the area-threshold method with optimum threshold in the 5–10 mm h^{-1} range performs best. This is in agreement with the analytical findings of Kedem et al. (1990). For even higher errors in the Z–R relationship, the area-threshold method with a low threshold provided the best results. This behavior is understandable if one notices that low thresholds have lower sensitivity to error in determination of the fractional area above the threshold. At the same time, the higher scatter associated with the lower threshold is averaged out if long-term performance statistics are calculated. Therefore, based on the results of this study, it seems that under the conditions of large uncertainties in reflectivity-to-rainfall conversion, a low threshold gives a more robust performance of the area-threshold method.

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