Estimating Rainfall in the Tropics Using the Fractional Time Raining

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ABSTRACT

The relationship between the fractional time raining and tropical rainfall amount is investigated using raingage data and a point process model of tropical rainfall. Both the strength and the nature of the relationship are dependent upon the resolution of the data used to estimate the fractional time raining. It is found that highly accurate estimates of rainfall amounts over periods of one month or greater can be obtained from the fractional time raining so long as high-time-resolution data are used. It is demonstrated that the relationship between the fractional time raining and monthly atoll rainfall is quasi-homogeneous within the monsoon trough region of the equatorial western Pacific.

1. Introduction

Rainfall is important in that the global atmospheric circulation is controlled by the latent heat released through water vapor condensation (Simpson et al. 1988). The initialization and evaluation of climate models requires knowledge of the global distribution of rainfall with a resolution of approximately one month and spatially averaged over areas of 75 000 km^2 (WMO 1985). Since 70% of the earth is covered by ocean, land-based rainfall-measuring networks are inadequate for global rainfall pattern evaluation. Satellite-based techniques are the only realistic means of monitoring global rainfall on these scales, but first they must be validated using surface observations (Arkin and Ardanuy 1989).

The availability of surface rainfall observations over the open ocean is limited. Coastal radars, which cover the zone influenced by ocean–land interactions, provide observations that do not represent open-ocean conditions. In addition, radars suffer from a host of other problems and their estimates of rainfall are associated with considerable uncertainty. Ocean raingage networks are limited primarily to atolls and islands (Morrissey and Greene 1991). Raingage measurements taken on islands with significant orography are also unlikely to represent open-ocean conditions. Another source of rainfall observations are ship reports. Unfortunately, ships of opportunity are rarely equipped with raingages. The data provided by the few ships that have carried raingages may be influenced by the effect of the ship's superstructure on the rain catchment (Reed and Elliott 1977).

This paper critically examines an alternative approach to obtaining tropical rainfall amounts. Rainfall amounts can be estimated using the fraction of time that rainfall occurs (i.e., FTR). The relationship between the FTR and rainfall amount in the tropics has been studied extensively (refer to Atkinson 1971). Martin (1964) found a consistent relationship between the cumulative percent amount of rainfall and the cumulative percent frequency of days with rain at 20 stations in central America. Harrison (1983), using daily rainfall accumulations from raingages, found a significant positive correlation between the number of days with rainfall and seasonal rainfall totals within the Orange Free State in South Africa. Reed (1979, 1980) explored the possibility of using rainfall occurrence from ships to derive monthly rainfall in the Pacific. However, as will be shown in this paper, these studies have used either biased estimators of the FTR or es-

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timators that contain significant amounts of random error. Thus, reliable assessments of the usefulness of the FTR as an estimator of rainfall amount have not been obtained.

It will be shown that FTR can provide precise estimates of rainfall amount over periods of one month or greater in the equatorial western Pacific. Moreover, the accuracy of the FTR estimator will be shown to depend largely on the time resolution of the data used in its determination. We will first illustrate the methodology with a set of atoll and island raingage data from the western tropical Pacific, then test the implications of the method with a second independent set of rainfall data from a recent small-scale array of moored optical raingages.

2. Data

Eight years (1984–91) of hourly raingage totals were obtained for two small island sites (Koror: 7°20′N, 134°29′E; Pohnpei: 6°58′N, 158°13′E) and two atoll sites (Majuro: 7°6′N, 177°24′E; Truk: 7°28′N, 151°51′E; refer to Fig. 1). The data were taken using standard 8-in. tipping-bucket raingages. The sites, which are influenced by the northern Pacific monsoon trough during the summer and the intertropical convergence zone (ITCZ) during winter (Sadler et al. 1987), lie along an east–west transect stretching approximately 4000 km. Applying cluster analysis to these data, Morrissey and Greene (1993) showed that these sites have similar rainfall statistics and could be classified into the same rainfall climate regime. The data cover both El Niño and non–El Niño periods.

A second set of rainfall data was obtained from an array of recently deployed optical raingages mounted on six TOGA (Tropical Ocean Global Atmosphere) moorings along the equator in the western Pacific (McPhaden and Milburn 1992; refer to Fig. 1). Nominal positions of the buoys are 2°N, 0° and 2°S, 156°E; and 154°E, 157°30′E and 165°E on the equator. The data are restricted to the period 1992–93, and for any given site are at most 1-yr long. A performance evaluation of optical raingages in laboratory tests indicates an instrument accuracy of ±20% compared to precision weighing gauges in rain rates up to 100 mm h^{-1} (McPhaden 1993). Besides accumulated rainfall, these instruments measure the percentage of an hour that rain has fallen. The raingages sample every 5 s, although the effective resolution of the time series is about 15 s, which is the response time constant for the instruments.

3. The FTR estimator

The theoretical relationship between the FTR and rainfall amount is described as follows. Rainfall accumulated over time T at a point is equal to

$$R = \mathrm{FTR}\langle[r|r > 0]\rangle T, \tag{1}$$

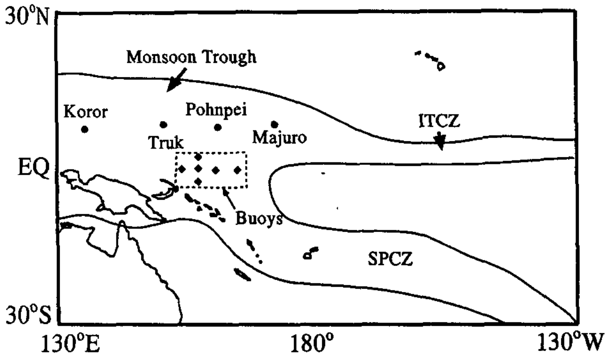

FIG. 1. A map showing the locations of the two island and two atoll sites used in this study. Also shown is the area encompassing the moored buoys used in this study. The climatological locations of the intertropical convergence zone (ITCZ), the northwest Pacific monsoon trough, and the Southern Pacific convergence zone (SPCZ) are also shown.

where $\langle[r|r > 0]\rangle$ is the average over T of the instantaneous rain rate r, conditional on rain. The FTR is equal to the average over T of an indicator variable,

$$\mathrm{FTR} = \langle I[r > 0]\rangle, \tag{2}$$

which is defined by

$$I[r > 0] = \begin{cases} 1, & \text{if } \; r > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The accuracy of the FTR estimator depends to a large extent on the resolution of the rainfall data used to estimate it. Generally, hourly or daily rainfall accumulations have been used in the estimation of the FTR (e.g., Martin 1964; Harrison 1983; Ananthakrishnan and Soman 1989). Since the rain rate r is defined at an instant in time, so is $I[r > 0]$. If accumulated rainfall measurements (e.g., daily amounts) are used to estimate the FTR, then $I[r_s > 0]$ is used instead of $I[r > 0]$, where r_s is the rain rate averaged over the accumulation time represented by a single measurement (i.e., t_a) and is defined by

$$r_s = \frac{1}{t_a}\int^{t_a} r dt. \tag{4}$$

Using r_s, the FTR within accumulation period T (e.g., one month) is equal to

$$\mathrm{FTR} = \frac{1}{N}\sum^{N} I[r_s > 0]P(r > 0|r_s > 0), \tag{5}$$

where $P(r > 0|r_s > 0)$ is the average fraction of the accumulation time that is raining associated with individual measurements that recorded rainfall (i.e., $r_s > 0$). The total number of measurements within T is N. Past studies have estimated FTR using

$$\mathrm{FTR} = \frac{1}{N}\sum^{N} I[r_s > 0]. \tag{6}$$

By comparing (5) and (6), it is obvious that the FTR estimated using (6) is biased by the factor $P(r > 0 | r_s > 0)$. This factor is a function of the mean duration and occurrence interval of rainfall events, as well as the resolution of the rainfall measurements. The value of $P(r > 0 | r_s > 0)$ approaches 1 as t_a approaches 0 (i.e., $r_s = r$). If the average percentage of the accumulation time associated with $r_s > 0$ is small, the bias in the FTR estimates will be substantial. The bias in the FTR estimates is evidenced through a comparison of monthly rainfall with the FTR per month estimated using hourly and daily rainfall observations from Majuro (Fig. 2). The FTR estimates using daily values (Fig. 2, top) are associated with much lower monthly rainfall amounts as compared with hourly values (Fig. 2, bottom).

The random error associated with the FTR estimates also increases with decreasing data resolution. This is revealed by the larger amount of scatter in the relationship when the FTR is estimated using daily rainfall values (Fig. 2, top).

The functional nature of the relationship is also dependent upon data resolution. Using a least-squares method, both linear and nonlinear equations were fit to the points shown in Fig. 2 (top and bottom). Although the precise functional relationship is unknown, the selected power equation provided an adequate description of the relationship at each site. Many different nonlinear equations were tested and those selected had the highest value for the F statistic and the coefficient of determination (i.e., R^2). Since it is physically reasonable to assume zero rainfall corresponding to zero FTR, only equations without y intercepts were tested.

The nonlinearity of the relationship observed for both hourly and daily data suggests that the averaged conditional rain rate (i.e., $\langle [r | r > 0] \rangle$) may be a function of the monthly rainfall amount, implying that higher rain rates occur during months with large accumulations. Harrison's (1983) results lead him to this conclusion. Further inspection suggests that the nonlinearity may also result from an increase in the probability of the occurrence of multiple rain events or increased rain durations within the accumulation time associated with an individual measurement (e.g., per hour or per day) during months with heavy rainfall. In other words, during months with heavy rainfall, a larger percentage of the accumulation time given $r_s > 0$ is associated with rainfall. This leads to a decrease in the bias in the FTR estimates during these months. Thus, the observed nonlinearity may not result from increased conditional rain rates but may be an artifact resulting from the use of rainfall accumulation measurements to estimate the FTR. That the exponent of the fitted nonlinear equation decreases with increasing data resolution supports this hypothesis.

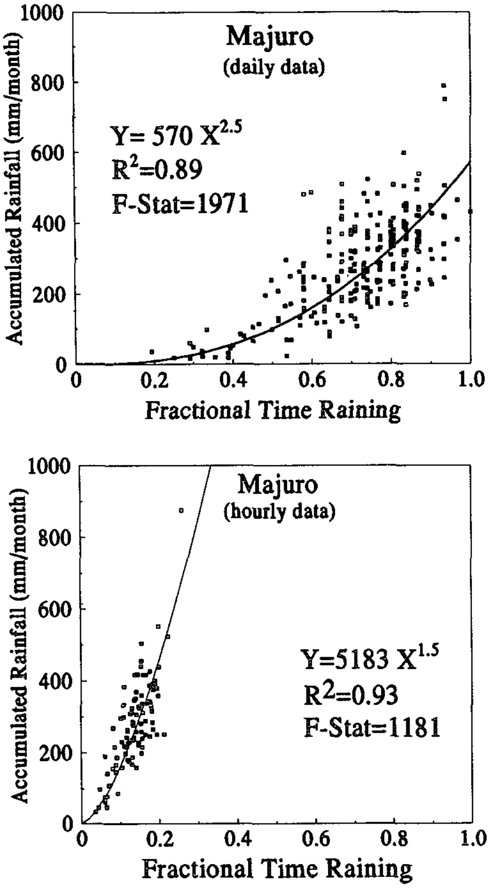

FIG. 2. A comparison of monthly rainfall accumulations with the fractional time raining as estimated using daily data (top) and hourly data (bottom).

The above results suggest that very high time resolution rainfall data are required to assess the usefulness of the FTR for estimating rainfall amount. Since sufficient measurements at this scale are not available in the tropical Pacific, a modeling approach was used instead. A Poisson point process model of tropical rainfall (Morrissey and Krajewski 1993) was fit to the hourly rainfall accumulations taken at Koror. A feature of these models are their ability to simulate rainfall measurements representing a specified temporal resolution (Rodriguez-Iturbe et al. 1984, 1987; Entekhabi et al. 1989).

Rainfall statistics computed using Koror hourly data were used to develop the model parameters. A simulated time series consisting of 4 million 1-min rainfall accumulations was then generated (i.e., 100 months). Comparisons (not shown) of model and observed statistics (i.e., mean, variance, autocorrelation, etc.) indicated that the model provided an adequate fit to the data. The FTR estimates were then computed from the simulated time series of hourly accumulations and compared to integrated simulated monthly rainfall totals (i.e., 735 h). By comparing Fig. 3 (top) with Fig. 2 (bottom), it can be observed that the model reproduces the expected behavior of the relationship.

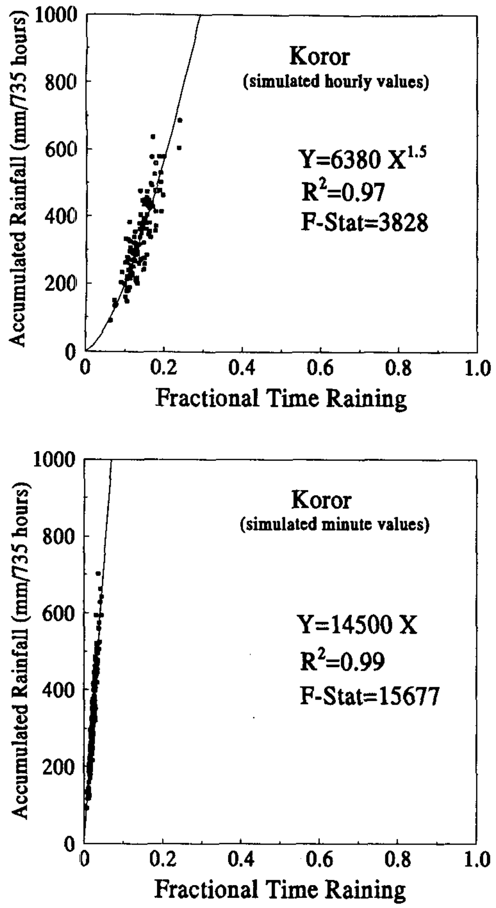

FIG. 3. A comparison of monthly rainfall accumulations with the fractional time raining as estimated using simulated hourly rainfall accumulations (top) and simulated 1-min accumulations (bottom). The point process model was fit to Koror hourly accumulations.

The potential usefulness of the FTR estimator using high-time-resolution data is observed through a comparison of FTR estimates per month using the simulated 1-min values with simulated monthly rainfall. The results (Fig. 3 bottom) reveal that the relationship becomes essentially linear and the scatter significantly decreases. This again reinforces the hypothesis that the nonlinearity observed using the hourly and daily data is not due to an increase in the conditional rain rate during months with large accumulations but is instead an artifact of the data resolution. A correlation coefficient of 0.99 and a standard error about the regression line of 20 mm month^{-1} indicates that monthly rainfall can be accurately obtained from the FTR if high-resolution data are used in its estimation.

4. Temporal and spatial stability of the conditional time-averaged rain rate

Another factor to consider when using the FTR to estimate rainfall amount is the natural variation in time and space of the conditional time-averaged rain rate, $\langle [r|r>0] \rangle$. Equation (1) indicates that the relationship between the FTR and rainfall amount is dependent upon the stability in time of $\langle [r|r>0] \rangle$. If $\langle [r|r>0] \rangle$ is relatively constant (i.e., stationary), then rainfall amount may be accurately estimated using the FTR. Since $\langle [r|r>0] \rangle$ is a time-averaged quantity, the central limit theorem suggests that its stability in time is a function of the length of the averaging period [i.e., T from (1)]. The optimal value of T was investigated for the equatorial western Pacific region by comparing the standard deviation of $\langle [r_s|r_s>0] \rangle$, computed using hourly data from Koror, with the length of the averaging period. The results (Fig. 4) show an exponential decrease of the standard deviation with increasing averaging period. The averaging period corresponding to the e-folding value is approximately 580 h. From concepts developed in the previous section, hourly data will produce biased estimates of the conditional rain rate, $[r|r>0]$. However, $\langle [r_s|r_s>0] \rangle$ may or may not be a biased estimate of $\langle [r|r>0] \rangle$ depending on the standard deviation of $[r|r>0]$, the coherency of the rainfall time series, and the FTR, all of which are unknown. Therefore, it may be assumed to a first approximation that $\langle [r|r>0] \rangle$ becomes relatively stable for averaging periods one month or greater.

If $\langle [r|r>0] \rangle$ can be shown to be stable in space (i.e., homogeneous), a climatological value of $\langle [r|r>0] \rangle$ may be used to relate the FTR to rainfall amount. This is investigated for the equatorial western Pacific by comparing the coefficient of the regression equation relating the FTR per month to monthly rainfall for each of the four island sites. From Fig. 5, it can be observed that the regression coefficient is very similar in magnitude among all four sites. Although there may be some orographic influence on the rain rates at

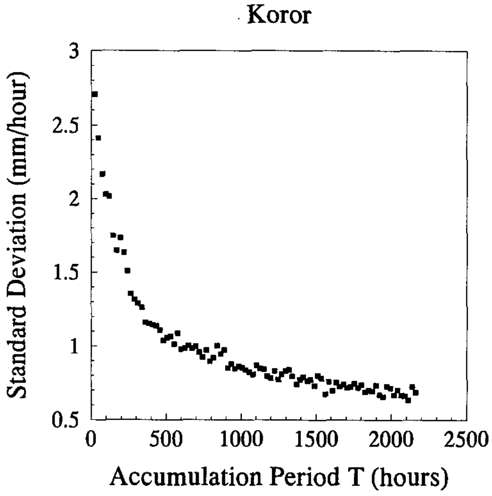

FIG. 4. The standard deviation of the conditional rain rate, based on hourly data, averaged over various aggregation periods.

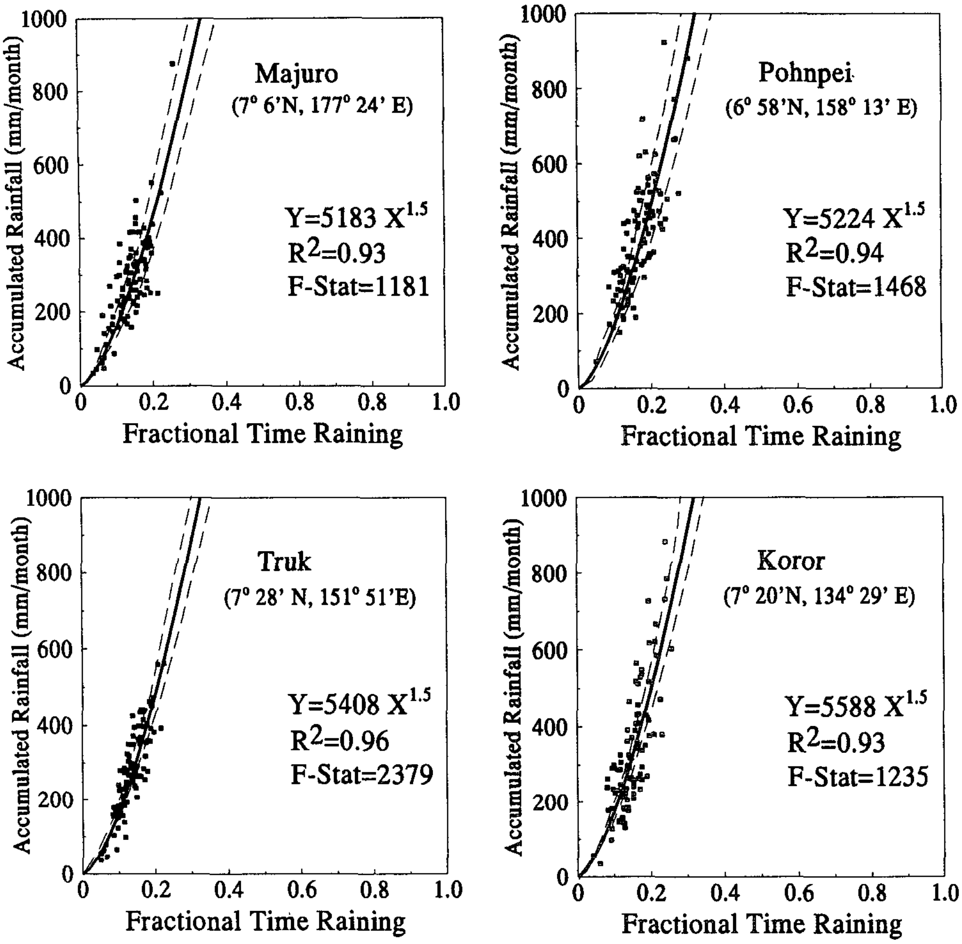

FIG. 5. A comparison of monthly rainfall accumulations with the fractional time raining as estimated using hourly raingage data from four island and atoll sites in the equatorial western Pacific. The 95% confidence limits are shown by the dashed lines.

Pohnpei and Koror, which are somewhat higher than on the atolls, a statistically significant difference (at the 95% level) was not found among the coefficients associated with the different sites. Thus, it appears that $\langle [r_s | r_s > 0] \rangle$ and, again to a first approximation, $\langle [r | r > 0] \rangle$, is quasi-homogeneous in space, as well as time, within the monsoon trough region of the equatorial western Pacific.

5. Discussion

The use of rain frequency as an estimator of rainfall amount requires an instrument from which an on–off indication of rainfall is the only variable required. It is likely that FTR measurements from such instruments would be relatively unbiased compared to standard raingage measurements, since significant biases have been associated with standard rain catchment systems (Snow and Harley 1988). Currently, the array of moored optical raingages deployed in the western equatorial Pacific measures rainfall with an effective temporal resolution of 15 s. At this resolution, accurate estimates of the FTR per month may be obtained.

A comparison of the FTR estimated from the moored optical raingage data with monthly rainfall at three different resolutions (Fig. 6) supports the conclusions found earlier in this paper using the point process model. That is, as the data resolution increases to 1-min accumulations or better, the accuracy of the FTR estimate improves to a point where precise estimates of monthly rainfall can be obtained. The results also support the conjecture that the nonlinearity in the relationship observed with daily and hourly data is an artifact of the data resolution rather than a functional relationship between the conditional rain rate and monthly rainfall.

A comparison of the a coefficient of the power equation (i.e., $Y = aX^{1.5}$) fitted to hourly data (Table 1) reveals a significant difference (by a factor of 2) in this coefficient between the ocean-sited optical raingages and the land-based tipping-bucket raingages. The source of this difference could result from many factors including instrument biases, orographic effect, or natural differences in the climatological rain rate between the monsoon trough region and the equator. This is an important issue and deserves further study. Regardless of these differences, however, the data from the moored optical raingages clearly demonstrate the accuracy with which the FTR can be used to estimate monthly rainfall totals and the dependency of the accuracy on the temporal resolution of the data.

6. Conclusions

It was demonstrated that the FTR can be used as an accurate estimator of rainfall accumulated over periods of one month or greater within the equatorial western Pacific region provided that high-time-resolution data are used to estimate the FTR. The FTR method was illustrated first using tipping-bucket raingages from the islands and atolls, and the relationship was shown to be quasi-homogeneous within the monsoon trough region of the north equatorial Pacific. As only four sites were tested, definitive conclusions about the homogeneity of the relationship must await further comparisons using data from other open-ocean climate regimes. The dependence of FTR estimator accuracy on temporal resolution of the data was demonstrated by an analysis of moored optical raingage data with an effective resolution of 15 s.

The resolution of the data used to estimate the FTR affects both strength and the nature of its relationship with rainfall amount. An analogous relationship in the spatial domain is the area threshold method (ATI; Atlas et al. 1990; Kedem et al. 1990), which relates the fraction of area containing rain to the rainfall averaged over the area. Since the areal fraction of rainfall is generally estimated using radar (Atlas and Wolff 1990; Atlas and Bell 1992) or satellite data (Arkin and Miesner 1987; Arkin and Ardanuy 1989), the results of this paper suggest that the spatial resolution (i.e., pixel size) also affects the ATI method as well. For example, the application of an ATI calibration coefficient to radar data of differing spatial resolution will produce biased results.

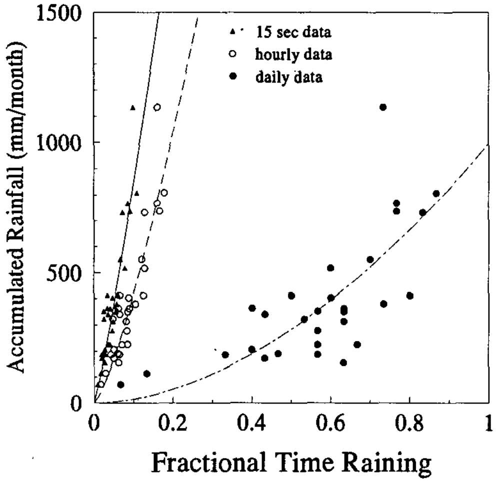

FIG. 6. A comparison of monthly rainfall accumulations with the fractional time raining estimated using hourly and daily data, and a direct computation with a resolution of 15 s. The data were taken using optical raingages mounted on six moored buoys located along the equator in the western Pacific during 1992–93.

TABLE 1. The regression coefficients, the coefficient of determination R^2, and the F statistics for the fitted equation $Y = aX^{1.5}$ for hourly data from the land-based and the buoy sites, where N is the number of months of data used in determining the fit. The lower F statistics in the case of the buoy dataset results from the smaller N compared to the land-based datasets.

Site	a (mm month^{-1})	R^2	F statistic	N
Majuro	5183	0.93	1181	94
Pohnpei	5224	0.94	1468	93
Truk	5408	0.96	2379	94
Koror	5588	0.93	1235	94
Buoys	12 656	0.94	406	29

The current spatial density of point rainfall measurements indicates that the FTR estimator cannot be used to obtain ocean basinwide estimates of rainfall such as those required by general circulation models. With the deployment of additional optical buoy raingages and higher temporal resolution raingages, however, adequate samples of ocean rainfall may soon be available to validate satellite rainfall algorithms within the various open-ocean climate regimes.

We hope that issues raised in this paper will serve as an aid in designing and planning activities of global rainfall estimation by national and international organizations such as the National Oceanic and Atmospheric Administration and the National Aeronautics and Space Administration in the United States, the World Meteorological Organization in Switzerland, and the Global Precipitation Climatology Center in Germany.

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