A Climatological Model for 1-min Precipitation Rates

PAUL TATTELMAN

Phillips Laboratory/Geophysics Directorate, Hanscom AFB, Massachusetts

KEVIN P. LARSON AND ANDREW J. MAZZELLA JR.

RDP Incorporated, Waltham, Massachusetts

(Manuscript received 11 March 1994, in final form 14 November 1994)

ABSTRACT

A model for estimating mean monthly total time occurrence for 1-min precipitation rates from monthly climatological variables has been developed. The model has two components: an estimation algorithm for the mean monthly percentage of time in which precipitation occurs and a set of algorithms to derive the mean cumulative distribution function of precipitation rates for a calendar month. Both components were developed using stepwise linear regression analysis applied to a database containing 10 years of 1-min precipitation data from 34 sites throughout the 48 contiguous states of the United States. The required climatological variables are the mean monthly temperature, the mean monthly temperature range, the mean monthly precipitation, and the mean number of days per month with precipitation (based on three commonly used threshold values to define a day with precipitation).

1. Introduction

Precipitation is an important consideration in the design of systems that must operate in natural environments. For satellite communication systems employing EHF (extremely high frequency), rain is one of the major environmental causes of outages (Crane 1980; Ajayi and Ofoche 1983). Short-duration precipitation rates, especially 1-min rates, are recognized as the most practical for signal attenuation calculations. Tattelman and Grantham (1985) pointed out the lack of data for modeling 1-min rates. This prompted an effort to build a database by extracting 1-min rates from weighing rain gauge recordings, as described by Tattelman and Knight (1988), and the subsequent development of the climatological model described in this article.

An earlier model relating precipitation occurrence to precipitation rates (Tattelman and Scharr 1983) employed separate formulas for each of six discrete occurrence percentages and defined some explicit constraints on applicability of the formulas. The model developments described here were performed to remove the restrictions to specific occurrence percentages while expanding the domain of applicability for the model. The structure of this model substantially complies with those requirements, but the restricted geographic domain used for this model development may impose an implicit restriction of the model to midlatitude regions.

This precipitation rate occurrence model was formulated as two components that together allow the determination of the monthly percentage of time of occurrence of precipitation exceeding a specified rate (as equivalent rainfall) or the inverse problem of determining a precipitation rate that is exceeded for a specified percentage of time. One component is a scalar quantity, which represents the mean percentage of time with precipitation (PTP) for a month. This quantity is referenced to a threshold precipitation rate of 0.001 mm min^{-1}, designated as "trace" precipitation. The other component is a function, the cumulative distribution function (CDF) for precipitation, representing the relative time occurrence of precipitation at or below a designated precipitation rate. Consequently, the CDF varies monotonically from zero to unity as a function of precipitation rate. Together these two components define the *absolute* percentage time occurrence for precipitation exceeding a specified threshold rate r according to the following equation:

$$\mathrm{PTE} = \mathrm{PTP}[1 - \mathrm{CDF}(r)]. \quad (1)$$

Actual precipitation percentages and cumulative distributions were calculated on a calendar-month basis for 34 sites (the "regression stations") within the United States, using 1-min precipitation rate data for the 10-

Corresponding author address: Paul Tattelman, OL-AA-PL/GPAA, Phillips Laboratory (AFMC), 29 Randolph Road, Hanscom AFB, MA 01731-3010.

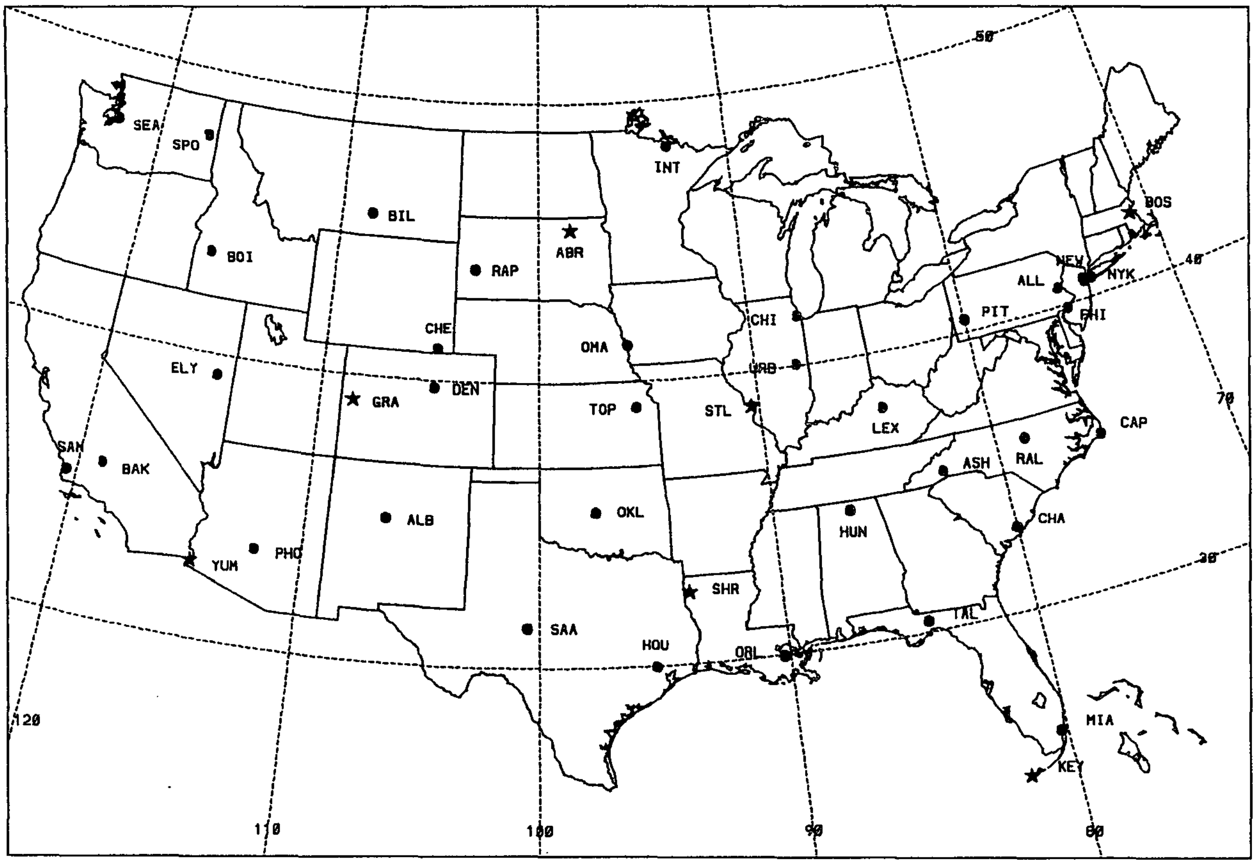

FIG. 1. Precipitation data sites, with the regression stations identified as circles and the reserved stations identified as stars.

yr period 1970–79, as described by Tattelman and Knight (1988). Thus, the total number of months used for the model development is 408 (i.e., 12 calendar months at each site times 34 sites). The precipitation percentages and cumulative distributions were used for the development of the formulas for the PTP and CDF, employing stepwise regression analysis to determine the coefficients and functional forms for their dependence on climatological variables.

The 1-min precipitation rate data for the same 10-yr period from an additional seven sites (the "reserved stations") were used to calculate precipitation percentages and cumulative distributions that were compared to model predictions for these sites as a separate validation analysis of the model. Tabulation of the monthly precipitation percentages and cumulative distributions for the "reserved station" sites was deferred until the climatological model development was completed in order to minimize the influence of these sites on the model development and thus achieve a truly unbiased evaluation of the model. The site locations are presented in Fig. 1, and the sites are identified in Table 1.

2. Model development

a. *Percentage of time with precipitation (PTP) model*

The climatological variables chosen as potential predictors were selected based on their general availability, not only within the United States but also globally. These variables are the mean monthly temperature T (°C), the mean monthly temperature range δT (°C), the mean monthly precipitation P (mm), and the mean number of days per month D with a threshold precipitation of 0.254 mm (0.01 in.). Sets of predictors were also used for alternative threshold precipitation levels. The alternative levels, 1.0 and 2.54 mm, are used in some countries to define a rainy day. The values for these variables were obtained from the Local Climatological Data tables published by the Environmental Data Service of the United States Department of Commerce and the precipitation rate database. An auxiliary variable that can be derived from these is the monthly precipitation index I (mm day^{-1}), which is the ratio of the mean monthly precipitation to the mean number of days per month with precipitation (P/D). The precipitation index is also associated with the threshold

TABLE 1. Precipitation data sites, identifying the locations used for model development and model validation.

Regression Stations	
ALB: Albuquerque, NM	NEW: Newark, NJ
ALL: Allentown, PA	NYK: JFK Airport, NY
ASH: Asheville, NC	OKL: Oklahoma City, OK
BAK: Bakersfield, CA	OMA: Omaha, NE
BIL: Billings, MT	ORL: New Orleans, LA
BOI: Boise, ID	PHI: Philadelphia, PA
CAP: Cape Hatteras, NC	PHO: Phoenix, AZ
CHA: Charleston, SC	PIT: Pittsburgh, PA
CHE: Cheyenne, WY	RAL: Raleigh, NC
CHI: Chicago, IL	RAP: Rapid City, SD
DEN: Denver, CO	SAA: San Angelo, TX
ELY: Ely, NV	SAM: Santa Maria, CA
HOU: Houston, TX	SEA: Seattle, WA
HUN: Huntsville, AL	SPO: Spokane, WA
INT: International Falls, MN	TAL: Tallahassee, FL
LEX: Lexington, KY	TOP: Topeka, KS
MIA: Miami, FL	URB: Urbana, IL
Reserved Stations	
ABR: Aberdeen, SD	STL: St. Louis, MO
BOS: Boston, MA	SHR: Shreveport, LA
GRA: Grand Junction, CO	YUM: Yuma, AZ
KEY: Key West, FL	

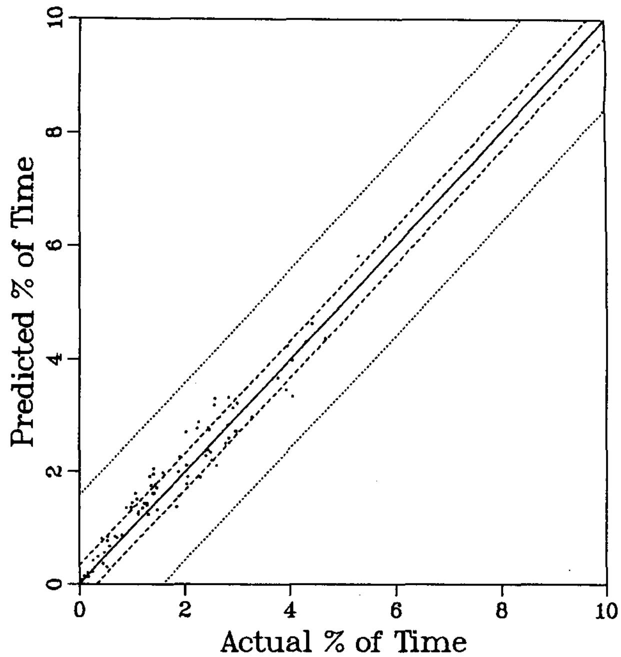

FIG. 3. PTP model evaluation for reserved sites.

precipitation level used for the mean number of days with precipitation.

The PTP is a scalar quantity for a given site and calendar month and is directly suitable for regression modeling against the climatological variables for each site and calendar month. The simplest regression model employs the individual climatological variables in a linear fashion, but this proved unsatisfactory in predicting the PTP values. Subsequent analysis was therefore performed to determine an appropriate set of auxiliary variables for incorporation into the regressions. These auxiliary variables would be based solely on the

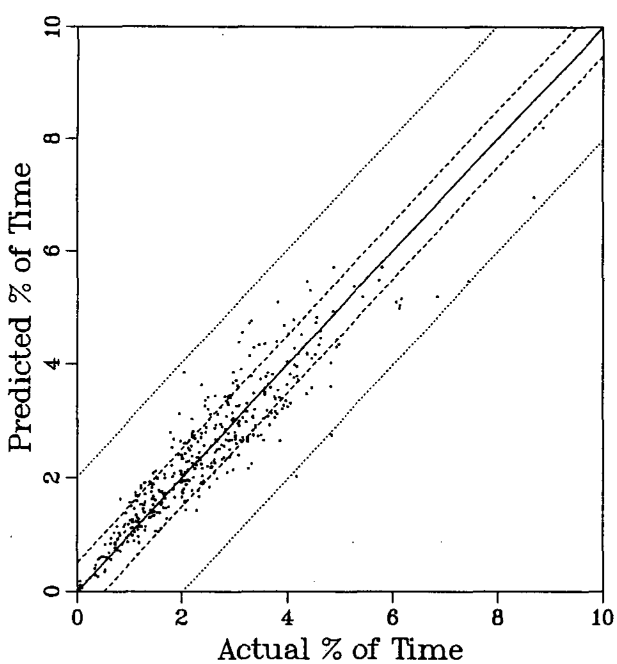

FIG. 2. PTP model evaluation for regression sites.

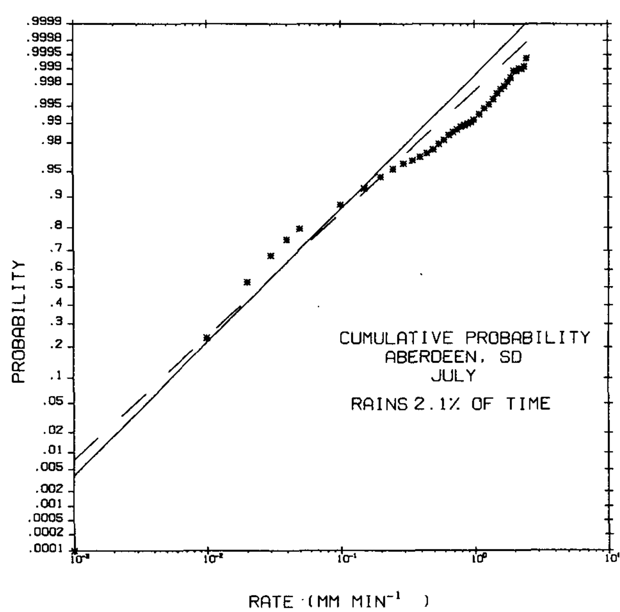

FIG. 4. Sample cumulative distribution, with least-squares linear fit (solid line) and CDF model (dashed line).

TABLE 2. Evaluation of the cumulative distribution function model.

Comparison	Mean error	Standard deviation	Skewness (dimensionless)	Kurtosis (dimensionless)
CDF model versus linear fit (units are Gaussian standard deviations)				
Regression sites	0.0964	0.3665	0.9011	5.4894
Reserved sites	0.2524	0.5172	0.6309	3.6378
CDF rates: model versus linear fit (units are logarithmic rates)				
Regression sites	−0.0383	0.1654	−0.2507	3.7062
Reserved sites	−0.1058	0.2369	0.0431	4.4530
CDF model versus tabulation (units are Gaussian standard deviations)				
Regression sites	0.0964	0.4386	0.8973	4.6968
Reserved sites	0.2524	0.5808	0.5461	3.3385
CDF rates: model versus tabulation (units are logarithmic rates)				
Regression sites	−0.0383	0.2019	−0.5239	3.5141
Reserved sites	−0.1058	0.2656	−0.1035	3.9395

original set of climatological predictors but could contain nonlinear combinations of these predictors, which is a circumstance not intrinsically accounted for in linear regression modeling.

A set of tables was constructed for the number of occurrences of PTP within a designated range of values versus designated ranges in each of the climatological variables. These tables represent the complete multivariate distribution of PTP with respect to the climatological variables, but only subsets representing the distribution of PTP with respect to any two climatological variables could be displayed. The bivariate subsets were examined for possible coordinated contributions of climatological variables to PTP, and a number of quadratic and cross-product terms involving the mean monthly temperature, the mean monthly temperature range, and the mean monthly precipitation were incorporated into the stepwise linear regression, with considerable improvement.

Using the extended set of 12 climatological terms, with the threshold precipitation level of 0.254 mm, a stepwise regression analysis selected eight terms as significant predictors for the percent of time with precipitation. The relative variation explained by the predictors (R^2) is 86.85%, with an estimated standard deviation of model error of 0.495 as percent of time with precipitation.

Review of the steps in the regression process indicated that only minor improvement (0.88% in R^2) was achieved by the introduction of the final three variables of the stepwise sequence. A simpler model without these three variables was considered a reasonable alternative. The expression for PTP for the simpler model is

$$\begin{aligned}\mathrm{PTP} = 2.018 \times 10^{-2} + 1.9164 \times 10^{-1} D \\ - 1.076 \times 10^{-3} TP + 3.565 \times 10^{-2} P \\ - 6.2106 \times 10^{-4} P\delta T. \quad (2)\end{aligned}$$

In this equation, and in all subsequent regression model equations, the terms are listed in the order in which they entered the stepwise regression. The estimated

TABLE 3. Evaluation of the percentage time exceeded model.

Comparison	Mean error	Standard deviation	Skewness (dimensionless)	Kurtosis (dimensionless)
PTE model versus tabulation (units are percentage occurrence)				
Regression sites	0.0365	0.1705	1.1650	28.2240
Reserved sites	0.0300	0.1150	2.3120	13.7967
PTE rates: model versus tabulation (mm $\min^{-1}$)				
Regression sites	−0.0531	0.4474	3.2972	58.3677
Reserved sites	−0.1051	0.9576	15.4043	424.0162
Hybrid PTE model versus tabulation (units are percentage occurrence)				
Regression sites	0.0375	0.1193	4.8674	40.1282
Reserved sites	0.0162	0.0678	3.1647	16.7668
Hybrid PTE rates: model versus tabulation (mm $\min^{-1}$)				
Regression sites	−0.0519	0.4728	3.1154	47.0874
Reserved sites	−0.1163	1.0195	15.8093	437.4248

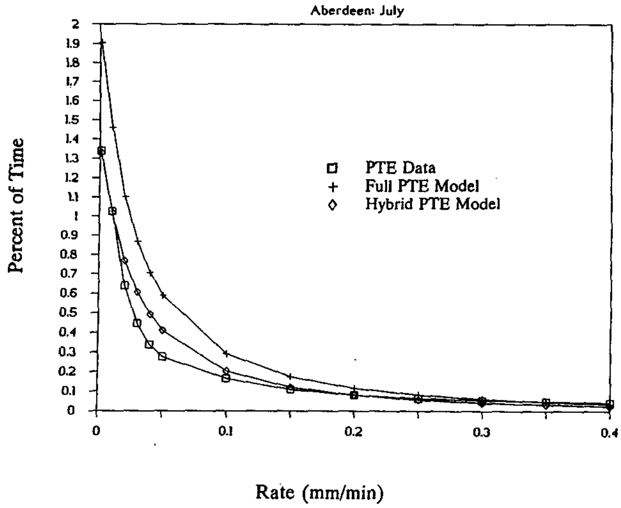

FIG. 5. Sample PTE using full model prediction or actual PTP compared to actual PTE data.

standard deviation of model error for this simpler model is 0.510 PTP, only 0.015 greater than for the initial model. The R^2 value is 85.97%.

A summary evaluation of the model performance is displayed in Fig. 2, which plots the predicted values of PTP from the simple regression model against the actual values for each month and each site. The dashed lines parallel to the central diagonal line indicate the standard deviation error (0.51%) for the model, while the dotted lines parallel to the central diagonal line indicate the standard deviation (2.02%) of all of the individual monthly values of the measured percent of time with precipitation. The complete variability at a site from year to year for a given calendar month is incorporated in this latter quantity. Thus, the year-to-year variability of the percent of time with precipitation for a calendar month far exceeds the model error.

Further evaluation of the PTP model was performed by examining the residuals for the model predictions with respect to the actual values for each of the individual predictor variables. This process would indicate whether higher powers of the single variables beyond quadratic were appropriate. No systematic effects justifying higher power terms were observed.

At the conclusion of the modeling process, the PTP model predictions were compared to actual values by using data from the reserved sites listed in Table 1. These results are displayed in Fig. 3 together with the model standard deviation (0.33%) and the monthly standard deviation of the measured percent of time with precipitation (1.58%), as calculated for the reserved sites. It is evident that the PTP values for the reserved sites are adequately predicted by the PTP model.

A similar regression process was performed using climatological variables based on precipitation thresholds of 1.0 and 2.54 mm for both the number of days with precipitation and the precipitation index. For the threshold level of 1.0 mm, the prediction expression for PTP becomes

$$\text{PTP} = 8.480 \times 10^{-2} + 2.8242 \times 10^{-1} D - 1.0527 \times 10^{-3} TP + 2.924 \times 10^{-2} P - 4.9554 \times 10^{-4} P\delta T. \quad (3)$$

In performing this regression, the particular months of July and August for Santa Maria and July for Bakersfield were excluded because these months had no days with precipitation above the specified threshold. The R^2 value for this model is 87.11%, and the standard deviation is 0.489 PTP.

When the threshold level for precipitation is increased to 2.54 mm, the prediction expression for PTP becomes

$$\text{PTP} = 2.9659 \times 10^{-1} + 3.9161 \times 10^{-1} D - 1.0690 \times 10^{-3} TP + 2.661 \times 10^{-2} P - 6.1040 \times 10^{-4} P\delta T. \quad (4)$$

The additional month of June for Santa Maria was excluded for this case, again because of the absence of days with precipitation above the specified threshold. The R^2 value for this model is 86.65%, and the standard deviation is 0.497 PTP.

b. Cumulative distribution function (CDF) model

The CDF is a function of precipitation rate, defined individually for each site and calendar month. Consequently, an intermediate step of defining an appropriate parameterized functional form for the CDF is required, following which a regression analysis similar to that for the PTP can be performed for each of the functional variables of the CDF.

Cumulative distributions were tabulated for each month for each of the 34 sites designated for model development (see Table 1). Six of the 408 total months were excluded from further study due to a lack of sufficient precipitation to define a CDF. These cases were Bakersfield, California, for the months of June, July, and August, and Santa Maria, California, for the same three months. Only one reference point at the trace precipitation rate could be tabulated for these cases, for which the associated PTP was below 0.1%. Thus, a minimum PTP of 0.1% was accepted as a necessary requirement for applicability of the CDF model.

The rate values at which the cumulative distributions were tabulated consisted of the trace rate of 0.001 mm min^{-1}; the rates from 0.01 to 0.05 mm min^{-1} at intervals of 0.01 mm min^{-1}; the rates from 0.1 to 1.0 mm min^{-1}, at intervals of 0.05 mm min^{-1}; and the rates from 1.1 to 2.5 mm min^{-1} at intervals of 0.1 mm min^{-1}—giving a total of 40 rate values. This set of rate values was determined to adequately sample the variety of precipitation distributions encountered, although precipitation at rates greater than 2.5 mm min^{-1} did occur for some sites in individual calendar months. For individual calendar months in which the maximum precipitation rate encountered was below 2.5 mm min^{-1}, the higher rate samples were disregarded, as these would have cumulative distribution values identically (and redundantly) equal to unity.

Nonlinear fits for the CDF were considered but were unsatisfactory. The linear form that was developed for the CDF corresponds to the following equation:

$$\text{CDF} = \frac{1}{2}\left\{1 + \text{erf}\left[\frac{T_0 + T_1 \log_{10}(r)}{\sqrt{2}}\right]\right\}, \quad (5)$$

where erf is the error function, r is the 1-min precipitation rate, T_0 is the constant coefficient for the functional fit, and T_1 is the first-order coefficient for the functional fit. The linear fits are Gaussian approximations in $\log_{10}(r)$ to the distribution functions (lognormal approximations). Each of the coefficients was subjected to an individual regression analysis in the same manner that was performed for the PTP.

Initial regression results for the CDF coefficients in terms of climatological variables, using the same 12 climatological terms originally employed for the PTP, were not as good as the PTP results when comparing predicted to actual values. A multivariate analysis yielded no further evidence of coordinated contributions of the climatological variables to the CDF fit coefficients, but the regression analysis itself could be used as an investigatory tool. Therefore, eight additional quadratic or cross-product climatological terms (I^2, $T \times I$, $P \times I$, $\delta T \times I$, D^2, $D \times T$, $D \times P$, $D \times \delta T$) were incorporated as possible candidate expressions for the stepwise linear regression. Five of these terms were selected by the stepwise regression as significant predictors for the CDF fit coefficients. The relative variation explained by these predictors is $R^2 = 81.79\%$ for T_0 and $R^2 = 74.43\%$ for T_1, with an estimated standard deviation of model error of 0.5430 for T_0 and 0.2700 for T_1.

As was the case with the PTP model, several terms were introduced near the end of the stepwise regression process that only marginally increased R^2 so that simpler representations with nearly the same accuracy could be obtained. These representations are

$$T_0 = 7.29223 - 4.3048 \times 10^{-1} I + 1.487 \times 10^{-2} I^2 - 5.5562 \times 10^{-3} T\delta T - 3.971 \times 10^{-2} D \quad (6)$$

$$T_1 = 3.99834 - 2.1162 \times 10^{-1} I + 5.9456 \times 10^{-3} I^2 - 2.2935 \times 10^{-3} T\delta T - 5.73 \times 10^{-3} D\delta T + 5.0191 \times 10^{-4} P\delta T. \quad (7)$$

For the case of T_0, the relative variation explained by the predictors is decreased by only $\delta R^2 = -1.49\%$, and the standard deviation is increased by only 0.0196; while for the case of T_1, the relative variation explained by the predictors is decreased by only $\delta R^2 = -2.14\%$, and the standard deviation is increased by only 0.0096.

The correspondence between predicted and actual values for the CDF fit coefficients does not necessarily imply that the resulting predicted CDF fits will closely match the tabulated cumulative distributions, because the appropriate correspondence between the slope T_1 and intercept T_0 coefficients must also be achieved to produce a representative fit. Thus, the predicted CDF for each site and month was plotted together with the associated tabulated cumulative distribution and the least-squares linear fit to evaluate the appropriateness of the model prediction. The predicted CDF generally approximates the fitted cumulative distribution reasonably well, but both of these can differ significantly from the actual tabulated cumulative distribution. This conclusion applies equally to the reserved sites used for the model validation as to the regression sites used for the model development. (See Fig. 4 for a sample linear fit and regression model.)

The quality of the climatological CDF model can be quantified by examining the distribution of errors for the model predictions with respect to either the original data or the least-squares linear fits. For consistency, these errors were tabulated in the domain of the fitting variables (the logarithm of the precipitation rate and Gaussian standard deviations) at the tabulated distribution values. Furthermore, because of applications in which the precipitation rate at a specified probability is the desired result, the error distributions for estimated logarithmic rates were also tabulated. These results are summarized in Table 2 for both the comparison of the CDF model to the original linear fits and for the comparison of the CDF model to the original tabulated values. For reference, the typical range of CDF errors of ±0.3 standard deviations is at most ±0.14 in linear probability units, while the typical range of logarithmic rate errors of ±0.18 corresponds to a factor of about 1.5 for linear rate units. The overall range of the CDF values is approximately 7.4 standard deviations (0.0001 to 0.9999 in linear probability units), while the overall range of logarithmic rates is approximately 3.4.

As with the PTP model, the CDF model was developed for alternative thresholds for the number of days with precipitation and the associated precipitation index. For the threshold level of 1.0 mm, the prediction expressions for the CDF coefficients become

$$T_0 = 8.67914 - 5.1783 \times 10^{-1} I + 1.085 \times 10^{-2} I^2 - 4.448 \times 10^{-3} T\delta T - 1.629 \times 10^{-2} D\delta T + 1.1509 \times 10^{-3} P\delta T - 1.503 \times 10^{-3} T^2 + 2.8576 \times 10^{-3} TI \quad (8)$$

$$T_1 = 4.14102 - 1.9439 \times 10^{-1} I + 4.8167 \times 10^{-3} I^2 - 1.8122 \times 10^{-3} T\delta T - 7.44 \times 10^{-3} D\delta T + 5.2609 \times 10^{-4} P\delta T - 2.9051 \times 10^{-4} T^2. \quad (9)$$

When the threshold level for precipitation is increased to 2.54 mm, the prediction expressions for the CDF coefficients become

$$T_0 = 8.55514 - 1.566 \times 10^{-2} D\delta T + 2.4709 \times 10^{-5} TP\delta T - 7.4520 \times 10^{-3} T\delta T - 4.0274 \times 10^{-1} I + 7.1748 \times 10^{-3} I\delta T + 6.38 \times 10^{-3} I^2 \quad (10)$$

$$T_1 = 4.14586 + 9.0705 \times 10^{-6} TP\delta T - 1.6615 \times 10^{-1} I - 8.88 \times 10^{-3} D\delta T + 2.6146 \times 10^{-3} I^2 - 3.1047 \times 10^{-3} T\delta T + 3.7354 \times 10^{-3} I\delta T + 3.59 \times 10^{-3} D^2. \quad (11)$$

3. Composite PTP–CDF model

The CDF model can be used in conjunction with the PTP model to evaluate the percentage of total time for which the precipitation rate exceeds a specified value. The expression defining the percentage of total time for which the precipitation rate exceeds a value r (mm min^{-1}) is given by Eq. (1), with the CDF defined by Eq. (5).

The error distributions for both the PTE as a function of precipitation rate and the threshold precipitation rate for a specified PTE were tabulated with respect to the actual data, and the results are summarized in Table 3. Note that these error evaluations are in terms of actual time percentages or rates, respectively, in contrast to the standard deviation units or logarithmic rates used for the CDF error evaluations.

It should be noted that actual PTP values can be used in Eq. (1) instead of model values, in circumstances where the actual PTP values are available. The typical improvement in the calculated PTE is then approximately 0.012%. A summary evaluation of this hybrid model also appears in Table 3.

The effects of using the actual PTP value for the PTE prediction are demonstrated in Fig. 5, which displays an example of the fully modeled PTE (using the model PTP and CDF), the hybrid-modeled PTE (using actual PTP values with the model CDF), and the tabulated PTE data values. The significance of an accurate PTP value can be seen at the lower precipitation rates where the PTP is the dominant factor in determining the scale of the PTE variation. The (partially displayed) asymptotic convergence of the model predictions and actual data is reflected in the error distributions as a large number of very small errors over the precipitation rate domain from about 0.3 mm min^{-1} to the maximum tabulated rate of 2.5 mm min^{-1}.

The typical PTE variation with precipitation rate, as displayed in Fig. 5, also clarifies the basis for the improvement from the CDF model evaluation statistics to the PTE model evaluation statistics. Large standard deviation errors that occur in the tail of the PTE distribution produce minimal linear PTE errors, while large logarithmic rate errors occurring at low precipitation rates produce minor rate error differences.

4. Summary

Models for estimating monthly total time occurrence for 1-min precipitation rate thresholds from local climatological variables were developed using stepwise multiple regression analysis. The models consist of one algorithm to estimate the percent of time with precipitation and a second algorithm to estimate the cumulative distribution of precipitation with respect to precipitation rate. These can be used to calculate the total percentage of time that a specified precipitation rate is exceeded, according to Eq. (1). The cumulative distribution estimation algorithm is not valid for situations where the percent of time with precipitation is less than 0.1%, and neither algorithm has been validated beyond the midlatitude regions corresponding to the extent of the 48 contiguous states of the United States. Used solely as a model to estimate the percent of time with precipitation, the model has a standard deviation of approximately 0.5%. Typical errors for total percentage of time exceeded by a specified rate are 0.01%–0.04%, while typical errors for critical rates determined from threshold time percentages are 0.01–0.10 mm min^{-1}.

Acknowledgments. We are grateful to Peter Dickson for his programming and processing support and to Dr. John Willett and Mrs. Penelope Yepez of the Geophysics Directorate of Phillips Laboratory for their reviews and comments. The model developments were supported in part under Contract F19628-89-C-0079 for the Data Systems Division of Phillips Laboratory.

APPENDIX

Definition of R^2

The quantity R^2, reported as a measure of the relative variation accounted for by the model, is descriptively defined as

$$R^2 = \frac{\text{variation explained by the independent variables}}{\text{total variation in dependent variable}}$$

and can be explicitly defined mathematically as

$$R^2 = \frac{\sum (Y' - \bar{Y})^2}{\sum (Y - \bar{Y})^2},$$

where Y is the dependent variable being modeled, Y' is the model prediction for the dependent variable, and $\bar{Y}$ is the average value of the measured dependent variable.

REFERENCES

Ajayi, G. O., and E. B. C. Ofoche, 1983: Some tropical rainfall rate characteristics at Ile-Ife for microwave and millimeter wave applications. *J. Climate Appl. Meteor.*, **22**, 562–567.

Crane, R. K., 1980: Prediction of attenuation by rain. *IEEE Trans. Comm.*, **8**, 1717–1733.

Tattelman, P., and K. G. Scharr, 1983: A model for estimating one-minute rainfall rates. *J. Climate Appl. Meteor.*, **22**, 1575–1580.

——, and D. D. Grantham, 1985: A review of models for estimating 1 min rainfall rates for microwave attenuation calculations. *IEEE Trans. Comm.*, **7**, 361–372.

——, and R. W. Knight, 1988: Analysis of 1-min rain rates extracted from weighing raingage recordings. *J. Appl. Meteor.*, **27**, 928–938.