Polarimetrically Tuned $R(Z)$ Relations and Comparison of Radar Rainfall Methods

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ABSTRACT

The following rainfall measurements are compared: 1) the reflectivity factor–rain rate or $R(Z)$ relation, whereby the rain is estimated point by point for mapping or area integration; 2) use of a specific differential phase K_{DP} (between vertical and horizontal polarization) in a relation with rainfall rate for point-by-point mapping and subsequent integration over areas and time; 3) use of a $R(K_{DP})$ relation together with a relation between K_{DP} and Z to derive a polarimetrically tuned or matched $R(Z)$ relation; and 4) use of empirical relations between the rainfall volume and the time integral of the storm area in which reflectivity is larger than a selected threshold. These methods are tested on five cases—two summer-type convections, one winter convective case, and two events of stratiform rain with embedded convection. Accumulations of rain in a dense gauge network in Oklahoma are used as a standard for comparison with radar measurements. In four of the five cases the rain totals obtained from the $R(K_{DP})$ relation agree very well with actual gauge accumulations. This is significantly better than the Marshall–Palmer $R(Z)$ relation, which agrees well with gauges for only one event. Matching Z to K_{DP} brought the $R(Z)$ derived rain total to better agreement with gauges in three more cases.

1. Introduction

Most of those concerned with radar measurements of rainfall recognize that the reflectivity factor Z–rain rate R relations vary with precipitation type. Those arguing to the contrary suggest that adjusting the $R(Z)$ relation for type is difficult to do or that it makes little difference in the final result—that is, the measurement of total rainfall. Others have suggested that in the final analysis one may simply tune the result to measurements made by surface rain gauges. For these reasons the present-day rainfall algorithm used in the modern Next Generation Weather Radar (WSR-88D) is a simple $R(Z)$ power law. In this paper it is shown that for hydrologic purposes it is especially important to adjust the $R(Z)$ relation to the precipitation type.

Besides the direct application of the $R(Z)$ relation to rainfall measurements, there are methods of more recent origin that utilize areas of rainfall to estimate the total amounts. If applied with radars, these area–time integral (ATI) techniques depend on radar observables such as the reflectivity factor or polarimetric variables. Recently polarimetric variables have become available on sufficient spatial and temporal scales to allow comparisons of rainfall measurements with gauge networks. The principal purpose of this paper is to compare rain measurements based on the reflectivity factor with those based on polarimetric measurements.

The methods of rainfall measurement that we compare are 1) the reflectivity factor–rain rate or $R(Z)$ relation, whereby the rain is estimated point by point for mapping or area integration; 2) use of a specific differential phase K_{DP} between vertical and horizontal polarization in a relation with rainfall rate for point by point mapping and subsequent integration over areas and time; 3) use of a $R(K_{DP})$ relation together with a relation between K_{DP} and Z to derive a polarimetrically tuned (PT) or matched $R(Z)$ relation; and 4) use of empirical relations between the rainfall volume and the time integral of the storm area over which reflectivity is larger than a selected threshold.

The PT method is novel, and in our implementation it depends on K_{DP}. Nevertheless, we use it to illustrate what we perceive would be close to optimum performance if a probability matching method, which does not

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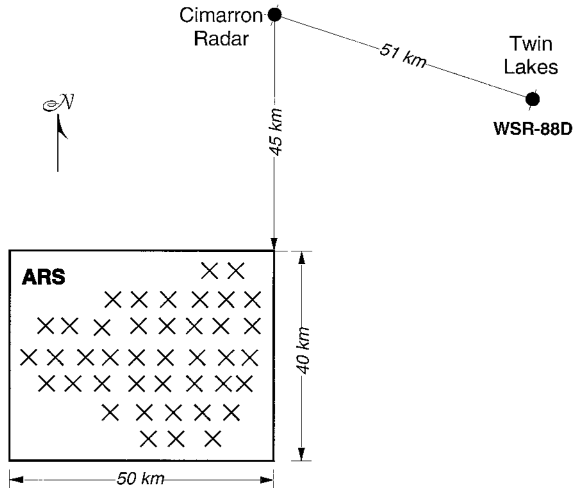

FIG. 1. Location of the Cimarron polarimetric radar, the WSR-88D radar, and the Little Washita River rain gauge network.

require polarimetric data, were implemented. Gorgucci et al. (1995) show that a multiparameter radar algorithm (which uses Z and differential reflectivity) can match the probability distribution functions better than the reflectivity-based algorithms.

Our comparison is made possible by the availability of a 10-cm polarimetric radar (Zahrai and Zrnić 1993) operated by the National Severe Storms Laboratory at Cimarron, Oklahoma. As a standard for comparison we use a dense rain gauge network (about one gauge per 25 km^2) operated by the Agricultural Research Service (ARS) in the Little Washita Basin. Figure 1 shows the locations of the radar and the gauge network.

2. Methods

Pertinent methods are summarized and/or reviewed in this section, and a suggestion on how to use polarimetry to establish a valid $R(Z)$ relation is given.

a. The R(Z) relation

The conventional $R(Z)$ scheme is derived from the basic work of Marshall and Palmer (1948, hereafter MP), who proposed the formula

$$Z = 200R^{1.6}, \qquad (1)$$

where Z is in its standard units (mm^6 m^{-3}) and R is in millimeters per hour.

b. Probability matching method

This method was introduced by Calheiros and Zawadzki (1987) and has been greatly elaborated on by Rosenfeld et al. (1994, 1995). In essence one equates the cumulative probabilities of Z and R so that

$$\int_{R_t}^{R_i} P(R)\,dR = \int_{Z_t}^{Z_i} P(Z)\,dZ, \qquad (2)$$

where $P(R)$ and $P(Z)$ are the conditional probabilities of R and Z (where or when it is raining). The pairs R_i, Z_i comprise the matched values, provided that the thresholds Z_t and R_t correspond to one another; otherwise one would match the Z's and R's at unequal percentiles.

Rosenfeld et al. (1995) have generalized and strengthened the method by classifying the rain types according to the physical characteristics that can be observed by radar in a 3D domain centered around a surface rain gauge. Each class is found to be associated with characteristic probability distribution functions (PDFs) of Z and R, and thus with its own $R(Z)$ relation. This local storm typing is particularly suitable for radar rainfall measurements over small areas and short periods such as are applicable in hydrology, provided that the sample is sufficiently large to attain representative PDFs.

At the outset of our study we realized that there were no representative climatological PDFs for different types of rain in Oklahoma. Therefore, we decided to use rain-rate estimates from the specific differential phase to "match" Z to R; details of this procedure are discussed in section 2e.

c. The area–time integral

The ATI was developed empirically by Doneaud et al. (1984) and justified theoretically by Atlas et al. (1990). Following individual storm clusters through their lifetimes with radar, Doneaud et al. (1984) found a linear relation between the volume of rain (for the entire storm) and the average area of the storm, bounded by a selected reflectivity threshold. Atlas et al. (1990) have shown that this relation is equivalent to

$$\langle R \rangle = S_t F_t, \qquad (3)$$

where $\langle R \rangle$ is the average rain rate over the area A_o in which $R > 0$, and $F_t = A_t/A_o$ is a fractional area where the rain rate is larger than a threshold R_t. Here, S_t is a proportionality factor related to R_t, and it can be expressed as

$$S_t = \frac{\displaystyle\int_0^\infty RP(R)\,dR}{\displaystyle\int_{R_t}^\infty P(R)\,dR}. \qquad (4)$$

The numerator in this equation is the first moment of the PDF of R, while the denominator represents the cumulative probability that $R > R_t$—that is, the equivalent of the average fractional area of the storm within the threshold.

To estimate rainfall with (3), the sample frequency distribution of R must approach the population PDF.

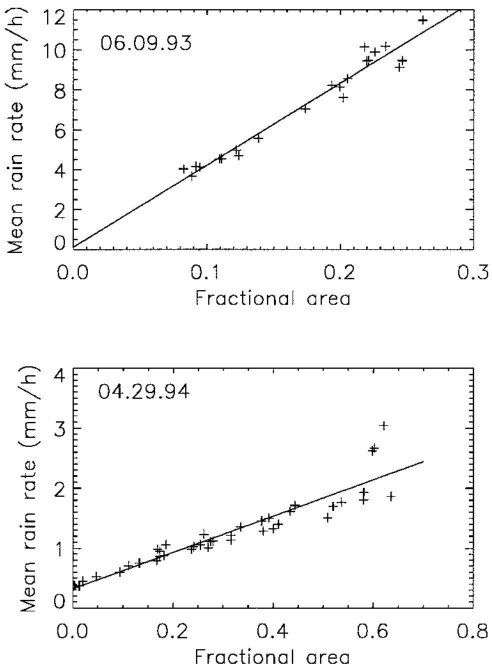

FIG. 2. Scattergram of average rain rate and fractional rain area for (a) 9 June 1993, a convective case, and (b) 29 April 1994, a stratiform case with embedded convection. The reflectivity threshold is 40 dBZ for the June case and 25 dBZ for the April case.

This means that for the particular rain type the space–time domain must be sufficiently large to obtain a representative sample. Thus, if $P(R)$ is known, the use of radar implies the existence of a known $R(Z)$ relation to relate R_t to a corresponding threshold Z_t. This can be done either by the use of a conventional relation or one obtained by other means, such as probability matching or polarimetric tuning (section 2e).

As stated earlier, there are no reliable $P(R)$ relations for rain in Oklahoma. Therefore, we have used the following methodology for testing the ATI. For each case the S_t was determined from the reflectivity factor data. An $R(Z)$ was used in the rainy area A_o (that was defined as the area with $Z > 10$ dBZ) to obtain a mean rain rate $\langle R_i \rangle$ for each successive radar scan. The fraction F_i of the area where rainfall was larger than $R(Z_t)$ was computed; Z_t was changed until a best linear relation between $\langle R_i \rangle$ and F_i was obtained. The slope of this linear relation is the optimum S_t. Note that S_t depends on the choice of $R(Z)$.

Examples of regression between $\langle R_i \rangle$ and F_i for two cases are shown in Fig. 2; the rain rates were obtained from the MP $R(Z)$ relation. In both cases the points are tightly clustered about straight lines. In our implementation of the ATI we used a 40-dBZ threshold for convective cases to obtain A_t within the watershed area, and 25- and 30-dBZ thresholds for the stratiform rain with embedded convection.

d. Specific differential phase

The suggestion that K_{DP} be used to measure rainfall in a single parameter relation $R(K_{DP})$ was made by Sachidananda and Zrnić (1986), who compared this estimator with other polarimetric rainfall estimators. The relative insensitivity of $R(K_{DP})$ to drop size distribution (DSD) variations is due to the following. The K_{DP} is influenced by forward scattering so that it is approximately proportional to the 4.24th moment of the DSD (for Rayleigh scattering). The rain rate is approximately proportional to the 3.67th moment. These two exponents of D in the integrals for computing R and K_{DP} are close to each other so that the integrals (i.e., R and K_{DP}) are almost linearly related. However, while this holds for a good variety of distributions, the extreme cases of light rain require further scrutiny.

DSD variability is only one of the factors that impede accurate radar rain measurements. Conceptually, this variability is simple to model and has therefore been studied intensively. There are, however, other factors implied in rainfall measurements that relegate drop size distribution variability to a secondary role (Zawadzki 1984). These are beam blockage, erroneous radar calibration, attenuation by precipitation and wet radome, incomplete beam filling, drop evolution on the way to the ground, and the influence of vertical air motions, among others. Measurement of rainfall using differential phase is immune to most of these except for the last three effects.

Throughout this paper we use the following $R(K_{DP})$ relation (Sachidananda and Zrnić 1987):[1]

$$R(K_{DP}) = 40.56|K_{DP}|^{0.866}\,\mathrm{sgn}(K_{DP}), \quad (5)$$

where R is in millimeters per hour and K_{DP} is in degrees per kilometer. The purpose of the sign term is to unbias (5) by assigning negative rain rates to negative estimates of K_{DP} that might be generated by noise in cases of very light rain. Here, K_{DP} is obtained by estimating the range derivative of the total differential phase Φ_{DP} over a fixed range interval. A formula for the standard deviation of K_{DP}, given by Balakrishnan and Zrnić (1990), can be expressed as

$$\mathrm{SD}(K_{DP}) = \frac{\sqrt{3}\mathrm{SD}(\Phi_{DP})}{N^{3/2}\Delta} \quad (6)$$

and applies if K_{DP} is obtained by least squares fitting a slope to the differential phase Φ_{DP} dependence on range. Here, $\mathrm{SD}(\Phi_{DP})$ (deg) is the standard deviation of the differential phase, Δ (km) is the range gate spacing, and N is the number of range samples in the least squares fit. Note that the accuracy of the K_{DP} estimate is independent of K_{DP} itself and is proportional to $N^{-3/2}$.

[1] Note that in Sachidananda and Zrnić (1987) K_{DP} denotes two-way specific differential phase, as opposed to the standard one-way phase shift.

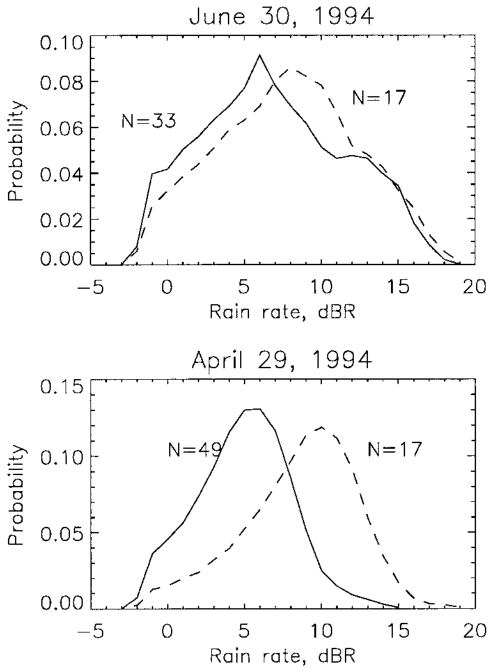

FIG. 3. Probability density distribution of rain rate for (a) 30 June 1994 and (b) 29 April 1994. The number of range samples N used to compute K_{DP} is indicated.

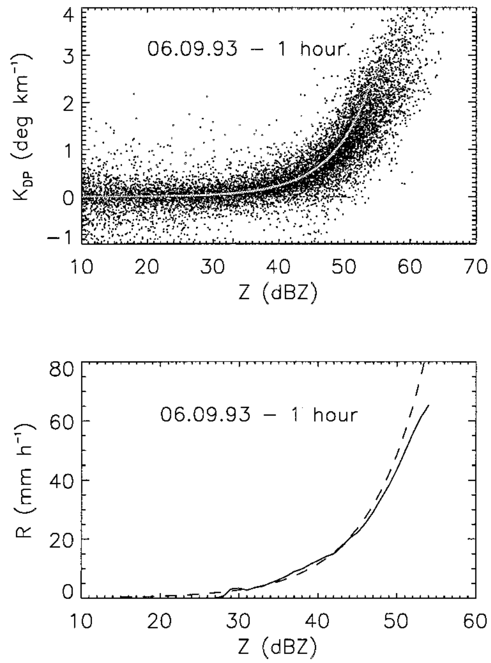

FIG. 4. (a) The K_{DP}–Z scattergrams for 9 June 1993 for the first hour of data collection during the time of intense convection. The light curve within the scattergram depicts the MP law. (b) Polarimetrically tuned $R(Z)$ relation (solid curve) and the MP curve (dashed).

In light rain the specific differential phase is small, and hence statistical errors of estimates are relatively large (Chandrasekar et al. 1990). Therefore, two range-averaging intervals are used to compute rain rates from (5). For $Z > 40$ dBZ the range-averaging interval is 2.4–3.8 km, depending on the range spacings Δ; otherwise (i.e., $Z < 40$ dBZ), the range-averaging interval is three times larger.

e. Polarimetrically tuned R(Z) relations

Here, we explain how K_{DP} can be used as a proxy for rain gauge data to obtain "polarimetrically tuned" $R(Z)$ relations. Initially we attempted to estimate $P(R)$ from K_{DP}. This PDF estimate $P_k(R)$, obtained from $R(K_{DP})$, is distorted by processing noise in (6); it is a convolution of the true PDF with the PDF of processing noise. If the sample contains relatively high rain rates this distortion is not significant, as can be seen in Fig. 3a, which illustrates the PDFs for two averaging intervals (N = 17 and 33, Δ = 180 m). This case is from a squall line (see section 3a), and the units of dBR refer to 10 log$R$ if R is in millimeters per hour; the log is to base 10. At high rain rates (>12 dBR or 16 mm h^{-1}) the two PDFs are almost identical, but at lower rain rates the differences are apparent. The $P_k(R)$ at these low values is contaminated by processing noise. Figure 3b is an example from a light rain event; a factor of 3 change in the range-averaging interval causes a shift in the $P_k(R)$ maximum by 5 dBR. Therefore, at low rain rates either deconvolution of the estimated PDF should be made or alternative means to obtain matched $R(Z)$ are required. Because deconvolution is inherently ill conditioned, we opted for an alternative method.

We chose to relate Z data to K_{DP} data (see Fig. 4a for example) and combine this relation with (5) to obtain $R(Z)$. For each reflectivity factor increment of 1 dB, we computed the average K_{DP}. This K_{DP} was used to relate its Z to rain rate through (5). Henceforth we call such an approach polarimetric tuning (PT). For the Marshall–Palmer $R(Z)$, K_{DP} and Z are related by

$$K_{DP} = 3.03 \times 10^{-4} Z^{0.722}, \qquad (7)$$

which we call the "MP curve." In (7), K_{DP} is expressed in degrees per kilometer and Z is in its standard units (mm^6 m^{-3}).

To separate the effects of drop size distribution variation from attenuation we have corrected the reflectivity factor using the formula for the negative reflectivity bias ΔZ (Bringi et al. 1990),

$$\Delta Z\ (\mathrm{dB}) = -a\Phi_{DP}\ (\mathrm{deg}), \qquad (8)$$

where Φ_{DP} is the total differential phase. In our calculations a = 0.025 dB deg^{-1} except for the case of 9 June 1993, when a = 0.036 dB deg^{-1} due to the presence of hail (Ryzhkov and Zrnić 1995b). The corrected values of Z are used for polarimetric tuning and rainfall estimation. For matching the Z to K_{DP} the shorter range averaging interval (2.4–3.8 km) is used for all values of Z.

TABLE 1. Rain accumulations over the ARS rain gauge network. The number in parentheses indicates accumulation if reflectivity values are capped by a 53-dBZ threshold, as done for WSR-88D algorithms. Here, MP refers to the Marshall–Palmer $R(Z)$ relation. Similarly, PT refers to the $R(Z)$ obtained with polarimetric tuning.

Date Type of storm	9 June 1993 Squall line with hail	30 June 1994 Squall line without hail	19 February 1994 Squall line with very intense rain	22 April 1995 Stratiform rain with embedded convection	29 April 1994 Stratiform rain with embedded convection
Accumulation time (h)	2	2	1	2	4
Number of valid gauges	42	37	33	42	40
Mean rain rate (mm h^{-1})	11.3	3.9	13.3	4.4	3.7
Gauge total (mm)	**945**	**286**	**438**	**367**	**591**
MP $R(Z)$	1275 (1086)	268	202	228	182
$R(K_{DP})$	984	292	409	346	253
MP ATI	1263	302	177	224	236
PT $R(Z)$	1140	—	404	352	282
PT ATI	1077	—	376	359	368

3. Comparisons

Data from five rain events in Oklahoma were selected for the analysis. Two are mesoscale convective systems (MCS) that occurred in June; one of these had a squall line with hail (9 June 1993), and the other one did not have hail (30 June 1994). The third case is a winter squall line accompanied by very intense rain (19 February 1994); the forth (29 April 1994) and fifth (22 April 1995) are widespread stratiform rains with embedded convection. The results are summarized in Table 1, where the last five rows depict radar measurements.

The radar-estimated rainfall accumulations are obtained by integrating the rain rate at each rain gauge in time and summing the results over all rain gauges. The time between radar volume scans was about 6 min. The gauge accumulation is the sum of rainfall measured by each gauge.

Accumulations from gauges do not provide a check of the validity of $R(Z)$ relations because these relations are often variable during typical integrating periods. Furthermore, the same accumulation can be obtained from many different relations. Nevertheless, we had no other choice because the gauges in the network measured only the totals. The reader is therefore cautioned not to regard the total gauge accumulations in Table 1 as an unequivocal standard of comparison.

a. June cases

Detailed analysis of the pointwise radar–gauge comparison for the June events has been previously performed (Ryzhkov and Zrnić 1995a,b; Zrnić and Ryzhkov 1996). The summary for the MP $R(Z)$ and the $R(K_{DP})$ algorithm is in Table 1.

It is evident from Table 1 that for both June cases the $R(K_{DP})$ relation yields rainfall estimates that compare very well with the gauges. The MP estimator works well for the 30 June 1994 case but overestimates total rainfall for the 9 June 1993 case, most likely due to the presence of hail. Capping Z at the 53-dBZ threshold (as done in the WSR-88D algorithm) improves rainfall estimate but still yields an overestimated rain total.

We have noticed that in the 9 June 1993 case the K_{DP}–Z scattergrams for the first and second hours of observations are different with respect to the line representing the MP curve in (7). The K_{DP}–Z scattergram (Fig. 4a) for the first hour of observation (when the contribution from the squall line dominated) is consistent with the MP relation (7) for reflectivities below 45 dBZ, but indicates positively biased Z in high-reflectivity regions where hail is dominant. Therefore, the PT $R(Z)$ relation for this period agrees with the Marshall–Palmer formula (1) for $Z < 45$ dBZ, as can be seen in Fig. 4b. During the second hour of data collection the convective part was gradually replaced with a stratiform rain in the gauge area. In the changing convective part (Fig. 5a), the MP relation between K_{DP} and Z no longer matches the data well for reflectivities below 45 dBZ. This is also clearly depicted by the difference between the PT and MP $R(Z)$ relations for reflectivities between 30 and 50 dBZ (Fig. 5b). Physical reasons for the different characteristics of rain during the first and second hour of data collection are not known. We speculate that melting of hail and shedding of drops during the first hour of observation broadened the DSD (Ryzhkov and Zrnić 1995a,b). The use of the polarimetrically tuned $R(Z)$ relation improves rainfall estimate for the 9 June 1993 case (see Table 1).

It is interesting to note that if radar reflectivity data are not corrected for attenuation in accordance with (8), then the radar rain total for 9 June 1993 matches the gauge total well if the MP $R(Z)$ is used. The rain accumulation obtained from radar is 940 mm (not shown in Table 1). The good performance of the MP relation applied to noncorrected Z data is, however, fortuitious because two factors, attenuation and hail contamination, counterbalance each other (Ryzhkov and Zrnić 1995a,b).

In the case of 30 June 1994, the polarimetric algo-

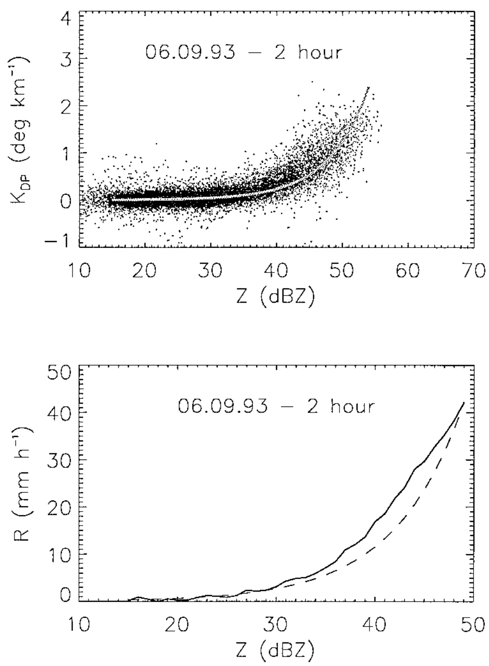

FIG. 5. As in Fig. 4 except for the second hour of data collection, when convection was decreasing and changing to stratiform rain.

rithm $R(K_{DP})$ exhibits excellent performance, and the MP $R(Z)$ is also very good (Table 1). Isopleths of accumulations (Fig. 6) provide a visual comparison between gauges and radar estimates. The accumulation patterns obtained from $R(Z)$ and $R(K_{DP})$ are consistent and well matched with the pattern from gauges. We conclude that the spatial and temporal characteristics of rainfall are captured equally well with the radar and the gauges. Thus, the MP relation that was derived for rain in the Montreal area happens to match the rain in an Oklahoma squall line, at least in this case. This finding might be surprising, but it is not in variance with the current $R(Z)$ relation ($Z = 300R^{1.4}$) used in the WSR-88D. The two relations are quite similar for rain rates below 50 mm h^{-1}.

The ATI algorithm based on the MP relation to set the threshold performs reasonably well for the 30 June 1994 storm and compares favorably with the two methods that use the point measurements. However, the ATI algorithm based on the MP relation overestimates actual rain total for the 9 June 1993 case in the same manner as the regular MP $R(Z)$ relation. The ATI method based on the polarimetrically tuned $R(Z)$ relation yields better agreement with gauges (see Table 1).

b. February case

The MP $R(Z)$ algorithm leads to a very large (more than twofold) underestimate of rainfall for the February event (Table 1). The ATI method based on the MP relation similarly underestimates rainfall. To check whether the radar calibration was erroneous, we have compared the reflectivity field with the field obtained by a nearby WSR-88D radar. The reflectivity factors agreed to within 1 dB, which suggests that a significant calibration error was not likely.

Polarimetric estimates agree much better with the gauges (Fig. 7a). The actual rain total is 438 mm; Cimarron radar yields 202 mm if the MP Z–R is used, whereas the $R(K_{DP})$ estimate (409 mm) is much closer to the accumulations measured by the gauges. It does, however, underestimate the total rain by about 7%. An independent rain total estimate with WSR-88D (185 mm, not shown) is in agreement with the Cimarron result that uses Z. We have identified two possible reasons for the bad performance of the MP algorithm in this case. These are (i) larger attenuation than was accounted for and (ii) the DSD was skewed toward smaller sizes (less than 1.5 mm in diameter). A detailed analysis of this case substantiating hypothesis (ii) is in Ryzhkov and Zrnić (1996b).

We again use the K_{DP}–Z scattergram (Fig. 8a) to establish an appropriate $R(Z)$ relation, as in section 3a (Fig. 8b). Although the attenuation was accounted for according to (8), the scattergram exhibits a very large deviation from the MP curve. By applying a tuned relation we obtain 404 mm for the rain total, which, as expected, agrees well with rain gauge total. The tuned $R(Z)$ relation helps to substantially improve the ATI estimate of rainfall as well for this case. The power law that fits the tuned relation can be approximated by $Z = 39R^{1.76}$. Although the multiplying coefficient in this relation is unusually low, it is still higher than the extreme values found by Blanchard (1953).

To test how realistic this relation is we modeled various DSDs and checked the consistency between computed and observed Z, K_{DP}, and differential reflectivity data. We found good consistency if a nearly monodisperse DSD was assumed with a large number of small ($<$1.5 mm) raindrops.

c. April cases

The 22 April 1995 case had light rain with embedded convection. In spite of the relatively light rain the $R(K_{DP})$ performs very well (Table 1); however, the MP $R(Z)$ underestimates rainfall by 38%. Examination of the scattergram (Fig. 9a) reveals that the MP curve in (7) is offset from the average and thus is not representative of the rain regime.

To determine the effects of range averaging on K_{DP} estimates and polarimetric tuning, we have used two range averaging intervals; one is 3.8 km, and the other is 11.5 km. Figure 9b shows the PT $R(Z)$ relations for these two range averaging intervals and also for the MP relation (1). Note that the PT relations diverge at reflectivity factors larger than about 40 dBZ. This is because cells with stronger reflectivities have larger gradients than weaker cells, and hence the shorter range averaging should retrieve a more representative K_{DP};

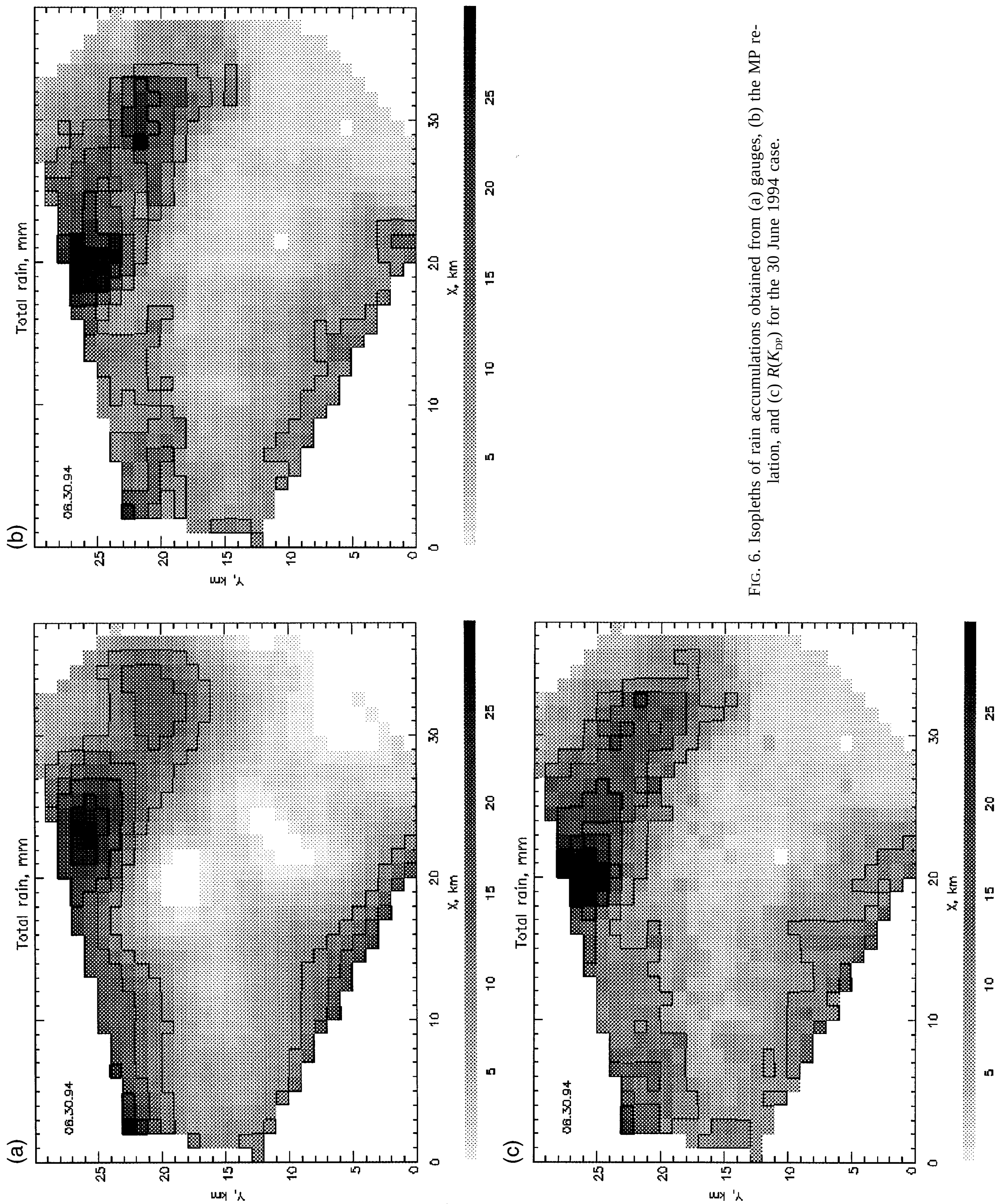

FIG. 6. Isopleths of rain accumulations obtained from (a) gauges, (b) the MP relation, and (c) $R(K_{DP})$ for the 30 June 1994 case.

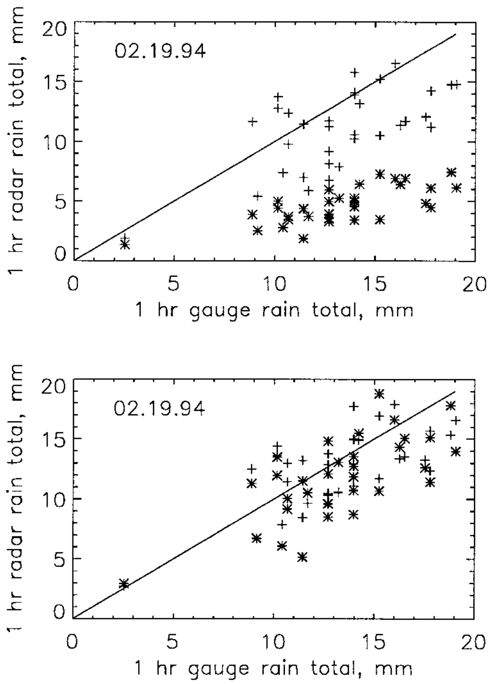

FIG. 7. (a) Rain accumulations obtained by radar and rain gauges for the 19 February 1994 case. The asterisk stands for the Marshall–Palmer $R(Z)$ relation, and crosses represent the $R(K_{DP})$ relation. (b) Same as in (a) except the $R(Z)$ was obtained by polarimetric matching.

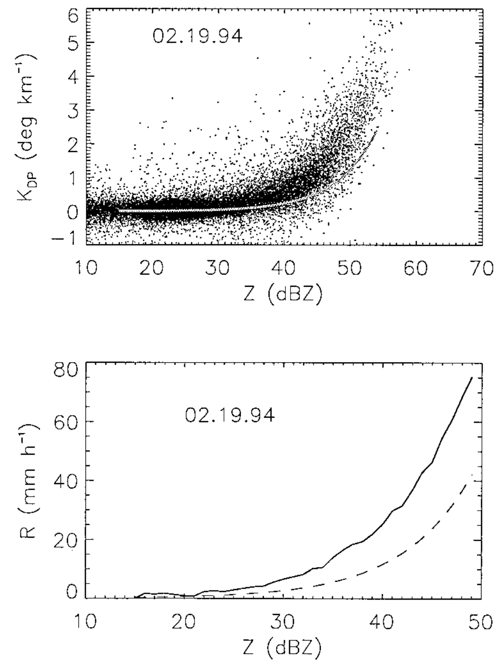

FIG. 8. (a) Scattergram K_{DP}–Z for 19 February 1994. The smooth curve corresponds to the Marshall–Palmer law. (b) Polarimetrically tuned $R(Z)$ (solid curve) and the MP curve.

with the longer averaging intervals, Φ_{DP} is smoothed and therefore the corresponding K_{DP}'s are lower than actual. Because at low rain rates there is no difference between the tuned $R(Z)$ obtained from the two different averaging intervals, it is acceptable to use only the smaller interval for polarimetric tuning.

It is evident that the PT $R(Z)$ agrees with the gauges (Table 1). As far as ATI is concerned, it depends on which $R(Z)$ is used. Thus, the MP choice in this case is inadequate, but the PT choice results in very good agreement with the gauges.

Finally we present the case of 29 April 1994, for which none of the algorithms worked well. As in the previous April case, the rainfall was light with embedded convection. The discrepancy between the radar (Cimarron MP 182 mm in Table 1, WSR-88D MP 251 mm not shown) and the rain gauge 591-mm accumulations is rather large. Even the $R(K_{DP})$ that measures 253 mm considerably underestimates the accumulation.

The PT relation produces better estimates (Table 1), equivalent to $R(K_{DP})$, from which it is derived. It exhibits similar bias. Evidently very light rain presents a challenge to the polarimetric method as well, although to a lesser degree. Light rain consisting of a large number of almost spherical small drops and light rain composed of typical distributions producing the same accumulations cause different specific differential phases. Clearly polarimetric tuning is not well suited to the former case because K_{DP} fails. Nevertheless, polarimetrically tuned ATI outperforms the other estimators.

Although the mean rain rates for both April cases are comparable (Table 1), the rain regimes are different.

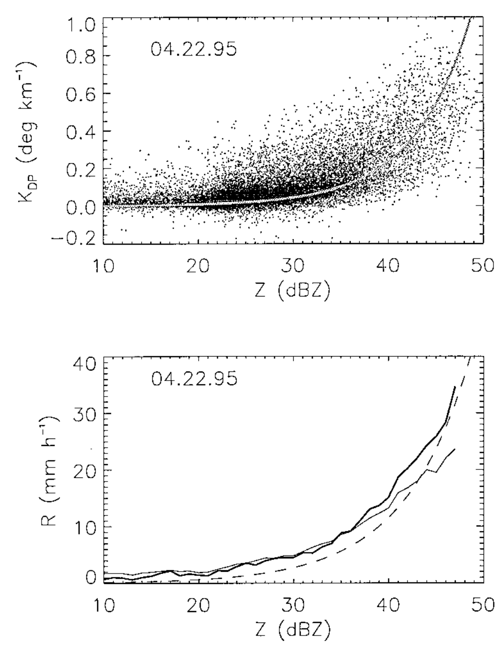

FIG. 9. (a) Same as in Fig. 8a but for 22 April 1995. (b) Polarimetrically tuned $R(Z)$ relation obtained by range averaging over 3.8 km (thick solid curve) and 11.5 km (thin solid curve). The MP curve is dashed.

Analysis of differential reflectivity and specific differential phase shows that, for a given K_{DP}, Z_{DR}'s were substantially lower in the April 1994 case (Ryzhkov and Zrnić 1996a). This would occur if the rain in April of 1994 were dominated by drizzle. Of 15 rain events observed in Oklahoma this was the only one for which the polarimetric rainfall estimate $R(K_{DP})$ failed.

4. Summary

Several methods of estimating rainfall by radar have been evaluated. These include the following: 1) the conventional $R(Z)$, 2) the use of specific differential phase in the $R(K_{DP})$ relation, 3) relating Z to K_{DP} to derive a polarimetrically tuned $R(Z)$ relation, and 4) the area–time integral (ATI) method.

Verification was provided by cumulative rain gauges in the Little Washita River basin. Gauge totals are not reliable means of validating $R(Z)$ or $R(K_{DP})$ relations because different relations can produce the same accumulations. Compounding the problem is the space–time variability of these relations. Nevertheless, this is what was available and what was used in the present study.

Because there were no independent measurements of the probability density functions of rain by gauges in any of the cases, we used R esimated from K_{DP} as a proxy for rain gauge estimates of R in the probability matching method. Simultaneous and collocated K_{DP} and Z over the entire network meet the basic requirement for probability matching. But processing noise in the K_{DP} data biases the PDF of K_{DP}, and this produces erroneously matched $R(Z)$ relations. To overcome this problem we have used Z–K_{DP} scattergrams from which we obtained a mean relation between Z and K_{DP}. We have then used this relation and $R(K_{DP})$ to obtain polarimetrically tuned $R(Z)$ relations.

In four of the five cases the rain totals obtained from $R(K_{DP})$ agree very well with gauge accumulations. This is significantly better than the Marshall–Palmer relation, which agrees well with gauges for only one event. Matching Z to K_{DP} brought the $R(Z)$-derived rain total to better agreement with gauges in three more cases. The same conclusions that apply for the $R(Z)$ method apply to the ATI method; this is expected because the ATI uses an $R(Z)$ algorithm to relate areas to rain accumulations.

All methods applied to the case of 29 April 1994 (mostly stratiform rain, averaging 3.7 mm h^{-1}) gave poor agreement with gauge estimates. All MP- and K_{DP}-based measurements grossly underestimated the network gauge total. Clearly the $R(K_{DP})$ relation does not apply to this light rain. It remains to be determined whether there is a suitable $R(K_{DP})$ to cover similar rain events.

As with other radar rainfall techniques, measurements of K_{DP} require that the DSD within the resolution volume correspond to that reaching the surface. This limits the range for accurate measurements, although the range can be made larger than that for reflectivity measurements. This is because the beam can be lowered to where it is partially blocked, but K_{DP} values, which are independent of absolute power, would still be valid (Zrnić and Ryzhkov 1996).

We have demonstrated that K_{DP} can be used to obtain an appropriate $R(Z)$ in the cases where $R(K_{DP})$ is accurate. Three possible implications follow. First, because the resolution of K_{DP} data is coarser than that of the Z data, the tuned $R(Z)$ would restore the intrinsic radar range resolution to rainfall measurement. Second, the polarimetrically tuned $R(Z)$ relations could be used for climatological purposes. Third, the tuned relations could be used on nonpolarimetric radars.

The analyzed cases amply reinforce the importance of using correct $R(Z)$ relations, which need adjustment even during single rain events. In the absence of a polarimetric radar, one should attempt to select the appropriate $R(Z)$ relation based on physical classifications of the precipitation type in the manner of Rosenfeld et al. (1994, 1995).

Although it was originally developed for large areas, we have shown here that the ATI method can be applied to small watersheds. The scheme performed as well as the $R(Z)$ method from which it was derived.

The comparisons made in this paper are for a small set of five events, three of which brought out differences in performance attributable to the rain type. Thus, further investigation of physical processes leading to these differences and a statistical evaluation of the methods is in order.

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