NOTES AND CORRESPONDENCE

A Local Parameterization Scheme for σ_w under Stable Conditions

Maithili Sharan and S. G. Gopalakrishnan*

Centre for Atmospheric Sciences, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi, India

R. T. McNider

Department of Mathematical Sciences, University of Alabama in Huntsville, Huntsville, Alabama

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ABSTRACT

Turbulence in stable conditions is local, that is, it is locally defined by small eddies. A local formulation for σ_w based on a level 2 approximation of Mellor and Yamada (1974) is proposed. The proposed formulation is able to describe the nondimensional profile of $(\sigma_w/U_*)^2$ against Z/H consistently when compared with the Minnesota observations, where H is the height of the turbulent stable boundary layer.

1. Introduction

For the modeling of the dispersion of air pollutants in the planetary boundary layer (PBL), a precise behavior of the standard deviations of vertical velocity fluctuations (σ_w) is required. The behavior of σ_w in the surface layer is fairly well understood (Merry and Panofsky 1976) under the neutral and unstable atmospheric conditions. The ratio σ_w/U_* obeys the Monin–Obukhov similarity (McBean 1971; Merry and Panofsky 1976; Hicks 1981) and has a value of 1.3 (±about 5%) under neutral conditions, where U_* is the frictional velocity scale at the surface. This value increases almost monotonically with the increase in instability.

Above the surface layer, under unstable conditions (Panofsky et al. 1977; Kaimal et al. 1976; Caughey 1984) and neutral conditions (Deardorff 1970; Arya 1984), the behavior of σ_w/U_* has been fairly well established. However, the behavior of σ_w is not clear in the stable boundary layer (SBL). Caughey et al. (1979), using limited Minnesota data, suggested a linear decrease in σ_w/U_* with Z/H, where H is the height of the turbulent SBL (Nieuwstadt 1985). The use of H as a scaling parameter can be justified if the height of the turbulent SBL can be taken as the representative length scale of turbulence (Nieuwstadt 1984). As the stability grows, the size of the turbulent eddies becomes small and with increasing stability, the eddy will no longer feel the effect of the surface. Under these conditions, it is more appropriate to use the concept of local scaling in the SBL (Nieuwstadt 1984; Sorbjan 1986). According to this scaling hypothesis, the ratio of σ_w to the local friction velocity ($U_* = \tau^{1/2}$, τ is the kinematic momentum flux) bears a constant that is $\sigma_w/\tau^{1/2} = 1.4$.

Application of the above scaling to dispersion models is limited by the fact that locally scaled profiles can be obtained only if the vertical distribution of turbulent fluxes of heat and momentum is available. For the purpose of direct application in numerical models, Nieuwstadt (1984) obtained the functional form of τ for a horizontally homogeneous terrain, that is,

$$\tau = U_*^2(1 - Z/H)^{\alpha_1/2}, \tag{1}$$

where α_1 is a parameter. This parameter depends on the stability and the type of the terrain. For the Minnesota observations, taken near sunset, α_1 is 4 (Sorbjan 1986) and in the case of the Cabauw observations it is 3 (Nieuwstadt 1984). Hence, there appears to be an uncertainty in the numerical value of α_1. Further, it should be noted that the functional form of σ_w derived from the scaling hypothesis (Nieuwstadt 1984) and Eq. (1) is expected to hold well only within the turbulent SBL. However, there is growing evidence of turbulence above the stable layer (Andre and Mahrt 1982; McNider et al. 1988), which develops due to near-neutral stability and a large wind shear in the residual layer above the SBL.

* Current affiliation: Department of Environmental Sciences, Cook College, Rutgers–The State University, New Brunswick, New Jersey.

Corresponding author address: Dr. Maithili Sharan, Centre for Atmospheric Sciences, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi 110016 India.
E-mail: mathilis@cas.iitd.ernet.in

Although this turbulence in the residual layer is not a part of the SBL, it is of vital importance for long-range transport of pollutants (McNider et al. 1988) and should thus be taken into account in mesoscale models.

A second-order closure model can be used to directly compute σ_w (Wyngaard 1975; Andre et al. 1978). However, the procedure to obtain the numerical solution becomes complex and computationally expensive. An alternative approach is to use an approximate form of governing equations for the turbulent fluxes and variance and then deduce a diagnostic relationship for σ_w. In an earlier attempt, Yamada (1979) obtained a formulation for σ_w in terms of Z/H. McNider (1981) deduced an approximate relationship for σ_w in terms of the local gradient Richardson number, mixing length and shear. However, his estimates deviated largely from the observations (McNider et al. 1988).

In this work, a simple local closure formulation for σ_w has been deduced based on the level 2 turbulence closure model of Mellor and Yamada (1974). This formulation was incorporated in the one-dimensional version of Pielke's mesoscale model (McNider et al. 1988; Sharan et al. 1995), and the computed values of $(\sigma_w/U_*)^2$ are compared with the Minnesota observations.

2. The formulation

The second-order closure theory (Mellor and Yamada 1974) for the variance and covariance equations has been used to deduce a local formulation for σ_w. We have applied the conditions of stationarity and have neglected the Coriolis term since their timescales are much larger than the turbulence timescale. Further, we have used the level 2 approximation (Mellor and Yamada 1974) and neglected the advective and diffusive terms in the equations for the second moments of the turbulent fluctuations. The remaining terms that involve pressure and molecular effects were parameterized (Rotta 1951; Mellor and Yamada 1974; Wyngaard 1975). The variance and covariance equations as given by Blackadar (1979) are

$$C_m \ell^{-1}\sigma_w \overline{W'U'} = -\alpha_m \sigma_w^2 \frac{\partial U}{\partial Z} + \frac{g}{\theta}\overline{U'\theta'}, \qquad (2)$$

$$C_m \ell^{-1}\sigma_w \overline{W'V'} = -\alpha_m \sigma_w^2 \frac{\partial V}{\partial Z} + \frac{g}{\theta}\overline{V'\theta'}, \qquad (3)$$

$$C_m \ell^{-1}\sigma_w \left(\sigma_w^2 - \frac{E^2}{3}\right) = \frac{4}{3}\frac{g}{\theta}\overline{W'\theta'} + \frac{2}{3}\left(\overline{U'W'}\frac{\partial U}{\partial Z} + \overline{V'W'}\frac{\partial V}{\partial Z}\right), \qquad (4)$$

$$C_h \ell^{-1}\sigma_w \overline{U'\theta'} = -\overline{W'\theta'}\frac{\partial U}{\partial Z} - \overline{W'U'}\frac{\partial \theta}{\partial Z}, \qquad (5)$$

$$C_h \ell^{-1}\sigma_w \overline{V'\theta'} = -\overline{W'\theta'}\frac{\partial V}{\partial Z} - \overline{W'V'}\frac{\partial \theta}{\partial Z}, \qquad (6)$$

$$C_h \ell^{-1}\sigma_w \overline{W'\theta'} = \alpha_h \frac{g}{\theta}\overline{\theta'^2} - \sigma_w^2 \frac{\partial \theta}{\partial Z}, \qquad (7)$$

$$C_E \ell^{-1}\sigma_w E^2 = \frac{g}{\theta}\overline{W'\theta'} - \left(\overline{W'U'}\frac{\partial U}{\partial Z} + \overline{W'V'}\frac{\partial V}{\partial Z}\right), \qquad (8)$$

and

$$C_m \ell^{-1}\sigma_w \overline{\theta'^2} = -\overline{W'\theta'}\frac{\partial \theta}{\partial Z}, \qquad (9)$$

where U, V, and θ are the mean field variables and g is the acceleration due to gravity. The cross-correlation terms of the form $\overline{W'U'}$, $\overline{W'\theta'}$, etc., appearing in Eqs. (2)–(8) are known as the kinematic momentum and heat fluxes and ℓ is the mixing length. Here, E^2 is defined as $(\overline{U'^2}, + \overline{V'^2} + \overline{W'^2})$ and α_m and α_h are terms that have their origin due to effects of vortex stretching and buoyancy adjustment on the pressure correlation terms (Blackadar 1979). The term α_h has been taken to be 1 by Mellor and Yamada (1974). The terms C_E, C_m, C_h, and C_θ are constants.

Following Blackadar (1979), we introduce a nondimensional parameter μ such that

$$\mu = \frac{\ell^2 S^2}{\sigma_w^2}\mathrm{Ri}, \qquad (10)$$

where Ri is the gradient Richardson number and S is the local shear defined as

$$S = \left[\left(\frac{\partial U}{\partial Z}\right)^2 + \left(\frac{\partial V}{\partial Z}\right)^2\right]^{1/2}. \qquad (11)$$

The mixing length ℓ is computed from the relationship (Blackadar 1962)

$$\ell = \frac{kZ}{\left(1 + \frac{kZ}{\lambda}\right)}, \qquad (12)$$

where k is the von Kármán constant taken to be 0.35 and

$$\lambda = 2.7 \times 10^{-4}\, |G|/|f|, \qquad (13)$$

in which

$$|G| = (U_g^2 + V_g^2)^{1/2} \qquad (14)$$

is the geostrophic speed and f is the Coriolis parameter.

Equation (12) indicates that the mixing length increases linearly with height near the surface and approaches a constant value λ far away from it. The parameter λ depends on the geostrophic wind.

If we assume a gradient relationship based on the K theory for the fluxes, Eqs. (7) and (9) give

$$\frac{K_H}{\ell\sigma_w} = \left(C_h + \frac{\alpha_h\mu}{C_\theta}\right)^{-1} = k_h \quad \text{(say)}. \tag{15}$$

Similarly, Eqs. (3) and (6) yield

$$\frac{K_M}{\ell\sigma_w} = \left(\frac{C_h\alpha_m - k_h\mu}{C_mC_h + \mu}\right) = k_m \quad \text{(say)}. \tag{16}$$

From Eqs. (4) and (8), E can be eliminated, and finally an independent expression can be obtained between μ and Ri, using Eqs. (10), (15), and (16); that is,

$$(A\text{Ri} - B)\mu^2 + (C\text{Ri} - D)\mu + E_1\text{Ri} = 0, \tag{17}$$

where

$$A = 3\alpha_hC_EC_m + C_\theta(4C_E + C_m), \tag{18}$$

$$B = (C_m - 2C_E)(\alpha_h\alpha_mC_h - C_\theta), \tag{19}$$

$$C = 3\alpha_hC_EC_m^2C_h + 3C_EC_mC_\theta C_h + C_mC_hC_\theta(4C_E + C_m), \tag{20}$$

$$D = C_h^2C_\theta\alpha_m(C_m - 2C_E), \tag{21}$$

and

$$E_1 = 3C_EC_m^2C_h^2C_\theta. \tag{22}$$

In Eq. (17), when Ri is zero, μ is also zero; thus σ_w, although indeterminate in Eq. (10), is not necessarily zero in neutral conditions. The solution of (17) reveals that μ always has the same sign as Ri. Further, μ becomes infinitely positive, as the Richardson number approaches a limiting value, called the critical Richardson number (Ric) and its value is given by (Blackadar 1979)

$$\text{Ric} = \frac{B}{A}. \tag{23}$$

Notice that the product of the roots of the quadratic equation in μ (17) is $E_1\text{Ri}/(A\text{Ri} - B)$, which has the negative sign as $(A\text{Ri} - B) < 0$ because Ri $\leq$ Ric in stable conditions. This implies that the roots of (17) have the opposite signs. Here, we are primarily interested in a positive root and it is given by (appendix)

$$\mu = \frac{E_1\text{Ri}}{D - C\text{Ri}}. \tag{24}$$

Equating the expressions for μ in (24) and (10) we get

$$\sigma_w^2 = \left(\frac{D - C\text{Ri}}{E_1}\right)\ell^2s^2. \tag{25}$$

Eliminating C, D, and E_1 in relations (18)–(23), we get

$$\sigma_w = \alpha\sqrt{1 - \frac{\beta\text{Ri}}{\text{Ric}}}\ell S, \tag{26}$$

where

$$\alpha = \sqrt{\frac{\alpha_m(C_m - 2C_E)}{3C_EC_m^2}} \quad \text{and}$$

$$\beta = \left\{\frac{1}{3C_EC_hC_\theta C_m} + \frac{1}{C_mC_h[3\alpha_hC_EC_m + C_\theta(4C_E + C_m)]}\right\} \times \left[\frac{\alpha_m}{3C_EC_m^2(\alpha_h\alpha_mC_h - C_\theta)}\right]^{-1}.$$

Here, both α and β depend on too many constants. Although these constants are known (Blackadar 1979), their exact values are questionable above the surface layer. However, a better estimate of these constants can be obtained on the basis of the known values of σ_w at the upper and lower boundaries.

At the upper boundary as Ri $\rightarrow$ Ric, σ_w/U_* vanishes. This implies that $\beta = 1.0$ in Eq. (26).

Similarly, under the neutral conditions, Ri $\rightarrow$ 0, $\ell S/U_* = \phi_m = 1$ and $\sigma_w/U_* = 1.3$ (Merry and Panofsky 1976). Using these conditions in (26) we find that $\alpha = 1.3$.

3. Boundary layer model

A one-dimensional version of Pielke's mesoscale model (McNider et al. 1988; Sharan et al. 1995) has been employed to study the behavior of σ_w in the SBL. The model was originally developed for a meso-β scale by Pielke (1974) and modified by Mahrer and Pielke (1977) and McNider and Pielke (1981). Some of the major aspects of the model are discussed here.

a. Planetary boundary layer

The PBL is composed of two layers, namely, the surface layer near the ground and the Ekman layer above it. In the surface layer, the surface fluxes of momentum, heat and moisture, and the turbulent exchange coefficients are calculated based on similarity approach proposed by Businger et al. (1971). The turbulent exchange coefficients are parameterized in the SBL on the basis of the local approach in terms of gradient Richardson number (Blackadar 1979).

b. Radiative and surface forcings

The longwave cooling (warming) is calculated on the basis of the radiative transfer equation simplified by the use of isothermal approximation proposed by Sasamori (1972). The contributions of carbon dioxide and water vapor are included in the computation of the longwave cooling (warming).

c. Model input parameters

A geostrophic wind of 10 m s^{-1} was imposed in this study. A neutral Ekman profile was generated within

TABLE 1. List of input parameters.

List of input parameters	Values
Albedo	0.20
Roughness length	10.0 cm
Latitude	40.0°
Day of (Julian) year	067
Surface pressure	987.0 mb
Surface temperature	290.0 K
Surface cooling rate	0.8 K h^{-1}
Surface specific humidity	27.3 (g kg^{-1})
Mean wind speed	10 m s^{-1}
Mean wind direction	105°
Geostrophic wind shear	0.00 s^{-1}
Time step	30 s
Simulation start time	At sunset
Total simulation time	14.0 h
Number of levels	22
Vertical levels	5, 15, 35, 65, 105, 155, 205, 275, 355, 445, 554, 655, 755, 905, 1045, 1195, 1355, 1525, 1705, 1895, 2095 and 2305 m

TABLE 2. List of the σ_w formulations.

Formulation number	Details of the formulation	Equation for σ_w
1	Proposed	$\sigma_w = \alpha\left(1 - \frac{\text{Ri}}{\text{Ric}}\right)^{1/2} \ell S$ (i) $\alpha = 1.3$ (theoretical) and (ii) $\alpha = 1.6$ (suggested)
2	Nieuwstadt (1984)	$\frac{\sigma_w}{U_*} = 1.4\left(1 - \frac{Z}{H}\right)^{3/4}$
3	McNider et al. (1988)	$\sigma_w = 1.2\left(1 - \frac{\text{Ri}}{\text{Ric}}\right)^{0.58} \ell S$
4	Hanna (1984)	$\frac{\sigma_w}{U_*} = 1.3\left(1 - \frac{Z}{H}\right)$

the PBL that was initially assumed to extend up to 1.2 km. Above this height, an inversion was introduced by increasing the potential temperature. This corresponds to a well-mixed ideal PBL at sunset. A constant profile of specific humidity was assumed within the PBL. Above the PBL, the specific humidity was decreased rapidly. The initial profiles of potential temperature and specific humidity have been simplified to minimize the number of variables on which the cooling due to radiation may depend. Finally, the surface was cooled at a constant rate of 0.8 K h^{-1} in all the cases. The simulation was started at sunset and the model was run for 12 h. The values of input parameters used for the simulation are given in Table 1.

4. Numerical experiments and results

Four different formulations (Table 2) for σ_w were incorporated in the one-dimensional boundary layer model (Sharan et al. 1995; Gopalakrishnan et al. 1998), and the performance of each of the formulations were tested against Minnesota observations (Caughey et al. 1979).

Figure 1 depicts the profiles of $(\sigma_w/U_*)^2$ plotted against Z/H computed using these formulations. The empirical relationship deduced by McNider et al. (1988) exhibits the most rapid decay with height. Also, there is a substantial difference between the modeled profiles and the observations. The value of α in the above relationship is 1.2 (Table 2). However, observations in the surface layer show that $\sigma_w/U_* = 1.3$ (Merry and Panofsky 1976). Hanna (1984) proposed a relationship (Table 2) that agrees well with the surface observations. However, this relationship also shows a rapid decay with height in the SBL. Also, such a relationship explicitly depends on the height of the SBL. The formulation for σ_w, deduced systematically on the basis of level 2 turbulence closure theory [Eq. (26) with $\beta = 1$], for $\alpha =$ 1.3, is able to produce better results, especially in the upper SBL. However, the difference is only marginal.

Although the surface layer observations show that (σ_w/U_*) is 1.3 in neutral conditions or when $Z \rightarrow 0$, both the Minnesota as well as the Cabauw observations show relatively high turbulent activity very near the surface. Taking into consideration the higher energies of the small eddies near the surface, we incorporated a larger value of α (=1.6) in Eq. (26). As depicted in Fig. 1, such a modification is able to simulate the behavior of σ_w closer to the observations in the SBL. In fact, this

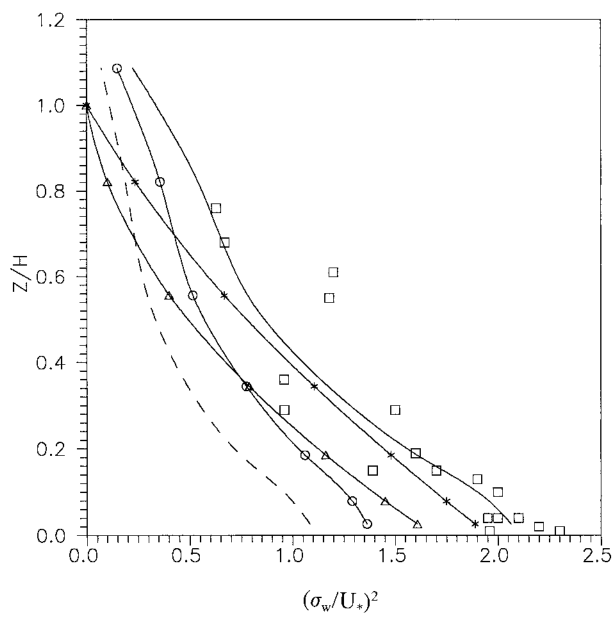

FIG. 1. Plot of $(\sigma_w/U_*)^2$ as simulated by the various parameterization schemes in the SBL compared against the Minnesota observations (Caughey et al. 1979). Observations (□), Nieuwstadt (★———★), Hanna (△———△), proposed formulation with α = 1.6 (———), proposed formulation with α = 1.3 (○———○), and McNider et al. (- - - -).

proposed formulation (Table 2) is even able to capture the essential features of turbulence in the upper SBL, which is locally defined by small eddies. However, more observations may be required to strengthen this conclusion.

The behavior of σ_w as modeled using the formulation proposed by Nieuwstadt (1984) is nearly the same as with the proposed formulation for $\alpha = 1.3$ (Fig. 1) except near the top of the turbulent boundary layer. For the sake of uniformity, the height of the SBL is computed from the relationship used by Nieuwstadt (1984)

$$H = 0.37(U_* L/f)^{1/2}, \tag{27}$$

where L is Monin–Obukhov length. This formula was first proposed by Zilitinkevich (1972). The formulations in terms of Z/H vanish at $Z = H$; however, Ri continues to be less than Ric, resulting in a nonzero value of σ_w/U_* at $Z = H$ in the formulations 1 and 3 in Table 2, which are the functions of Ri/Ric.

It should be noted that although the formulation proposed by Nieuwstadt (1984) is able to simulate the behavior of σ_w in the SBL well, the new formulation does not explicitly depend on the height of the nocturnal boundary layer, which makes it different from the former.

5. Conclusions

A new formulation for σ_w in the SBL is proposed. The formulation is local and based on the level 2 approximation of Mellor and Yamada (1974). The proposed formulation is able to describe the nondimensional profile of $(\sigma_w/U_*)^2$ against Z/H consistently when compared with the Minnesota observations. The discrepancy between the proposed formulation and the observations in the surface layer in near-neutral conditions or $Z \to 0$ still exists. The formulation needs to be validated in a weak wind SBL (Gopalakrishnan et al. 1998; Sharan and Gopalakrishnan 1997) as and when data become available.

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APPENDIX

Positive Root of Eq. (17)

In stable conditions, since $\mathrm{Ri} \le \mathrm{Ric} < 1$, we can expand the solution μ of Eq. (17) as a power series of Ri

$$\mu = \mu_1 \mathrm{Ri} + \mu_2 \mathrm{Ri}^2 + \cdots, \tag{A1}$$

where μ_1 and μ_2, $\cdots$ are the coefficients of order one. The term μ_0 in the expansion (A1) of μ is zero in view of the fact that μ vanishes as Ri approaches to zero and so the leading term will be of $O(\mathrm{Ri})$.

Substituting the expansion of μ from Eq. (A1) in Eq. (17) and equating the coefficients of Ri, Ri^2, . . . , we obtain the following relations:

$$O(\mathrm{Ri}): \qquad -D\mu_1 + E_1 = 0 \tag{A2}$$

and

$$O(\mathrm{Ri}^2): \quad -B\mu_1^2 + \mu_1 C - D\mu_2 = 0. \tag{A3}$$

From (A2), we obtain

$$\mu_1 = \frac{E_1}{D}. \tag{A4}$$

Using the numerical values of the parameters appearing in the expressions (18)–(22) for A, B, C, D, and E, from Blackadar (1979), we find, for example,

$$A = 1.49, \qquad B = 0.002, \qquad C = 6.6,$$
$$D = 1.2, \quad \text{and} \quad E_1 = 4.45 \quad \text{for } \alpha_h = 0.437.$$

It may be noted that the magnitude of B is at least one order lower than that of the parameters A, C, and D. In Eq. (A3), μ_1 and μ_2 are of order one and so the contribution of the term $B\mu_1^2$ will be at least one order lower than the other two terms and, thus, can be neglected. Accordingly from Eq. (A3), we have

$$\mu_2 = \frac{C}{D}\mu_1 = \frac{C}{D^2}E_1. \tag{A5}$$

Putting the values of μ_1 and μ_2 from Eqs. (A4) and (A5) in (A1), we get

$$\mu = \frac{E_1}{D}\mathrm{Ri} + \frac{CE_1}{D^2}\mathrm{Ri}^2 + O(\mathrm{Ri}^3), \tag{A6}$$

where $O(\mathrm{Ri}^3)$ includes the term of Ri^3 and its higher powers. The solution (A6) of Eq. (17) has the accuracy of order Ri^2.

The above analysis shows that in Eq. (17), the coefficient of μ^2 does not contribute up to $O(\mathrm{Ri}^2)$ and so for this order of accuracy, Eq. (17) reduces to

$$(C\mathrm{Ri} - D)\mu + E_1\mathrm{Ri} = 0, \tag{A7}$$

from which we obtain

$$\mu = \frac{E_1\mathrm{Ri}}{(D - C\mathrm{Ri})}. \tag{A8}$$

It may be noted that (A6) can be derived from (A8) by expanding its denominator in the powers of Ri.

REFERENCES

Andre, J. C., and L. Mahrt, 1982: The nocturnal surface inversion and influence of clear-air radiational cooling. *J. Atmos. Sci.*, **39,** 864–878.

——, G. DeMoor, P. Lacarrere, G. Therry, and R. DuVachat, 1978: Modeling the 24-hour evolution of the mean and turbulent structure of the planetary boundary layer. *J. Atmos. Sci.*, **35,** 1861–1883.

Arya, S. P. S., 1984: Parametric relations for the atmospheric boundary layer. *Bound.-Layer Meteor.*, **30,** 57–73.

Blackadar, A. K., 1962: The vertical distribution of wind and turbulent exchange in neutral atmosphere. *J. Geophys. Res.,* **67,** 3095–3102.

——, 1979: High resolution models of the planetary boundary layer. *Advances in Environmental and Scientific Engineering,* Vol. I, Gordon and Breach.

Businger, J. A., J. C. Wyngaard, Y. Izumi, and E. F. Bradley, 1971: Flux-profile relationships in the atmospheric surface layer. *J. Atmos. Sci.,* **28,** 181–189.

Caughey, S. J., 1984: Observed characteristics of the atmospheric boundary layer. *Atmospheric Turbulence and Air Pollution Modelling,* F. T. M. Nieuwstadt and H. Van Dop, Eds., D. Reidel Publishing, 107–158.

——, J. C. Wyngaard, and J. C. Kaimal, 1979: Turbulence in the evolving stable boundary layer. *J. Atmos. Sci.,* **36,** 1041–1052.

Deardorff, J. W., 1970: A three-dimensional numerical investigation of the idealized planetary boundary layer. *Geophys. Fluid Dyn.,* **1,** 377–410.

Gopalakrishnan, S. G., M. Sharan, R. T. McNider, and M. P. Singh, 1998: Study of radiative and turbulent processes in the stable boundary layer under weak wind conditions. *J. Atmos. Sci.,* **55,** 954–960.

Hanna, S., 1984: Applications in air pollution modeling. *Atmospheric Turbulence and Air Pollution Modeling,* F. T. M. Nieuwstadt and H. Van Dop, Eds., D. Reidel Publishing, 275–310.

Hicks, B. B., 1981: An examination of turbulence statistics in the surface boundary layer. *Bound.-Layer Meteor.,* **21,** 389–402.

Kaimal, J. C., J. C., Wyngaard, D. A., Hauger, O. R., Cote, Y. Izumi, S. J., Caughey and C. J. Readings, 1976: Turbulence structure in the convective boundary layer. *J. Atmos. Sci.,* **33,** 2152–2169.

Mahrer, Y., and R. A. Pielke, 1977: A numerical study of airflow over irregular terrain. *Beitr. Phys. Atmos.,* **50,** 98–113.

McBean, G. A., 1971: The variations of the statistics of wind, temperature and humidity fluctuations with stability. *Bound.-Layer Meteor.,* **1,** 438–457.

McNider, R. T., 1981: Investigation of the impact of topographic circulations on the transport and dispersion of air pollutants. Ph.D. dissertation, University of Virginia, 210 pp. [Available from University of Virginia, Department of Environmental Sciences, Charlottesville, VA 22903.]

——, and R. A. Pielke, 1981: Diurnal boundary layer development over sloping terrain. *J. Atmos. Sci.,* **38,** 2198–2212.

——, M. D. Moran, and R. A. Pielke, 1988: Influence of diurnal and inertial boundary layer oscillations on long-range dispersion. *Atmos. Environ.,* **22,** 2445–2462.

Mellor, G. L., and T. Yamada, 1974: A hierarchy of turbulence closure models for planetary boundary layers. *J. Atmos. Sci.,* **31,** 1791–1806.

Merry, M., and H. A. Panofsky, 1976: Statistics of vertical motion over land water. *Quart. J. Roy. Meteor. Soc.,* **102,** 255–260.

Nieuwstadt, F. T. M., 1984: The turbulent structure of the stable nocturnal boundary layer. *J. Atmos. Sci.,* **41,** 2202–2216.

——, 1985: Some aspects of the turbulent stable boundary layer. *Bound.-Layer Meteor.,* **30,** 31–56.

Panofsky, H. A., H. Tennekes, D. H. Lenschow, and J. C. Wyngaard, 1977: The characteristics of turbulent velocity components in the surface layer under convective conditions. *Bound.-Layer Meteor.,* **11,** 355–361.

Pielke, R. A., 1974: A three-dimensional numerical model of the sea breezes over south Florida. *Mon. Wea. Rev.,* **102,** 115–139.

Rotta, J. C., 1951: Statistische theorie nichthomogener turbulenz. *Z. Phys.,* **129,** 547–572.

Sasamori, T., 1972: A linear harmonic analysis of atmospheric motion with radiative dissipation. *J. Meteor. Soc. Japan,* **50,** 505–518.

Sharan, M., and S. G. Gopalakrishnan, 1997: Comparative evaluation of eddy exchange coefficients for strong and weak wind stable boundary layer modeling. *J. Appl. Meteor.,* **36,** 545–559.

——, R. T. McNider, S. G. Gopalakrishnan, and M. P. Singh, 1995: Bhopal gas leak: A numerical simulation of episodic dispersion. *Atmos. Environ.,* **29,** 2061–2070.

Sorbjan, Z., 1986: On similarity in the atmospheric boundary layer. *Bound.-Layer Meteor.,* **34,** 377–397.

Wyngaard, J. C., 1975: Modelling the planetary boundary layer: Extension to the stable case. *Bound.-Layer Meteor.,* **9,** 441–460.

Yamada, T., 1979: PBL similarity profiles determined from level-2 turbulence closure model. *Bound.-Layer Meteor.,* **17,** 333–351.

Zilitinkevich, S. S., 1972: On the determination of the height of the Ekman boundary layer. *Bound.-Layer Meteor.,* **3,** 141–145.