A Simplified Equation for Paulson's $\Psi_m(Z/L)$ Formulation for Overwater Applications

S. A. HSU AND B. W. BLANCHARD

Coastal Studies Institute, Louisiana State University, Baton Rouge, Louisiana

ZHONGDE YAN

Department of Mathematics, Southern University and A&M College, Baton Rouge, Louisiana

25 April 1998 and 18 September 1998

ABSTRACT

The mathematical formulation of Paulson's $\Psi_m(Z/L)$ that applies to the unstable atmospheric boundary layer has been simplified. The authors propose that $\Psi_m(Z/L) = a(-Z/L)^b$, where the coefficients a and b have been determined. Based on data provided in Panofsky and Dutton, $a = 1.0496$ and $b = 0.4591$. The correlation coefficient between $\Psi_m(Z/L)$ and $(-Z/L)$ is 0.99, so this equation can directly account for $(0.99)^2 = 98\%$ of the variation in $\Psi_m(Z/L)$. Comparisons between Paulson's and this proposed formula show that the difference between the two is negligible for overwater applications.

1. Introduction

In the atmospheric surface layer, the diabatic wind profile is described by (e.g., Panofsky and Dutton 1984, 134)

$$U_Z = \frac{u_*}{\kappa}\left[\ln\frac{Z}{Z_0} - \Psi_m\left(\frac{Z}{L}\right)\right], \tag{1}$$

where U_Z is the wind speed at height Z, u_* is the friction velocity, κ is the von Kármán constant, Z_0 is the aerodynamic roughness length, and L is the Monin–Obukhov stability length.

Under unstable conditions, Ψ_m has the form (Paulson 1970)

$$\Psi_m\left(\frac{Z}{L}\right) = \ln\left[\left(\frac{1+x^2}{2}\right)\left(\frac{1+x}{2}\right)^2\right] - 2\tan^{-1}x + \frac{\pi}{2}, \tag{2}$$

where $x = [1 - 16(Z/L)]^{1/4}$ and under stable conditions (Panofsky and Dutton 1984, 136),

$$\Psi_m\left(\frac{Z}{L}\right) = -5\frac{Z}{L}. \tag{3}$$

In the offshore environment, such as over the Gulf of Mexico, daily operations occurring on many tall platforms (>50 m) require wind-loading estimations. If the platform does not have in situ wind measurements, the wind information is generally extrapolated from buoy measurements at 5–10 m above the sea surface. The purpose of this research note is to provide offshore technicians with a quick and accurate alternative to Eq. (2) for estimating the wind shear. A comparison between shear values derived from Eq. (2) and those from our simplified formula will also be made.

2. The simplified formula

We propose that, under unstable conditions,

$$\Psi_m\left(\frac{Z}{L}\right) = a\left(-\frac{Z}{L}\right)^b, \tag{4}$$

where a and b are coefficients to be obtained from the values of $(-Z/L)$ versus Ψ_m (Businger), as provided by Panofsky and Dutton (1984, 135–136). Our results (see Fig. 1) show that

$$\Psi_m\left(\frac{Z}{L}\right) = 1.0496\left(-\frac{Z}{L}\right)^{0.4591}. \tag{5}$$

Since the correlation coefficient is 0.9912, Eq. (5) can directly account for $(0.9912)^2 = 98\%$ of the variability in $\Psi_m(Z/L)$. Because the total system accuracy for wind speed from data buoys is about ±10% (National Data Buoy Center 1990), our Eq. (5) is well within the measurement accuracy and thus should be useful since it is much easier to compute with a calculator by field operators.

Corresponding author address: Dr. S. A. Hsu, Coastal Studies Institute, Howe-Russell Geoscience Complex, Louisiana State University, Baton Rouge, LA 70803.
E-mail: sahsu@antares.esl.lsu.edu.

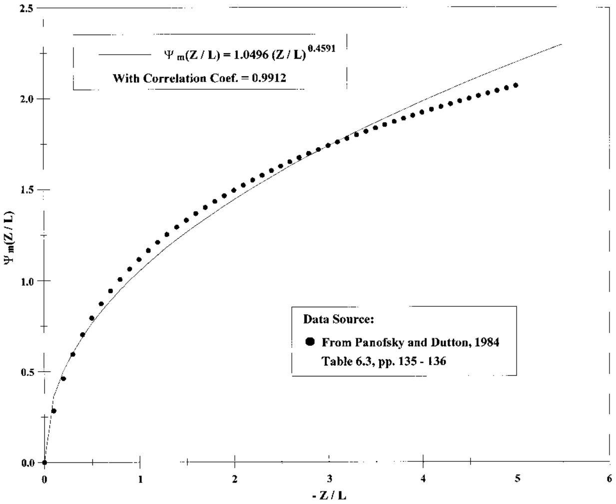

FIG. 1. Determination of coefficients *a* and *b* for Eq. (4).

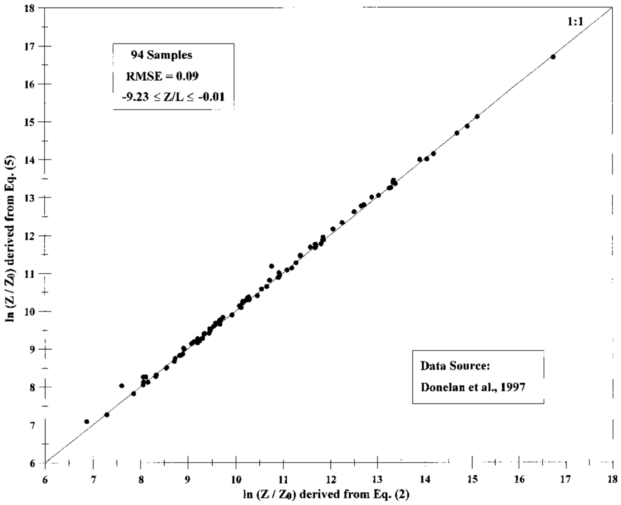

FIG. 2. A comparison between Eqs. (2) and (5) for determining the aerodynamic roughness length, Z_0.

3. A comparison between Eqs. (2) and (5)

In order to compare Eqs. (2) and (5), we rearrange Eq. (1) as follows:

$$\ln\frac{Z}{Z_0} = \frac{\kappa U_Z}{u_*} + \Psi_m\left(\frac{Z}{L}\right). \tag{6}$$

The dataset used for this comparison is provided by Donelan et al. (1997). Since U_Z is measured at 12 m above the mean sea surface, we set $Z = 12$ m. The von Kármán constant κ is taken to be 0.4 (Donelan et al. 1997).

Figure 2 shows our results. Note that Z/L ranges between -0.01 and -9.23 and that the rmse is only 0.09 for the range of $\ln(Z/Z_0)$ between approximately 7 and 17. Thus the difference between Eqs. (2) and (5) is almost negligible so that for offshore applications, Eq. (5) should be useful.

4. Further comparison between Eqs. (2) and (5) for operational use

Most of the time for offshore applications, the shear velocity u_* is needed. For example, the wind-drift sea surface velocity, $u_s \approx 0.55u_*$ (Wu 1975; Garratt 1992, 98). To estimate u_* one may ‘‘bypass’’ Z_0 by using the drag coefficient formulation

$$C_{\mathrm{DN}} = \left(\frac{u_*}{U_{10N}}\right)^2, \tag{7}$$

where C_{DN} is the near-neutral drag coefficient and U_{10N} the near-neutral wind speed at 10 m. Since under near-neutral conditions, $-L \to \infty$ so that $Z/L \to 0$. Thus the limit of Eq. (1) when $\Psi_m(Z/L) \to 0$ is

$$\frac{U_Z\kappa}{u_*} = \ln\frac{Z}{Z_0} \tag{8}$$

and

$$\therefore \quad \frac{\kappa}{\sqrt{C_{\mathrm{DN}}}} = \ln\frac{Z}{Z_0}. \tag{9}$$

Then Eq. (1) becomes

$$u_* = \kappa U_Z\left[\frac{\kappa}{\sqrt{C_{\mathrm{DN}}}} - \Psi_m\left(\frac{Z}{L}\right)\right]^{-1}, \tag{10}$$

where C_{DN} is obtained from Garratt (1992, 101) as follows:

$$C_{\mathrm{DN}} = (0.75 + 0.067U_{10N})10^{-3}. \tag{11}$$

Since values of U_{10N} are also provided by Donelan et

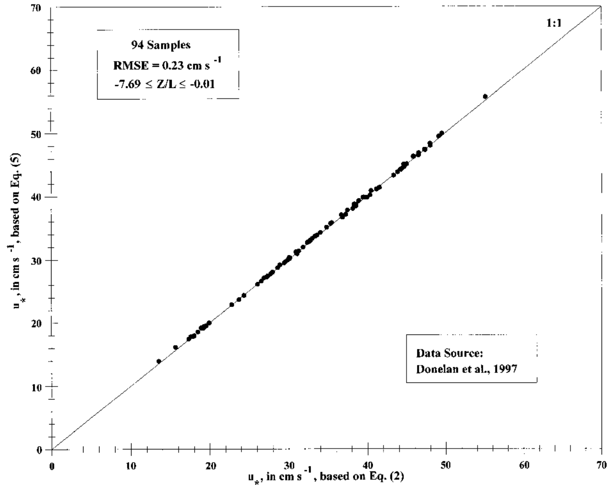

FIG. 3. A comparison between Eqs. (2) and (5) for determining the shear velocity, u_*.

al. (1997), Eq. (10) is used to compare the difference between Eqs. (2) and (5) (setting $Z = 10$ m and $U_Z = U_{10N}$). Our results are shown in Fig. 3. The rmse here is only 0.23 cm s^{-1} for the u_* range between about 10 and 60 cm s^{-1}, therefore we can say that Eq. (5) is nearly identical to Eq. (2) and thus it is applicable for operational use.

5. Conclusions

The mathematical representation of $\Psi_m(Z/L)$ used for the wind speed profile in the unstable atmospheric surface layer originally developed by Paulson in 1970 has been simplified. Comparisons between Paulson's [shown as Eq. (2)] and our simplified formula [shown as Eq. (5)] indicate that, for overwater aerodynamic roughness length and shear velocity determination, their difference is negligible for offshore applications.

REFERENCES

Donelan, M. A., W. M. Drennan, and K. B. Katsaros, 1997: The air–sea momentum flux in conditions of wind sea and swell. *J. Phys. Oceanogr.*, **27,** 2087–2099.

Garratt, J. R., 1992: *The Atmospheric Boundary Layer.* Cambridge University Press, 316 pp.

National Data Buoy Center, 1990: Climatic summaries for NDBC buoys and stations, Update 1, 454 pp.

Panofsky, H. A., and J. A., Dutton, 1984: *Atmospheric Turbulence.* Wiley, 397 pp.

Paulson, C. A., 1970: The mathematical representation of wind speed and temperature profiles in the unstable atmospheric surface layer. *J. Appl. Meteor.*, **9,** 857–861.

Wu, J., 1975: Wind-induced drift currents. *J. Fluid Mech.*, **68,** 49–70.