Reflectivity, Rain Rate, and Kinetic Energy Flux Relationships Based on Raindrop Spectra

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ABSTRACT

The relationships between radar reflectivity factor $Z$, rainfall rate $R$, and rainfall kinetic energy flux $E$ were analyzed based on a multiyear raindrop spectra dataset recorded by a Joss-Waldvogel disdrometer in the Goodwin Creek research watershed in northern Mississippi. Particular attention was given to the climatological variability of the relationships and the uncertainty by which one rainfall parameter may be estimated from another. Substantial variability for the coefficients of a power-law relationship $Y = A_b X^b$ between two rainfall parameters $Y$ and $X$ (where $Y$ and $X$ may stand for any paired combination of $Z$, $R$, and $E$) was found. The variability of the exponent $b$, however, was small enough to support approaches of climatologically fixed exponents to simplify radar rainfall estimation procedures. The multiplicative factor $A_b$ should typically be adjusted on a storm basis. The uncertainty of the estimation of one rainfall parameter from another, being a function of the difference in weighting of the drop size by the two parameters and the variability of raindrop spectra, was found to be approximately 50% for the $Z-R$ relation, 40% for the $E-R$ relation, and 25% for the $Z-E$ relation. For extreme precipitation intensities ($R \geq 100 \text{ mm h}^{-1}$), this drop spectra-based uncertainty reduced to approximately 20% for all three relationships. The results exhibited significant sensitivity to the choice of method applied to determine the relationship between two rainfall parameters. Appreciable sensitivity of the relationship between rainfall parameters (i.e., power-law coefficients and drop spectra-based uncertainty) to the number of raindrops registered per 1-min drop spectrum was also found.

1. Introduction

Hydrological processes at the land surface, such as infiltration, runoff, and soil erosion are driven by the rainfall rate and rainfall kinetic energy flux at small spatial and temporal scales (e.g., Dunne and Leopold 1978, chapters 6, 9, and 15; Brandt and Thornes 1987). Past efforts in monitoring these quantities and their spatial and temporal variability have been based largely on rain gauge observations. Networks of radar and rain gauges, possibly including raindrop spectrometers, have been advocated and used for decades to monitor rainfall rate (e.g., Wilson 1970; Harrold et al. 1974; Wilson and Brandes 1979; Collier et al. 1983; Collier 1986; Austin 1987; Joss and Waldvogel 1990). Recently Smith and DeVeaux (1992), Steiner (1992), and Steiner et al. (1997) pointed out the great potential of radar to concurrently monitor the rainfall kinetic energy flux in time and space. The radar, with its large area coverage ($10^4 \text{ km}^2$) at high spatial (1–2 km) and temporal (5–10 min) resolution, serves to fill the (mostly significant) obser-
measurements, differences in the space and time resolution, overlap, and area coverage of observations, could be substantial enough that rainfall estimated by radar and rain gauges may appear comparable by chance only. Similar conclusions have been reached by Messaoud and Pointin (1990), Lovejoy and Schertzer (1990), Barnston (1991), Anagnostou and Krajewski (1997), Anagnostou et al. (1999), and Ciach and Krajewski (1999). Measurement caveats are manifold and may be related to instrumental errors and uncertainties, such as calibration, sensitivity and precision, but also to how close the quantity measured by an instrument is related to the parameters of interest. Moreover, in combination with the previously mentioned factors, the weather itself may influence the measurements; for example, strong spatial precipitation intensity gradients in the horizontal and/or vertical direction, or the type and thermodynamic state of precipitation particles may lead to ambiguous observations (e.g., Battan 1973, chapters 7, 9, and 10; Wilson and Brandes 1979; Zawadzki 1984; Austin 1987; Joss and Waldvogel 1990; Doviak and Zrnić 1993, chapter 8).

This study is focused on aspects of the weather. In particular, we examine the uncertainty of relationships between the radar reflectivity factor \( Z \), rainfall rate \( R \), and rainfall kinetic energy flux \( E \), that is caused by the variability of the raindrop size distribution. There are an infinity of raindrop spectra that can produce the same rainfall rate. Moreover, raindrop size distributions can vary significantly among climatic precipitation regimes, from storm to storm, and dramatic changes may occur even within storms (e.g., Fujiwara 1965; Stout and Mueller 1968; Joss and Waldvogel 1970; Battan 1973, chapter 7; Waldvogel 1974; Carbone and Nelson 1978; Austin 1987; Short et al. 1990; Smith and Krajewski 1993; Sempere-Torres et al. 1994; Tokay and Short 1996; Steiner and Smith 1998; Uijlenhoet and Stricker 1999). The relation between any two of the above rainfall parameters, therefore, is not unique and the scatter of the data, in the parameter space spanned by the two rainfall parameters, indicates the raindrop spectra-based uncertainty that is inherent in the estimate of one parameter from the other (see section 2a).

The uncertainty of the \( Z-R \) relation (i.e., scatter of \( R \) for a given \( Z \) in the \( Z-R \) parameter space), caused by raindrop spectra variability, has received substantial attention in the past. This has been based on a strong interest in the use of radar for rainfall measurements, but also because this scatter could be described from raindrop spectra observations. For example, Atlas (1984) summarized that using radar reflectivity to estimate rain rate would consistently lead to uncertainties on the order of 40%–50% and larger, when assuming a given \( Z-R \) relationship. But even for favorable conditions (i.e., precise and accurate measurements and a priori known relationship between \( Z \) and \( R \)), the raindrop spectra-based uncertainties may exceed 30% (e.g., Ulbrich and Atlas 1984).

The relationship between the rainfall rate \( R \) and kinetic energy flux \( E \) has been investigated from a soil-erosional perspective of estimating kinetic energy based on rainfall intensity observations (e.g., Wischmeier and Smith 1958, 1965; Brown and Foster 1987; Renard and Freimund 1994; McGregor et al. 1995). Based on raindrop spectra observations made at different locations around the world, Carter et al. (1974), Kinnell (1981), Rosewell (1986), Smith and DeVeaux (1992), Sempere-Torres et al. (1992), and Cerro et al. (1998) discussed the \( E-R \) relationship and compared their results to those obtained by earlier studies. However, the uncertainty of this relationship to variations of raindrop spectra has not been treated in systematic and quantitative ways hitherto.

The relationship between the kinetic energy flux \( E \) and radar reflectivity factor \( Z \) has received particular attention from a radar measurement perspective of the kinetic energy of hailfall (Waldvogel et al. 1978a,b; Waldvogel and Schmid 1982; Ulbrich 1978). For rain, results of preliminary investigations of the \( Z-E \) relationship have been reported by Steiner and Waldvogel (1989), Smith and DeVeaux (1992), and Steiner et al. (1997). Similarly to the \( E-R \) relation, the uncertainty of the \( Z-E \) relationship to variations of raindrop spectra remains to be quantified.

Hence, the purpose of this study is to present a systematic and quantitative treatment of the relationships between radar reflectivity factor \( Z \), rainfall rate \( R \), and rainfall kinetic energy flux \( E \), and the related uncertainty that is caused by the natural variability of raindrop size distributions. Comparison of the \( Z-R \), \( Z-E \), and \( E-R \) relationships on a common and extensive dataset of raindrop spectra will highlight advantages and disadvantages for using each of these relationships and enable objective assessment of the relative scatter of data within the corresponding parameter spaces (section 3a). In addition, the sensitivity to differences in data processing (section 3b) and to the drop sample size (section 3c) is explored.

2. Data and analysis procedures

\subsection*{a. Data and basic theory}

This study is based on data recorded by a raindrop spectrometer of the Joss–Waldvogel RD-69 type (i.e., disdrometer; Joss and Waldvogel 1967) deployed in the center of the Goodwin Creek research watershed in Panola County, northern Mississippi (McGregor et al. 1995; Alonso 1996; Steiner et al. 1999). The disdrometer is an electro-mechanical instrument that classifies raindrops falling on its sensor according to their momentum into 20 bins ranging in size from 0.3 to 5.5 mm and larger (Joss and Waldvogel 1967; Waldvogel 1974). The observed raindrop size distributions (i.e., number of counts in each bin) were recorded with a time resolution of 1 min. The analyses presented here are based on data
extracted from the disdrometer record collected between March 1996 and June 1998.

Based on the number of drops $n(D_i)$ counted in each bin, the sampling area $F$ (0.005 m$^2$) and time $t$ (60 s), the raindrop size distribution $N(D)$ (m$^{-3}$ mm$^{-1}$)

$$N(D_i) = \frac{n(D_i)}{F n(D_i) \Delta D_i},$$

(1)

can be determined, where $D_i$ is the assumed spherical drop diameter (mm) and $v(D_i)$ is the corresponding terminal fall velocity (m s$^{-1}$); for example, determined by Gunn and Kinzer (1949) for raindrops. Here, $\Delta D_i$ is the diameter interval (mm) corresponding to drop size class $i$. Building on that, the radar reflectivity factor $Z$ (mm$^6$ m$^{-2}$),

$$Z = \sum_{i=1}^{20} N(D_i) D_i^6 \Delta D_i,$$

(2)

the rainfall intensity $R$ (mm h$^{-1}$),

$$R = \frac{6 \pi}{10^4} \sum_{i=1}^{20} N(D_i) D_i^3 v(D_i) \Delta D_i,$$

(3)

and the rainfall kinetic energy flux $E$ (J m$^{-2}$ h$^{-1}$),

$$E = \frac{3 \pi}{10^4} \sum_{i=1}^{20} N(D_i) D_i^3 v(D_i)^3 \Delta D_i,$$

(4)

can be obtained.

Based on using all raindrop spectra of the 1996–98 Goodwin Creek data, Figs. 1–3 show the relationships between $Z$, $R$, and $E$. Each point in these figures corresponds to a 1-min drop spectrum. Scatterplots of the $Z$-$R$, $Z$-$E$, and $E$-$R$ relationships, for any given storm, may exhibit significant variability that is caused by the variability of the raindrop size distribution during the storm. The scatter of the data is a reflection of the drop spectra variability amplified by the different weighting of the drop size by $Z$, $R$, and $E$. The disdrometer’s precision in determining the drop size (<5%; Joss and Waldvogel 1977) can be neglected as a contributing factor to this scatter. At least initially, we also assume that the sample size of drops collected by the disdrometer (Joss and Waldvogel 1969; Gertzman and Atlas 1977) may not affect the scatter of the data in Figs. 1–3. However, we will return to this question in section 3c to test the validity of that assumption.

The $Z$-$R$ (Fig. 1) and $E$-$R$ (Fig. 3) relationships are more sensitive to variations of the raindrop size distribution than the $Z$-$E$ relation (Fig. 2). This may be explained by comparing Eqs. (2) and (4): assuming proportionality between the raindrop fall velocity and the drop diameter, $v(D) \propto D$, the kinetic energy flux is proportional to the drop diameter to the power of six, $E \propto D^6$, which in essence is the radar reflectivity factor

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**Fig. 1.** Scatterplot of the 1996–98 Goodwin Creek raindrop spectra data within the $Z$-$R$ parameter space and boundaries, based on monodisperse raindrop size distributions. See section 2a for details.

**Fig. 2.** Scatterplot of raindrop spectra data within the $Z$-$E$ parameter space, similar to Fig. 1.

**Fig. 3.** Scatterplot of raindrop spectra data within the $E$-$R$ parameter space, similar to Fig. 1.
Z. In reality, the particle fall speed is not proportional to the diameter; for example, Atlas and Ulbrich (1977) suggest \( v(D) \) to be proportional to the drop diameter to a power of 0.67, which renders \( E \approx D^3 \). Nonetheless, the radar reflectivity factor is an effective proxy measure of the rainfall kinetic energy flux.

The scatter of the data in Figs. 1–3 is constrained by monodisperse raindrop size distributions

\[
n(D) = \begin{cases} 
  n_m & \text{for } D = D_m \\
  0 & \text{for } D \neq D_m 
\end{cases} 
\]  

in basically two ways:

1. Combinations of Eqs. (1), (2), (3), (4), (5), and using a cloud/raindrop-boundary drop size of \( D_m = 0.2 \) mm and \( v(D_m) = 0.72 \) m s\(^{-1} \) (McDonald 1958; Rogers 1979, p. 57), yield

\[
Z = \frac{10^4 D_m}{6\pi v(D_m)} R = 5.89R, 
\]

\[
Z = \frac{10^4 D_m^3}{3\pi v(D_m)^3} E = 22.7E, \quad \text{and} 
\]

\[
E = \frac{v(D_m)^2}{2} R = 0.26R. 
\]

These relationships are shown in Figs. 1–3 by the dotted lines. However, \( D_m = 0.35 \) mm and \( v(D_m) = 1.40 \) m s\(^{-1} \) are the mean drop diameter and corresponding fall velocity of the disdrometer’s smallest drop size bin (Joss and Waldvogel 1967; Waldvogel 1974), which is indicated by the solid line. This boundary is independent of the integration time and drop count, but dependent on the minimum drop diameter sensed by an instrument and the corresponding fall speed.

2. The other boundary is spanned by monodisperse raindrop spectra made up by a single drop of varying size (see also Fig. 1 of Doelling et al. 1998). The dashed lines shown in Figs. 1–3 are (hand-fitted) power-law approximations of that boundary, based on one raindrop of size \( D_m \) recorded per min by the disdrometer (i.e., \( n_m = 1 \)). This boundary is a function of the fall velocity with drop size and the sampling time.

The gray-shaded area in Figs. 1–3, therefore, outlines the domain of possible (disdrometer) raindrop spectrum-derived values within a given parameter space. At low precipitation intensities, the observed variability of raindrop spectra encompasses the range spanned by the monodisperse drop spectra. However, for moderate, and particularly for high intensities, dynamical (e.g., size sorting due to up- and downdrafts or wind shear) and microphysical processes (e.g., coalescence and breakup, or evaporation) constrain the naturally occurring raindrop spectra and thus the parameter values. This apparent limitation suggests a maximum rain rate in cloud systems. According to the envelope relationship for maximum rain rates as a function of duration provided by Linsley et al. (1982, p. 80), the maximum 1-min rainfall intensity could be beyond a stunning 3700 mm h\(^{-1} \). Barcelo et al. (1997) provide a recent compilation of record rainfall values for time intervals from 1 min to 2 yr.

A characterization of the raindrop size distribution of intense rainfall has been attempted by several investigators; for example, Blanchard and Spencer (1970) found that as the rain intensity exceeded 100 mm h\(^{-1} \), the total number of drops continued to increase rather than the drop size, which was an indication of breakup limiting the maximum drop size. Similar results were obtained by Srivastava (1971), List et al. (1987), and Willis and Tattelman (1989). Breakup may be caused either by drops reaching hydrodynamic instability or by collision among each other. For high rain rates, collisions are likely the dominant cause for breakup because the drop interactions increase approximately with the square of the rain rate (Srivastava 1988; Rogers 1989; McFarquhar and List 1991).

We are interested in characterizing the uncertainty of relationships between rainfall parameters that is caused by the variability of raindrop spectra, and how this uncertainty may vary from storm to storm, and possibly even within storms. Therefore, rainfall periods (i.e., storms) were determined based on a duration of the rain rate exceeding 0.1 mm h\(^{-1} \). Rainfall periods separated in time by nonraining periods (\( R < 0.1 \) mm h\(^{-1} \)) lasting less than a maximum of 6 h were combined into a single storm. The resulting discrete storm data comprises 102 rainfall periods that satisfied the requirement of a minimum accumulation of 2.5 mm (approximately one tenth of an inch) of rain. Table 1 and Fig. 4 summarize these data.

The 102 rainfall periods of the 1996–98 Goodwin Creek data comprise approximately 85 000 1-min raindrop spectra that accumulated more than 2200 mm of rain and 46 kJ m\(^{-2} \) of kinetic energy (Table 1). Sixty-four thousand of these spectra are “real” size distributions in that at least one raindrop was recorded during the corresponding 1-min period. The distributions of the 1-min rain rate and kinetic energy flux values of these data (i.e., excluding the no-raindrop spectra) are shown

<table>
<thead>
<tr>
<th>Rain rate (mm h(^{-1} ))</th>
<th>Drop spectra</th>
<th>Rainfall</th>
<th>Kinetic energy (kJ m(^{-2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>84 890</td>
<td>2215 mm</td>
<td>46 009 J m(^{-2} )</td>
</tr>
<tr>
<td>0.01</td>
<td>64 998 (100%)</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>0.10</td>
<td>51 788 (80.8%)</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>0.30</td>
<td>36 891 (57.6%)</td>
<td>99.5%</td>
<td>99.9%</td>
</tr>
<tr>
<td>0.60</td>
<td>18 759 (29.3%)</td>
<td>94.1%</td>
<td>97.9%</td>
</tr>
<tr>
<td>1.00</td>
<td>2663 (4.2%)</td>
<td>53.2%</td>
<td>64.8%</td>
</tr>
<tr>
<td>2.00</td>
<td>876 (1.4%)</td>
<td>33.0%</td>
<td>43.7%</td>
</tr>
<tr>
<td>10.00</td>
<td>47 (&lt;0.1%)</td>
<td>4.1%</td>
<td>5.7%</td>
</tr>
</tbody>
</table>

Table 1. Number of raindrop spectra and their contribution to the overall rainfall and rainfall kinetic energy accumulation as a function of rain rate for the 1996–98 data (102 rainfall periods) collected by the disdrometer in Goodwin Creek, MS. See section 2a for a discussion of this table.
Fig. 4. Distributions of instantaneous (1-min), positive (a) rainfall rate and (b) kinetic energy flux of the 1996–98 Goodwin Creek disdrometer data, and (c) distributions of rainfall and (d) kinetic energy accumulations, and (e) maximum rainfall rate and (f) maximum kinetic energy flux of the corresponding 102 rainfall periods. See section 2a for details.
in Figs. 4a and 4b. The distributions are almost uniform for rain rates up to 5 mm h\(^{-1}\) and kinetic energy flux values up to 100 J m\(^{-2}\) h\(^{-1}\), followed by a sharply decreasing tail for higher intensities. The 1-min maximum rain rate and kinetic energy flux values of the data are 176 mm h\(^{-1}\) and 5488 J m\(^{-2}\) h\(^{-1}\), respectively. For the rainfall periods investigated, approximately 30% of the raindrop spectra (i.e., those with \(R \geq 1.0\) mm h\(^{-1}\)) contributed 94% to the rainfall and 98% to the kinetic energy accumulation (Table 1). Drop spectra with rain rates exceeding one inch per h (\(R \geq 25.4\) mm h\(^{-1}\)), which make up 1.4% of the 64 000 1-min rain observations, contributed 33% to the rainfall and 44% to the kinetic energy accumulation.

The distributions of rainfall and kinetic energy accumulation of the selected 102 rainfall periods are shown in Figs. 4c and 4d, and those of maximum rain rate and maximum kinetic energy flux in Figs. 4e and 4f. The distributions of the rainfall and kinetic energy accumulations are highly skewed: there were many storms with relatively little and only few with major rainfall and kinetic energy accumulations. Approximately three quarters of the rainfall periods resulted in rainfall accumulations of 30 mm or less and kinetic energy accumulations of less than 500 J m\(^{-2}\). The maximum rainfall and kinetic energy accumulations of any of the 102 storms are 79 mm and 2010 J m\(^{-2}\), respectively. The interquartile range (i.e., center 50%) of the distribution resides between approximately 20 and 70 mm h\(^{-1}\) for the maximum 1-min rain rate (Fig. 4c), and between 540 and 2200 J m\(^{-2}\) h\(^{-1}\) for the maximum 1-min kinetic energy flux (Fig. 4f). The respective distribution medians are 46 mm h\(^{-1}\) and 1325 J m\(^{-2}\) h\(^{-1}\).

b. Power-law coefficients and uncertainty estimation

The relationship between two rainfall parameters \(Y\) and \(X\) (where \(Y\) and \(X\) stand for any paired combination of \(Z\), \(R\), and \(E\)) was approximated by a power law of the form \(Y = A_s X^b\). The multiplicative factor \(A_s\) and exponent \(b\) may be determined in a variety of ways. The most simple approach (case FIX), of the six investigated here, is the one where the exponent \(b\) is assumed to be known and fixed (see section 3a) and, for each of the 102 rainfall periods comprising \(m\) 1-min raindrop spectra, the multiplicative factor \(A_s\)

\[
A_s = \left( \frac{\sum_{j=1}^{m} Y_i b}{\sum_{j=1}^{m} X_j} \right),
\]

gets adjusted such that the mass (i.e., rainfall or kinetic energy) is correctly accumulated by using a relationship based on these two power-law coefficients. For four additional approaches, the multiplicative factors and exponents were obtained through the process of minimizing the residual standard deviation or root-mean-square error, either in the linear (two options) or logarithmic parameter space (two options).

Within the linear parameter space spanned by \(X\) and \(Y\), two distinct root-mean-square error criteria were used to minimize the residuals, one with an internal standardization (case INT)

\[
RMSE = 100\% \sqrt{\frac{1}{m-2} \sum_{j=1}^{m} \left( \frac{Y_j - (Y/A_s b)^{1/b}}{X_j} \right)^2}, \tag{10}
\]

and the other with external (case EXT)

\[
rmse = 100\% \sqrt{\frac{1}{m-2} \sum_{j=1}^{m} \left( \frac{Y_j - (Y/A_s b)^{1/b}}{X_j} \right)^2}, \tag{11}
\]

Case INT represents an instantaneous standardization based on the current parameter value, while case EXT reflects a standardization over an entire storm. In both cases, those coefficients (i.e., multiplicative factor \(A_s\) and exponent \(b\)) are sought that would minimize the RMSE and rmse, respectively. This was achieved iteratively through variation of \(b\), while \(A_s\) was given by Eq. (9).

The approaches of finding the power-law relationship coefficients \(A_s\) and \(b\), based on an optimization in linear parameter space, are different from a least-square-fit regression method often used. For example, in the logarithmic parameter space spanned by \(x = \log X\) and \(y = \log Y\) (case LSF1), the power-law relationship \(Y = A_s X^b\) takes on the linear form \(y = \alpha + \beta x\), where \(\alpha = \log A_s\) and \(\beta = b\) are the linear regression coefficients determined by

\[
\alpha = \bar{y} - \beta \bar{x}
\]

\[
\beta = \frac{\sum_{j=1}^{m} (x_j y_j) - \bar{m} \bar{x} \bar{y}}{\sum_{j=1}^{m} x_j^2 - \bar{m} \bar{x}^2} = \rho \frac{s_y}{s_x}, \tag{13}
\]

with \(\bar{y}\) and \(\bar{x}\) being the mean of \(y\) and \(x\), respectively. Alternatively, \(\beta\) may be expressed as the correlation \(\rho\) (between \(x\) and \(y\)) multiplied by the ratio of the standard deviations \(s_x\) and \(s_y\). The regression approach, however, is not unique in the sense that for a least-square-fit regression, there are two possible, generally different, solutions to the problem, depending on whether \(Y\) was regressed on \(X\) (this case) or vice versa. Identical relationships are obtained for a correlation \(\rho = 1\), in which case \(\beta\) is determined by the ratio of \(s_x\) and \(s_y\). In the case of \(X\) being regressed on \(Y\) (case LSF2), the parameter space is spanned by \(x = \log Y\) and \(y = \log X\), while the linear regression coefficients \(\alpha = -\beta \log A_s\) and \(\beta = b^{-1}\) are still obtained by Eqs. (12) and (13), respectively.

Last but not least, the sixth approach (case PMM) to
be tested is the Probability Matching Method (Miller 1972; Calheiros and Zawadzki 1987; Atlas et al. 1990; Rosenfeld et al. 1993, 1994), where the $Z$–$R$ relationship is derived by matching equal percentiles of the probability density functions of the radar reflectivities $Z$ and the rain rates $R$. That is, the time series of concurrent $R$s and $Z$s are converted into separate cumulative probability distributions for each parameter, and a new (no longer concurrent) relation between the radar reflectivities $Z$ and rain rates $R$ is established by connecting equal percentiles of the $Z$ and $R$ distributions. Probability-matched $Z$–$R$ relations were obtained for each storm, which may not have resulted in stable relationships (Krajewski and Smith 1991; Rosenfeld et al. 1993; Ciach et al. 1997); however, we attempted to place all approaches on the same basis (i.e., using the same storm data). The PMM approach does not yield multiplicative factors and exponents.

For each of the above six approaches, both the RMSE and rmse were used to characterize the uncertainty (i.e., scatter of $Y$ for a particular $X$ in the $X$–$Y$ parameter space) of a given relationship that resulted from variability of the raindrop size distributions. A single parameter (RMSE or rmse) though may not necessarily be an ideal measure of this uncertainty (see section 3). With the exception of case PMM, we obtained sets of $\{A_b, b, \text{RMSE}\}$ and $\{A_b, b, \text{rmse}\}$ for each of the rainfall periods, and this for each of the $Z$–$R$, $Z$–$E$, and $E$–$R$ relationships.

3. Results and discussion

The rainfall periods of the 1996–98 Goodwin Creek data were analyzed according to the procedures outlined in section 2. The results of the coefficient and parameter sets $\{A_b, b, \text{RMSE}\}$ for the rainfall periods and each of the $Z$–$R$, $Z$–$E$, and $E$–$R$ relationships are summarized hereafter. In section 3a, results that are based on using variable and fixed exponents $b$ are discussed. The sensitivity to differences in data processing is explored in section 3b and to the drop sample size in section 3c.

a. Variable and fixed power factors

First the results that are based on using variable, storm-adjusted exponents $b$ are discussed (case INT), as outlined in section 2. These results are shown in Figs. 5 and 6 by the light gray shaded histograms. In comparison, the dark shaded histograms indicate results obtained by using climatologically fixed exponents (case FIX). The parameter RMSE (Fig. 5), and power-law coefficients $A_b$ and $b$ (Fig. 6), follow approximately normal distributions for the three relationships investigated, except for the exponents $b$ of case FIX, which (by design) follow a monodisperse distribution. Characteristics summarizing the distributions for the various approaches and relationships are listed in Table 2.

The distributions of RMSE values for the $Z$–$R$, $Z$–$E$, and $E$–$R$ relationships are shown in Figs. 5a, 5c, and 5e, respectively. The mean of the RMSE distribution for the $Z$–$R$ relation is 47%, with the center half of the values ranging from 40% to 51% (Fig. 5a). The mean RMSE for the $Z$–$E$ and $E$–$R$ relationships are 23% and 36%, respectively, with interquartile ranges of 20%–26% and 32%–42%. The coefficient of variation, defined as 100% times the standard deviation divided by the mean, for the RMSE values is approximately 22% for all three relationships.

The multiplicative factors $A_b$ are distributed about a mean of 372 for the $Z$–$R$ (Fig. 6a), 24 for the $Z$–$E$ (Fig. 6c), and 12 for the $E$–$R$ (Fig. 6e) relationships, with standard deviations of 143, 4, and 3, respectively (Table 2). The coefficient of variation for the multiplicative factors ranges between 17% and 38%. Similarly, the exponents $b$ are distributed about a mean of 1.34 for the $Z$–$R$ (Fig. 6b), 1.10 for the $Z$–$E$ (Fig. 6d), and 1.23 for the $E$–$R$ (Fig. 6f) relationships (Table 2). The coefficient of variation for the exponents is small and ranges between 3% and 6%. Plotting the power-law coefficient and uncertainty parameter sets of $\{A_b, b, \text{RMSE}\}$ as a function of either rainfall or kinetic energy accumulations revealed no dependencies (not shown). In addition, we found little or no apparent dependence with the maximum precipitation intensity (i.e., radar reflectivity factor, rain rate, or kinetic energy flux) of a storm, which is in contrast to the findings of Austin (1987), but in agreement with the results of Smith and Krajewski (1993) and Anagnostou and Krajewski (1999).

The RMSE measure is not necessarily a uniform function of rain rate or kinetic energy flux, as mentioned in section 2b. Consequently, the RMSE was analyzed as a function of rain rate and kinetic energy, respectively, while using the same power-law relationship coefficients $A_b$ and $b$ for each rainfall period as before. The results of this analysis are shown in Figs. 5b, 5d, and 5f for the three relationships of interest. The RMSE for the $Z$–$R$ and $E$–$R$ relationships is largest for rain rates less than 1 mm h$^{-1}$ (median of 50% or larger), and decreases with increasing rainfall intensity (Figs. 5b and 5f). For extreme rain rates ($R \geq$ 100 mm h$^{-1}$) the RMSE is about 20% for the $Z$–$R$ and 10%–15% for the $E$–$R$ relationship. In contrast, for the $Z$–$E$ relation, the RMSE shows no trend and is almost uniformly distributed as a function of kinetic energy flux (Fig. 5d). (Note that this uniform distribution of data scatter with intensity, as depicted in Fig. 5d, results in significantly smaller differences between the approaches INT and EXT for the $Z$–$E$ relation compared to the $Z$–$R$ and $E$–$R$ relationships.) The RMSE values of the three relationships reduce to approximately 20% for extreme precipitation intensities.

The use of a fixed exponent $b$ has been suggested to simplify adjustment procedures for radar-based rainfall estimation (Joss and Waldvogel 1970; Cataneo and Vercellino 1972; Harrold et al. 1974; Smith and Joss 1997; Doelling et al. 1998). The present results (case INT) of the exponent distributions (Figs. 6b, 6d, and 6f) support
FIG. 5. Distribution of RMSE uncertainties for the $Z\rightarrow R$ (top), $Z\rightarrow E$ (middle), and $E\rightarrow R$ (bottom) relationships, shown overall (full histograms in panels to the left) and as a function of the rain rate or kinetic energy flux (interquartile range and median in panels to the right). The light gray shading indicates the results of the storm-adjusted exponent approach while the dark shading highlights results based on a fixed exponent for all storms. See section 3a for details.
this advice. Hence, the data were analyzed under the assumption that the most likely value for the exponent of each relationship would be known. For example, this may be the result of monitoring the raindrop size distributions over quite some time to establish a climatology of exponent values. Based on the distributions shown in Figs. 6b, 6d, and 6f, an exponent of $b = 1.35$ was selected for the $Z-R$ relation, $b = 1.10$ for the $Z-E$
relation, and $b = 1.25$ for the $E-R$ relation. The results of this fixed-exponent analysis (case FIX) are shown in Figs. 5 and 6 by the dark gray shaded histograms and are summarized in Table 2. The results of a climatologically fixed exponent versus those of a storm-adjusted exponent differ modestly. There is a shift in RMSE to slightly larger values, mainly contributed by justed exponent differ modestly. There is a shift in logistically fixed exponent versus those of a storm-ad-

**Table 2. Characteristics of the distributions of RMSE, multiplicative factors $A_r$, and exponents $b$ for different relationships and two processing scenarios.** Shown are the center half of the distribution (interquartile range), the median, mean, and standard deviation about the mean. The last column shows a test on the parameter distributions being approximately normal; the numbers indicate the percentage of the data within the range of the mean ±1/2/3 standard deviations (i.e., 68/95/100 = normal distribution). See section 3a for a discussion of this table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relation</th>
<th>Approach</th>
<th>1–3 Quartile</th>
<th>Median</th>
<th>Mean</th>
<th>St dev</th>
<th>Normality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>$Z-R$</td>
<td>INT</td>
<td>40%–51%</td>
<td>46%</td>
<td>47%</td>
<td>10%</td>
<td>76/97/99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>44%–56%</td>
<td>50%</td>
<td>51%</td>
<td>12%</td>
<td>75/96/99</td>
</tr>
<tr>
<td></td>
<td>$Z-E$</td>
<td>INT</td>
<td>20%–26%</td>
<td>23%</td>
<td>23%</td>
<td>5%</td>
<td>75/93/100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>23%–32%</td>
<td>27%</td>
<td>29%</td>
<td>10%</td>
<td>85/95/97</td>
</tr>
<tr>
<td></td>
<td>$E-R$</td>
<td>INT</td>
<td>32%–42%</td>
<td>37%</td>
<td>36%</td>
<td>8%</td>
<td>65/96/100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>35%–47%</td>
<td>39%</td>
<td>41%</td>
<td>10%</td>
<td>72/97/99</td>
</tr>
<tr>
<td>$A_r$</td>
<td>$Z-R$</td>
<td>INT</td>
<td>351</td>
<td>372</td>
<td>143</td>
<td></td>
<td>75/96/99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>344</td>
<td>368</td>
<td>157</td>
<td></td>
<td>75/97/99</td>
</tr>
<tr>
<td></td>
<td>$Z-E$</td>
<td>INT</td>
<td>24</td>
<td>24</td>
<td>4</td>
<td></td>
<td>69/95/100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>24</td>
<td>25</td>
<td>6</td>
<td></td>
<td>73/98/99</td>
</tr>
<tr>
<td></td>
<td>$E-R$</td>
<td>INT</td>
<td>12</td>
<td>12</td>
<td>3</td>
<td></td>
<td>70/94/99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>10</td>
<td>11</td>
<td>2</td>
<td></td>
<td>77/92/99</td>
</tr>
<tr>
<td>$b$</td>
<td>$Z-R$</td>
<td>INT</td>
<td>1.30–1.39</td>
<td>1.34</td>
<td>1.34</td>
<td>0.08</td>
<td>76/95/97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>1.35–1.35</td>
<td>1.35</td>
<td>1.35</td>
<td>0.00</td>
<td>100/100/100</td>
</tr>
<tr>
<td></td>
<td>$Z-E$</td>
<td>INT</td>
<td>1.08–1.11</td>
<td>1.10</td>
<td>1.10</td>
<td>0.03</td>
<td>75/93/98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>1.10–1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>0.00</td>
<td>100/100/100</td>
</tr>
<tr>
<td></td>
<td>$E-R$</td>
<td>INT</td>
<td>1.19–1.26</td>
<td>1.23</td>
<td>1.23</td>
<td>0.05</td>
<td>72/95/99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FIX</td>
<td>1.25–1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>0.00</td>
<td>100/100/100</td>
</tr>
</tbody>
</table>

b. Sensitivity to different data processing scenarios

Five different approaches were detailed in section 2b on how to obtain the multiplicative factor and exponent of a relationship between any two of the rainfall parameters $Z$, $R$, and $E$. Two approaches have been discussed so far (cases INT and FIX). In this section we present a comparison of all five approaches and, for part of the discussion, include also case PMM (although this method did not yield multiplicative factors and exponents). For simplicity, the $Z-R$ relationship is exemplified hereafter; however, similar results may be obtained for the $Z-E$ and $E-R$ relations.

Comparison of the results obtained by the different approaches of estimating the $Z-R$ power-law relationship coefficients and the two raindrop spectra variability-based uncertainty parameters are shown in Fig. 7 (boxplot of interquartile range and median) and are summarized in Table 3. Table 3 lists the characteristics of the distribution of the 102 ratios of individual storm-based values of multiplicative factor $A_r$ and exponent $b$, RMSE and rmse, and the storm total rainfall accumulation, for pairs of approaches (e.g., case INT versus case FIX). On average, the cases FIX, INT, LSF1, LSF2, and PMM produced comparable RMSE values (Fig. 7a). Case EXT resulted in significantly larger RMSEs, the distribution median being approximately a factor of two larger than the one for case INT, which showed the smallest RMSE values (as expected because RMSE was used to tune the case INT power-law coefficients). [Note that the larger RMSE values for case EXT are the result of significantly larger residuals at rain rates less than 1 mm h$^{-1}$ compared to the other cases, while the residuals at rain rates exceeding 1 mm h$^{-1}$ were more similar among the approaches (not shown).] Evaluated on the basis of rmse, as anticipated, case EXT showed the smallest uncertainty values among the power-law relationship approaches (e.g., the median for case INT is 20% larger than the one for case EXT). The PMM approach, however, achieved the smallest rmse values overall, slightly outperforming case EXT. The distributions, though, are wide and overlapping for all approaches (Fig. 7b and Table 3). These results are in agreement with Rosenfeld and Amitai (1998) who found
that both $Z-R$ relations based on the Window Probability Matching Method (Rosenfeld et al. 1994) and based on bias-adjusted regression methods have similar rainfall estimation skills as long as the reflectivity and rain-rate observations are well in tune with each other. Krajewski and Smith (1991) illustrated bias problems that may arise from nonsynchronous (e.g., differences in measurement thresholds) radar and rain gauge observations. Because our analyses are based on synchronized data (disdrometer only), these problems were not addressed here.

The distributions of multiplicative factor $A_b$ are wide and overlapping for all approaches, with cases FIX and INT producing the largest and case EXT the smallest values (Fig. 7c). The distribution medians are 344 (case FIX), 351 (case INT), 234 (case EXT), 280 (case LSF1), and 291 (case LSF2), respectively. On average, the exponent $b$ for cases FIX, LSF1, and LSF2 was within 5% of the value obtained for case INT (Fig. 7d and Table 3). The results for case EXT are more widely distributed, and the average difference in exponent $b$ between case INT and case EXT was approximately three times larger (16%) than the differences among the other approaches. None of the above findings appears to be dependent on the amount of rainfall accumulation or maximum rainfall intensity of storms (not shown).

Krajewski and Smith (1991) pointed out that linear regression techniques will lead to biased power-law co-
efficiency estimates, and Ciach and Krajewski (1999) showed that the exponents \( b \) obtained by the LSF2 approach are larger than those obtained by LSF1, similar to our findings (Fig. 7b). In fact, based on using Eq. (13) it can be shown that the exponents of cases LSF1 (\( b_1 \)) and LSF2 (\( b_2 \)) are related through

\[
\frac{b_1}{b_2} = \rho^2,
\]

(J. Joss, personal communication 1999). The exponent \( b_2 \) is generally larger than \( b_1 \) because \( \rho^2 \leq 1 \). Identical exponents are obtained for a correlation of one (see section 2b).

Regression techniques may produce Z-R relationships that nicely fit the data; however, rainfall estimates based on those relationships tend to be biased and generally do not correctly accumulate the true rainfall (i.e., preserve the mass). Moreover, the results based on a regression of logZ on logR (case LSF1) differ widely from those based on a regression of logR on logZ (case LSF2)—the latter case being more relevant for the radar rainfall estimation problem, because the radar reflectivity factor Z is the independent and the rainfall rate R the dependent variable. We are using case INT as a standard to compare with because that approach, by definition (see section 2b), results in unbiased storm total rainfall accumulations. Table 3 shows the characteristics of the distributions of 102 ratios of storm total rainfall accumulated by the case INT approach over storm total rainfall accumulated by cases FIX, EXT, LSF2, and PMM, respectively, plus the ratio of LSF2 and LSF1. The interquartile range of the data shows rainfall accumulations for case LSF2 that were approximately 5%–10% lower than those of case LSF1 (Table 3). The use of Z-R relationships based on a regression of logR on logZ (case LSF2), on average, resulted in approximately 4% overestimates of the true rainfall amounts, while relationships based on a regression of logZ on logR (case LSF1) produced positive biases of 13% and more. In contrast, as pointed out before, the other approaches presented in section 2b (cases FIX, INT, EXT, including PMM) have an advantage over least-square-fit regression methods (cases LSF1 and LSF2), in that the former approaches by design result in unbiased rainfall accumulations (Table 3).

The procedure by which the multiplicative factor \( A_b \) and exponent \( b \) were determined has a significant effect on the values obtained (as discussed before), and it also affects the relation between \( A_b \) and \( b \). Figure 8 shows the 102 storm-based power-law coefficients within the parameter space spanned by \( A_b \) and \( b \) for four of the five power-law relationship approaches outlined in section 2b. The results of case FIX (fixed exponent \( b \)) are trivial and thus not shown. Three of the four cases presented in Fig. 8 (i.e., cases INT, LSF1, and LSF2) do not reveal any relationship among the two power-law coefficients. The fourth case (EXT), in contrast, indicates a trend of decreasing multiplicative factor \( A_b \) with increasing exponent \( b \) (Fig. 8b). Such a relationship between the power-law coefficients has been suggested by Atlas et al. (1999). In light of the significant sensitivity to the procedure of estimating the power-law coefficients, however, discussions about potential relationships between the factors \( A_b \) and \( b \) (e.g., Tokay and Short 1996; Atlas et al. 1999) have to be carefully examined. We do not exclude the possibility of a relationship between the type of storm and range of values for the multiplicative factor \( A_b \) and exponent \( b \) (e.g., Fujiwara 1965), but the influence of how these coefficients were derived may be as significant as the meteorological signal one is looking for.

c. Sensitivity to drop sample size

The effect of sample size on the rainfall parameters derived from drop spectra samples has been raised occasionally. For example, based on the assumption of Poissonian sample statistics, Joss and Waldvogel (1969) and Gertzman and Atlas (1977) showed that generally large raindrop samples are needed to get good estimates of Z and R (and consequently E as well). Smith et al. (1993), investigating sampling variability effects in raindrop size observations, concluded “The best way to deal with this basic problem is to make sure that only samples of substantial size are used in establishing Z-R relationships. In general, this will require that the fractional standard deviations of the quantities of interest

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ratio of</th>
<th>1–3 Quartile</th>
<th>Median</th>
<th>Mean</th>
<th>Std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>INT/FIX</td>
<td>0.963–1.028</td>
<td>0.993</td>
<td>0.993</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>INT/EXT</td>
<td>0.804–0.931</td>
<td>0.867</td>
<td>0.865</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>INT/LSF2</td>
<td>0.935–0.978</td>
<td>0.963</td>
<td>0.952</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>1.015–1.033</td>
<td>1.022</td>
<td>1.029</td>
<td>0.025</td>
</tr>
<tr>
<td>A_b</td>
<td>INT/FIX</td>
<td>0.932–1.112</td>
<td>1.026</td>
<td>1.041</td>
<td>0.201</td>
</tr>
<tr>
<td></td>
<td>INT/EXT</td>
<td>1.196–1.783</td>
<td>1.404</td>
<td>1.585</td>
<td>0.660</td>
</tr>
<tr>
<td></td>
<td>INT/LSF2</td>
<td>1.143–1.340</td>
<td>1.219</td>
<td>1.277</td>
<td>0.233</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>1.020–1.078</td>
<td>1.041</td>
<td>1.055</td>
<td>0.070</td>
</tr>
<tr>
<td>RMSE</td>
<td>INT/FIX</td>
<td>0.903–0.986</td>
<td>0.966</td>
<td>0.923</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td>INT/EXT</td>
<td>0.274–0.778</td>
<td>0.470</td>
<td>0.524</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td>INT/LSF2</td>
<td>0.830–0.917</td>
<td>0.877</td>
<td>0.871</td>
<td>0.094</td>
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<td></td>
<td>INT/PMM</td>
<td>0.813–0.971</td>
<td>0.903</td>
<td>0.915</td>
<td>0.170</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>0.967–0.998</td>
<td>0.983</td>
<td>0.975</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>0.962–1.071</td>
<td>1.000</td>
<td>1.026</td>
<td>0.153</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>1.054–1.545</td>
<td>1.204</td>
<td>1.375</td>
<td>0.469</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>0.820–1.066</td>
<td>0.968</td>
<td>0.932</td>
<td>0.254</td>
</tr>
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<td></td>
<td>LSF2/LSF1</td>
<td>1.080–1.642</td>
<td>1.235</td>
<td>1.475</td>
<td>0.609</td>
</tr>
<tr>
<td>Rainfall</td>
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<td>1.000–1.090</td>
<td>1.000</td>
<td>1.000</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>INT/LSF2</td>
<td>0.900–1.040</td>
<td>0.970</td>
<td>0.964</td>
<td>0.156</td>
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<td></td>
<td>INT/PMM</td>
<td>1.000–1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>LSF2/LSF1</td>
<td>0.899–0.948</td>
<td>0.929</td>
<td>0.910</td>
<td>0.061</td>
</tr>
</tbody>
</table>
be of the same order as (or smaller than) the natural variations to be investigated.” Most recently, Kostinski and Jameson (1997, 1999), Jameson and Kostinski (1998), and Jameson et al. (1999), studying the fluctuation and clustering properties of rainfall, have questioned the general applicability of Poissonian sample statistics to the raindrop spectra measurement problem, suggesting that for heavy rainfall or convective showers the use of geometric statistical descriptions (resulting in a much smaller reduction of uncertainty with increasing sample size than based on Poissonian statistics) would be more appropriate. Assessment of contributions of this kind to the scatter of data in Figs. 1–3, however, is beyond the scope of the current study.

In section 2a, our initial assumption was to neglect the sample size effect on the scatter of data in Figs. 1–3. We return now to this point to assess the validity of that assumption. Table 4 shows the effect of limiting the raindrop spectra to samples with drop counts exceeding a certain threshold number. Disregarding raindrop spectra with sample sizes of less than 10 drop counts reduced the ensemble of drop spectra significantly but did not

Table 4. Number of raindrop spectra and their contribution to the overall rainfall and rainfall kinetic energy accumulation as a function of the minimum drop count per 1-min spectrum for the 1996–98 data (102 rainfall periods) collected by the disdrometer in Goodwin Creek, MS. See section 3c for a discussion of this table.

<table>
<thead>
<tr>
<th>Drop counts</th>
<th>Drop spectra</th>
<th>Rainfall</th>
<th>Kinetic energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>all (incl. zero)</td>
<td>84 890 (100%)</td>
<td>2215 mm</td>
<td>46 009 J m⁻²</td>
</tr>
<tr>
<td>≥1</td>
<td>64 098 (75.5%)</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>≥10</td>
<td>53 151 (62.6%)</td>
<td>99.9%</td>
<td>100.0%</td>
</tr>
<tr>
<td>≥100</td>
<td>35 650 (42.0%)</td>
<td>97.8%</td>
<td>98.4%</td>
</tr>
<tr>
<td>≥1,000</td>
<td>2608 (3.1%)</td>
<td>24.9%</td>
<td>28.6%</td>
</tr>
</tbody>
</table>

Fig. 8. Scatterplots of 102 storm-based Z−R relationship coefficients within the parameter space spanned by the multiplicative factor $A_b$ and exponent $b$. Shown are the results of four different approaches to estimate power-law coefficients. See section 3b for details.
affect the rainfall or the kinetic energy accumulations. Increasing that threshold beyond a minimum of 100 raindrops per 1-min spectrum, however, started to show increasing effect (Table 4). In particular, the scatter of data in Figs. 1–3 was increasingly eroded approximately in parallel to the dashed (single drop count) boundary shown in those figures.

Figure 9 highlights the effect of restricting the raindrop spectra sample to those that contain a minimum drop count on the storm-based power-law coefficients \((A_b\) and \(b\)) and the uncertainty parameter (RMSE), defined in section 2b. Although analyses are presented only for the \(Z-R\) relationship, similar results may be obtained for the \(Z-E\) and \(E-R\) relations. Figure 9 shows boxplots (interquartile range, with median marked by the dot) of the distributions of 102 storm-based values of RMSE (Fig. 9a), relative reduction in drop spectra sample size for a given storm (Fig. 9b), multiplicative factor \(A_b\) (Fig. 9c), and exponent \(b\) (Fig. 9d), based on the case INT approach. Limiting the spectra to those with drop counts exceeding a minimum number did noticeably affect the analyses. The sample size for a given storm rapidly decreased with increasing minimum drop count requirement, but the effect varied significantly from storm to storm, as indicated by the wide interquartile range of the distributions (Fig. 9b). The effect of a sample restriction on the RMSE (Fig. 9a) and power-law coefficients \(A_b\) and \(b\) (Figs. 9c and 9d) became appreciable if we required drop spectra to be composed of 100 drop counts or more. [Note that spectra with less than 100 drop counts contributed little to the rainfall or kinetic energy accumulation (Table 4).] An increase of
the minimum drop count per spectrum reduced the scatter of data (as mentioned before), which resulted in smaller RMSE values (Fig. 9a). The multiplicative factors decreased also (Fig. 9c), while the exponents showed an increasing trend (Fig. 9d).

Our analyses revealed a significant effect of raindrop sample size on the coefficients of power-law relationships. A requirement of a minimum drop count will focus the analyses on the good-quality raindrop spectra (see first paragraph of this section); however, such a limitation may seriously affect the representativeness of the storm’s meteorological (raindrop spectra) range because weak rain rate samples are disregarded. A variable time integration may help alleviate the problem of small drop samples in raindrop spectra and thus possibly compensate for the effects discussed above. On the other hand, time averaging may result in raindrop spectra that are less meaningful physically (Jameson and Kostinski 1998) or, as Jameson et al. (1999) phrased it, “Over-sampling destroys information just as effectively as undersampling misses it.” The solution to this dilemma is beyond the scope of this paper and clearly needs further investigation.

Future analyses of the sample size effect may incorporate Monte Carlo simulations with random modifications of the collected raindrop size distributions (rather than discarding certain samples) that may be based on Poisson mixture statistics (Kostinski and Jameson 1997; Jameson and Kostinski 1998), or possibly distinguish between stratiform (Poisson statistics) and convective rainfall (geometric statistics), as advocated by Kostinski and Jameson (1999) and Jameson et al. (1999).

A final comment: Krajewski and Smith (1991) pointed out that large samples (1000–10 000) of raindrop spectra may be required to obtain good estimates of regressed $Z-R$ power-law coefficients. Our record shows that 98% of the analyzed storms were composed of less than 5000 spectra and 77% of the storms had even less than 1000 spectra. This, together with a significant sensitivity to the choice of analysis method discussed in section 3b (Fig. 7), implies that power-law coefficients of $Z-R$ (and similar) relationships have to be carefully examined.

4. Summary and conclusions

This study was concerned with relationships between radar reflectivity factor $Z$, rainfall rate $R$, and rainfall kinetic energy flux $E$ derived directly from disdrometer-measured raindrop size distributions. In particular, we investigated the uncertainty inherent in the estimation of one parameter from another, caused by the variability of the raindrop size distribution within and between storms. In addition, we explored the climatological variability of the estimated relationship coefficients, studied the sensitivity to the estimation procedure, and discussed the effect of raindrop sample size. The analyses were based on a sample of 102 storms extracted from data collected between March 1996 and June 1998 in the Goodwin Creek research watershed, Mississippi.

Estimates of one rainfall parameter from another are burdened by a significant uncertainty that arises from variability of the raindrop size distribution within storms and from storm to storm. The degree of uncertainty is a function of the difference in weighting of the drop size by the respective two rainfall parameters. The drop size variability within storms affects the precision by which a given rainfall parameter may be estimated (under the favorable condition that the relationship between two rainfall parameters is known for a given storm), while the variability among storms affects the appropriateness of a climatological choice of relationship (given that there is no way of knowing ahead of time the best relationship on a storm-to-storm basis; for example, for operational applications).

Climatological analyses of the coefficients of power-law relationships, $Y = A_{YX}X^b$ (where $Y$ and $X$ may stand for any pairing of radar reflectivity factor $Z$, rainfall rate $R$, or rainfall kinetic energy flux $E$), showed that the exponents $b$ were narrowly distributed about their mean (section 3a). The coefficient of variation describing this variability about the mean was approximately 5% for the $Z-R$, $E-R$, and $Z-E$ relationships. It is concluded that the use of power-law relationships with climatologically fixed exponents is a viable option to simplify rainfall estimation procedures.

The multiplicative factors $A_y$ were more widely distributed about the respective means, with a coefficient of variation of approximately 20% or more (section 3a). This significant variability from storm to storm renders a climatological choice of relationship difficult and the use of such a relationship may result in appreciable uncertainty. Thus, storm-based adjustments of the multiplicative factor are recommended whenever possible. Variable adjustments within a storm (e.g., different for convective and stratiform rainfall) may be desired but are typically not practical.

The uncertainty by which one rainfall parameter may be estimated from another (caused by the drop spectra variability within storms) was gauged using criteria based on residual root-mean-square differences (section 3a). On average, this uncertainty was approximately 50% for the $Z-R$, 40% for the $E-R$, and 25% for the $Z-E$ relationship. The coefficient of variation (expressing the variability from storm to storm of this within-storm variability of drop spectra) was approximately 20% for all three relationships.

The drop spectra-based uncertainty of relationships involving the rain rate $R$ (i.e., $Z-R$ and $E-R$), however, is a function of the rainfall intensity and found to be most significant at low intensities (section 3a). For extreme precipitation intensities ($R \geq 100$ mm h$^{-1}$), this uncertainty was reduced to approximately 20%. The $Z-E$ relation, in contrast, showed uniform scatter of residuals of approximately 25% over the range of precip-
itation intensities. Hence, the radar reflectivity factor provides an effective means to monitor rainfall kinetic energy flux.

Based on synchronized 1-min raindrop spectra, power-law and probability-matched relationships showed comparable rainfall parameter estimation skills, as measured by residual root-mean-square difference criteria (section 3b). However, the choice of method significantly influenced the derived values for the multiplicative factor and exponent of power-law relationships. This uncertainty is approximately 10% for the exponent and 20% or larger for the multiplicative factor. Moreover, the use of least-square-fit-based power-law relationships results in positively biased rainfall and kinetic energy estimates.

There is an appreciable sensitivity of the rainfall parameters and the relationships between them (e.g., power-law coefficients) to the sample of raindrops registered per 1-min drop spectrum (section 3c) and to the sample of raindrop spectra. Thus, drop spectra-based relationships between rainfall parameters have to be carefully examined.

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