NOTES AND CORRESPONDENCE

Areal Rainfall Estimates Using Differential Phase

ALEXANDER RYZHKOV

Cooperative Institute for Mesoscale Meteorological Studies/University of Oklahoma and National Severe Storms Laboratory, Norman, Oklahoma

DUSAN ZRNIĆ

National Severe Storms Laboratory, Norman, Oklahoma

RICHARD FULTON

Hydrologic Research Laboratory, NOAA/NWS, Silver Spring, Maryland

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ABSTRACT

A radar polarimetric method for areal rainfall estimation is examined. In contrast to the polarimetric algorithm based on specific differential phase K_{DP}, the proposed method does not require rain-rate estimation from K_{DP} inside the area of interest, but it utilizes only values of total differential phase Φ_{DP} on the areal contour. Even if the radar reflectivity and differential phase data inside the area are corrupted by ground clutter, anomalous propagation, biological scatterers, or hail contamination, reliable areal rainfall estimate is still possible, provided that correct Φ_{DP} estimates are available at a relatively small number of range locations in or at the periphery of the contour of this area.

This concept of areal rainfall estimation has been tested on the Little Washita River watershed area in Oklahoma that contains 42 densely located rain gauges. The areal rainfall estimates obtained from the polarimetric data collected with the 10-cm Cimarron radar are in good agreement with the gauge data, with the standard error of about 18%. This accuracy is better than that obtained with the algorithm utilizing areal averaging of pointwise estimates of K_{DP} inside the watershed area.

1. Introduction

Radar polarimetric methods for rainfall measurements have received increasing attention in recent years. The one based on the estimate of specific differential phase K_{DP} uses the relation

$$R = aK_{DP}^{b}, \tag{1}$$

where R is rain rate (Sachidananda and Zrnić 1987; Chandrasekar et al. 1990). This method has several advantages compared to the conventional method, which utilizes radar reflectivity factor Z. Differential phase is immune to radar miscalibration, microwave attenuation, and partial beam blockage. It is less contaminated by hail and is less affected by drop size distribution variations (Zrnić and Ryzhkov 1996).

The K_{DP} estimate is usually obtained either as a slope of a least square linear fit of the total differential phase Φ_{DP} along a radial or as a slope of spatially filtered data (Hubbert et al. 1993). In both cases the standard deviation of the K_{DP} estimate depends on the range-averaging interval. To obtain acceptable accuracy of rain rate derived from (1), this averaging interval has to be relatively large. The optimal width of this averaging window is a compromise between the need to have low estimation errors and the desire to reproduce correctly the radial rainfall profile. Ryzhkov and Zrnić (1996) suggest averaging in range over 17 successive gates if $Z > 40$ dBZ, and 49 gates otherwise. For typical gate spacing of 0.15 km, the effective radial resolution of the polarimetric estimates of rainfall is about 2.5 km for moderate and heavy rain and 7.3 km for light precipitation; this is considerably coarser than the radial resolution of the conventional $R(Z)$ rainfall estimator. In addition, large changes of Z or Φ_{DP} within the radar resolution volume can lead to local biases in the K_{DP} field that are most pronounced at distances far from the radar. Therefore, the polarimetric estimator $R(K_{DP})$ distorts the shape of an isolated rain cell more than the

Corresponding author address: Alexander Ryzhkov, CIMMS/NSSL, 1313 Halley Circle, Norman, OK 73069.
E-mail: ryzhkov@nssl.noaa.gov

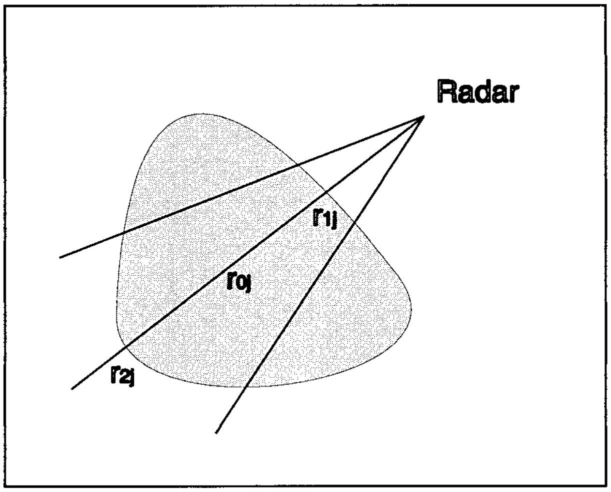

FIG. 1. Relative location of the radar and watershed area.

conventional $R(Z)$ algorithm (Ryzhkov and Zrnić 1998). Nonetheless, despite relatively low radial resolution and local disturbances of the $R(K_{DP})$ estimate in the presence of strong beamfilling nonuniformities, the polarimetric method based on K_{DP} yields almost unbiased integrated rainfall over areas with linear dimensions of a few tens of kilometers.

Because K_{DP} is a radial derivative of the total differential phase Φ_{DP} ($K_{DP} = \frac{1}{2}d\Phi_{DP}/dr$), and the exponent b in (1) is close to unity, the rainfall integrated over the radial interval (r_1, r_2) is approximately proportional to the difference between Φ_{DP} values at the ends of the interval: $\Phi_{DP}(r_2) - \Phi_{DP}(r_1)$. Similarly, areal rainfall is determined by the values of differential phase on the areal contour and, therefore, is not affected by distribution of differential phase inside the area of interest. This idea was first suggested by Raghavan and Chandrasekar (1994) as a useful technique with the potential to obtain area–time integral rain accumulations. In this paper, we examine this technique using data obtained with the 10-cm-wavelength polarimetric radar and rain gauges located in the Little Washita River watershed area.

2. Theory

Assume that rain-rate R and specific differential phase K_{DP} are related, as specified by (1). Then, for an area of interest (Fig. 1), the areal rainfall AR (i.e., rain rate integrated over the area) can be expressed as

$$\mathrm{AR} = \int_S a[K_{DP}(r,\ \theta)]^b r\ dr\ d\theta$$

$$= a \int_S r\left[\frac{d\Phi_{DP}(r,\ \theta)}{2dr}\right]^b dr\ d\theta, \qquad (2)$$

where r is range, and θ is azimuth angle. Assuming that K_{DP} is constant for a given θ, we can represent the integral AR as a sum of the contributions from individual radials:

$$\mathrm{AR} \approx a \sum_j \Delta\theta_j \int_{r_{1j}}^{r_{2j}} r \frac{[\Phi_{DP}(r_{2j},\ \theta_j) - \Phi_{DP}(r_{1j},\ \theta_j)]^b}{[2(r_{2j} - r_{1j})]^b}\ dr$$

$$= \frac{a}{2} \sum_j \Delta\theta_j r_{0j}[2(r_{2j} - r_{1j})]^{1-b}[\Delta\Phi_{DP}^{(j)}]^b, \qquad (3)$$

where r_{1j} and r_{2j} are boundary points of the area along the radial corresponding to the jth azimuth, r_{0j} is the middle point of the interval (r_{1j}, r_{2j}), $\Delta\Theta_j$ is the azimuthal difference between two adjacent radials, and $\Delta\Phi_{DP}^{(j)}$ is the differential phase difference between r_{1j} and r_{2j} along the jth radial. Note that the sum (3) is an accurate approximation of the areal rainfall AR even if K_{DP} is not uniform along the radar beam because the exponent b in (1) is close to unity (Sachidananda and Zrnić 1987; Chandrasekar et al. 1990), and $\Delta\Phi_{DP} = 2 \int K_{DP}(r)\ dr$ for an arbitrary radial profile of K_{DP}. Thus, integral rainfall depends only on boundary values of total differential phase.

If the signal-to-noise ratio at one of the boundary points, r_{1j} or r_{2j}, is close to or less than 0 dB, then Φ_{DP} has a wide distribution within the phase interval (0, π) and cannot be used in (3). A similar problem occurs in the case of differential phase aliasing [see Fig. 1 in Ryzhkov and Zrnić (1995)], or if the differential phase data are contaminated by ground clutter or anomalous propagation [Fig. 10 in Zrnić and Ryzhkov (1996)] or by biological scatterers such as insects and birds [Fig. 1 in Zrnić and Ryzhkov (1998)]. To avoid this problem, editing and smoothing of the Φ_{DP} data must be performed prior to application of (3). After dealiasing the Φ_{DP} data (the unambiguous angle is 180°), estimation of the standard deviation SD of Φ_{DP} at each range location is made. Normally, 17 consecutive range samples are used to compute the SD(Φ_{DP}), which is then assigned to the center of a selected radial interval. If the SD(Φ_{DP}) exceeds a specified threshold (12° for the Cimarron radar), the data are classified as noise or ground clutter. Otherwise, they are assigned to the ''weather signal'' category and are smoothed. The gaps between ''signal'' intervals, where noise or ground clutter are present, are bridged by linear interpolation of the smoothed Φ_{DP} data. An example of original and smoothed Φ_{DP} data along a radial is in Fig. 2.

Statistical errors of the areal rainfall estimates for the suggested [$R(\Phi_{DP})$] and the original [$R(K_{DP})$] algorithms are discussed in the appendix. It is shown that these errors are of the same order for both methods; furthermore, the errors are small provided the watershed area is large (>10 km along the range axis) and areal rainfall accumulation is estimated over a period of 1 h or more.

3. Observations

To verify the suggested method, we have augmented the dataset we have previously examined (Ryzhkov and

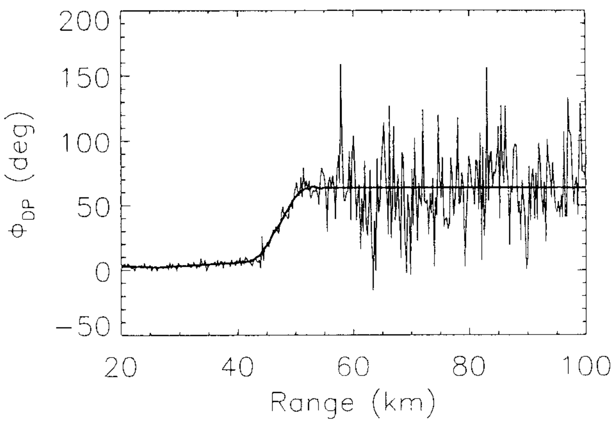

FIG. 2. An example of the raw (thin line) and smoothed (thick line) Φ_{DP} data along a radial.

Zrnić 1996) by five new cases. This combined set includes 20 Oklahoma storms observed with the Cimarron polarimetric radar (10-cm wavelength; Zahrai and Zrnić 1993) over the test area in Central Oklahoma, where 42 closely spaced rain gauges are located. The set consists of eight squall lines, four pure stratiform events, and eight cases of stratiform rain with embedded convective cells. Duration of single events varied between 1 and 7 h and radar update time was between 5 and 9 min. Radar polarimetric data collected from the elevation of 0.5° were used for analysis. In the previous paper (Ryzhkov and Zrnić 1996), we estimated the specific differential phase K_{DP}, converted the K_{DP} data from a polar to a 1 km × 1 km Cartesian grid, and computed total rainfall over the test area and duration of the event using the relation

$$R(K_{DP}) = 40.6\,|K_{DP}|^{0.866}\,\mathrm{sign}(K_{DP}). \qquad (4)$$

The mean rain rate was estimated for each storm event as a gauge (radar) total areal accumulation divided by a product of the watershed area and rain duration time.

In the proposed new procedure, we do not convert radar data from polar to cartesian coordinates, but compute areal rainfall directly from (3) using quality-controlled and smoothed Φ_{DP} data along the radials. Azimuthal spacing $\Delta\theta_j$ between adjacent radials varies between 1° and 2° for different events. For areal rainfall estimations from (3), we use the same coefficients (a = 40.6 and b = 0.866) as in the previous study.

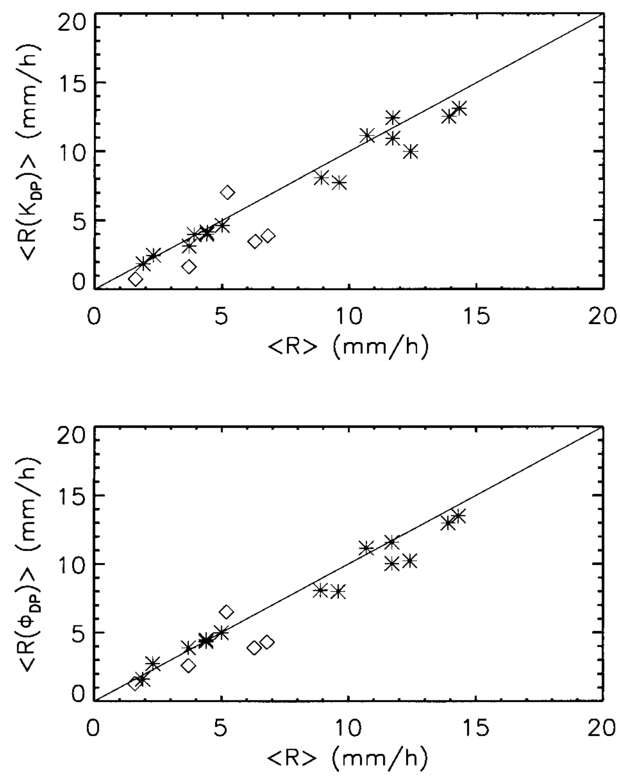

FIG. 3. Radar mean basin-averaged rain rate vs gauge mean basin-averaged rain rate for 20 Oklahoma storms obtained using (top) $R(K_{DP})$ estimates [(4)] and (bottom) $R(\Phi_{DP})$ estimates [(3)]. The stars denote ''normal'' cases, and the diamonds denote outliers.

4. Discussion

Figure 3 and Table 1 illustrate the performance of both algorithms for 20 Oklahoma storms. In this figure we have divided the whole dataset into two categories: ''normal'' cases (denoted by stars in Fig. 3, top) for which the $R(K_{DP})$ algorithm performs quite well, and outliers (denoted by diamonds in Fig. 3, top) for which this algorithm produces significantly larger biases in rainfall. There are five outliers in the augmented dataset. Two of these belong to the older part of the set that has been previously examined (Ryzhkov and Zrnić 1996). The outliers usually occur outside of the months typically associated with the heaviest convective rain in Oklahoma (May–August). We recently observed that the differential reflectivity Z_{DR} is unusually low (for the same radar reflectivity factor Z) if the $R(K_{DP})$ algorithm significantly underestimates rainfall, and it is unusually

TABLE 1. The biases and the fractional standard errors (FSE) in the areal rainfall estimates for the $R(K_{DP})$ and $R(\Phi_{DP})$ algorithms.

Algorithm type	All cases		''Normal'' cases		Outliers	
	Bias (%)	FSE (%)	Bias (%)	FSE (%)	Bias (%)	FSE (%)
$R(K_{DP})$	−12.7	25.1	−6.1	10.2	−32.4	47.0
$R(\Phi_{DP})$	−8.2	18.3	−4.1	10.7	−19.9	30.8

high in the opposite case (Ryzhkov and Zrnić 1996; Fulton et al. 1999). This suggests that both biases are related to the mean drop size.

It is evident from Fig. 3 (bottom) that the suggested algorithm (3) outperforms the original $R(K_{DP})$ algorithm, especially for outlier cases. The mean biases and the fractional standard errors (FSE) of the mean rain-rate estimates computed for both methods are shown in Table 1. Computations have been made separately for the normal cases, outliers, and for the whole dataset. The magnitude of negative bias decreases from 32.4% to 19.9% and the FSE drops from 47.0% to 30.8% for outlier cases if the relation $R(\Phi_{DP})$ is utilized instead of the relation $R(K_{DP})$. The corresponding improvement for the whole dataset is from −12.7% to −8.2% in terms of the mean bias and from 25.1% to 18.3% in terms of the FSE.

The new version of the polarimetric algorithm for areal rainfall estimation saves a considerable amount of computations. First, computation of a specific differential phase K_{DP} is not required. Second, only correctly estimated values of total differential phase Φ_{DP} along the area contour are necessary to compute the areal rainfall. Radar data inside the area of interest might be contaminated by ground clutter, anomalous propagation, presence of hail, or biological scatterers, but this does not affect the performance of the suggested algorithm. Strong azimuthal gradients of radar reflectivity factor Z or differential phase Φ_{DP} that can cause local disturbances of the K_{DP} fields do not affect areal rainfall estimates on larger scales unless those disturbances lie on the areal boundary. The integral method is valid even for light rain where pointwise estimates of K_{DP} are very noisy. All these advantages contribute to the overall improvement of the polarimetric estimate of the areal rainfall over that from averages of pointwise K_{DP} estimates, although the dominant contributors change from case to case.

Neither estimator $R(K_{DP})$ nor $R(\Phi_{DP})$ is totally immune to drop size distribution variations. That is why outliers are not completely eliminated but are reduced if the $R(\Phi_{DP})$ algorithm is used. The simplicity of the suggested algorithm and data quality factors (mentioned above) make it more robust than the original $R(K_{DP})$ method. To bring outliers into the fold, it appears that other polarimetric variables (e.g., Z_{DR}, Z) might need to be included in the formulas that use differential phase (Fulton et al. 1999).

The suggested algorithm is especially well suited for estimating areal rainfall accumulation over watersheds within the radar coverage area and for other hydrological applications.

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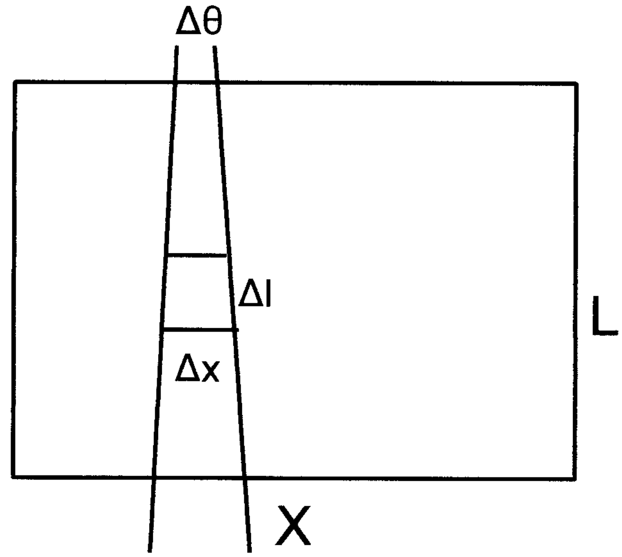

FIG. A1. Illustration of the difference between point (within the interval Δl) and path-integrated rainfall in the watershed area.

APPENDIX

Statistical Errors of Areal Rainfall Estimation

Areal rainfall accumulation ARA is proportional to the sum of areal rain integrals $\mathrm{AR}^{(m)}$ computed from each radar scan performed during an observation period

$$\mathrm{ARA} = \sum_{m=1}^{M} \mathrm{AR}^{(m)}\Delta t, \qquad \text{(A1)}$$

where Δt is an update time, M is a number of consecutive radar scans, and AR is defined by (3). Consequently, the mean rain-rate $\langle R \rangle$ is equal to the ratio ARA/ST, where S is the area of watershed, and $T = M\Delta t$ is total observation time.

Let's assume for the sake of simplicity that rain rate is constant over the whole area during the time of observation, and the exponent b in (1)–(3) is equal to unity. We assume also the simplified geometry of the problem shown in Fig. A1 (rectangular shape of the area with radar radials almost parallel to two of the area's boundaries). These assumptions allow quantitative comparison between pointwise and integral rainfall measurements. Although not precise, the comparison can show interrelation and trade-off among the variables that affect the estimates. According to (3), for any radar scan the areal rain integral AR is a sum of contributions from each radial of path-integrated rainfall estimates $\mathrm{LR}^{(j)}$:

$$\mathrm{AR} = \sum_{j=1}^{J} \mathrm{LR}^{(j)}. \qquad \text{(A2)}$$

The path-integrated rainfall can be expressed as

$$\mathrm{LR} = \frac{a}{2}\Delta x \Delta \Phi_{\mathrm{DP}}, \qquad \text{(A3)}$$

if the $R(\Phi_{\mathrm{DP}})$ algorithm is used [see (3)], or as

$$\mathrm{LR} = \sum_{i=1}^{I} a \Delta x \Delta l K_{\mathrm{DP}}^{(i)}, \qquad \text{(A4)}$$

if the traditional $R(K_{\mathrm{DP}})$ algorithm is used. In the latter case rain rates are estimated directly from K_{DP} for each ith interval Δl along the radial. In (A3) and (A4) $\Delta x = r_0 \Delta \theta = X/J$; $\Delta l = L/I$ (Fig. A1).

The path-integrated rainfall LR varies because of statistical variations of the differential phase difference $\Delta\Phi_{\mathrm{DP}}$ or specific differential phase K_{DP}. It can be easily shown from (A1) and (A2) that the standard deviation of the areal rainfall accumulation ARA (or the mean rain rate $\langle R \rangle$) is proportional to the standard deviation of the LR estimate [SD(LR)]:

$$\mathrm{SD}(\langle R \rangle) = \sqrt{\frac{J}{M}}\frac{\mathrm{SD(LR)}}{S}, \qquad \text{(A5)}$$

SD(LR) for the $R(\Phi_{\mathrm{DP}})$ algorithm (hereinafter SD_1) can be derived from (A3) as

$$\mathrm{SD}_1 = \frac{a}{\sqrt{2}}\Delta x \mathrm{SD}(\Phi_{\mathrm{DP}}), \qquad \text{(A6)}$$

and SD(LR) for the $R(K_{\mathrm{DP}})$ algorithm (hereinafter SD_2) can be written as

$$\mathrm{SD}_2 = a \Delta x \Delta l \sqrt{I} \mathrm{SD}(K_{\mathrm{DP}}). \qquad \text{(A7)}$$

$\mathrm{SD}(K_{\mathrm{DP}})$ is proportional to $\mathrm{SD}(\Phi_{\mathrm{DP}})$ (Ryzhkov and Zrnić 1996; Gorgucci et al. 1999):

$$\mathrm{SD}(K_{\mathrm{DP}}) = \frac{(3)^{1/2}\mathrm{SD}(\Phi_{\mathrm{DP}})}{\Delta l (\Delta N)^{1/2}} \qquad \Delta N \gg 1, \qquad \text{(A8)}$$

where $\Delta N = N/I$ is a number of range gates in the interval Δl, and N is a total number of gates in the interval L. Therefore, the ratio $\mathrm{SD}_2/\mathrm{SD}_1$ is equal to $I(6/N)^{1/2}$; that is, it is directly proportional to the number of subintervals I or inversely proportional to the spatial resolution of the K_{DP} estimates along the radial. If rain is uniform along the radial direction, then it is advantageous to use coarser radial resolution to reduce the statistical error in the areal rainfall estimate provided the traditional $R(K_{\mathrm{DP}})$ algorithm is applied. In reality, however, rain is nonuniform and this imposes a limitation on the minimal number I. In our particular case L is about 30 km, range gate spacing is 0.24 km for most cases examined, $\Delta N = 16$ if $Z > 40$ dBZ, and $\Delta N = 48$ otherwise. Thus, the ratio $\mathrm{SD}_2/\mathrm{SD}_1$ is about 0.6 for light precipitation ($Z < 40$ dBZ) and 1.7 for moderate or heavy precipitation ($Z > 40$ dBZ). Overall, the statistical errors in the areal rainfall estimation for both methods are comparable because the majority of the examined rain events contain both high-reflectivity and low-reflectivity components.

As can be deduced from (A5)–(A8), the magnitude of the statistical error of the mean rain rate $\langle R \rangle$ is quite small given the size of the watershed area and relatively large number of consecutive radar scans used for the areal rainfall computation. Indeed, combining (A5) and (A6) we can obtain the following expression for $\mathrm{SD}(\langle R \rangle)$ if the proposed $R(\Phi_{\mathrm{DP}})$ algorithm is used:

$$\mathrm{SD}(\langle R \rangle) = \frac{a\mathrm{SD}(\Phi_{\mathrm{DP}})}{L(2MJ)^{1/2}}. \qquad \text{(A9)}$$

The number J is determined by the size X of the watershed area (Fig. A1) and the angular resolution $\Delta\theta$. For $X = 40$ km the parameter J is about 34 if $\Delta\theta = 1°$ and is about 17 if $\Delta\theta = 2°$. The number of radar scans M is determined by the storm duration (1–7 h in our dataset) and the update time (6 min on average). Thus, M varies from 10 to 70. According to Ryzhkov and Zrnić (1996) the standard deviation of the Φ_{DP} estimate is about 2°–4° for the Cimarron radar (if no smoothing of the raw data is performed). Even in the worst scenario, when no preliminary smoothing of the Φ_{DP} data is made [and $\mathrm{SD}(\Phi_{\mathrm{DP}}) = 4°$], and M and J are minimal (10 and 17, respectively), the standard error of the mean rain-rate estimate is only about 0.3 mm h^{-1}. Preliminary processing and smoothing of the raw differential phase data (that were actually carried out in our data analysis) further reduces the statistical error in the mean rain rate or the areal rainfall estimation. From this we can conclude that the differences in the estimates of the mean rain rates obtained from the radar and gauges are mainly due to biases caused by drop size distribution variations rather than statistical errors even for rain rates as small as 2 mm h^{-1} (Fig. 3). The assumption of rain uniformity within the test area is fairly realistic for stratiform light precipitation but might not hold for convective storms. In the latter case the statistical error of the areal mean rain rate will be higher.

REFERENCES

Chandrasekar, V., V. N. Bringi, N. Balakrishnan, and D. S. Zrnić, 1990: Error structure of multiparameter radar and surface measurements of rainfall. Part III: Specific differential phase. *J. Atmos. Oceanic Technol.,* **7,** 621–629.

Fulton, R., A. Ryzhkov, and D. Zrnić, 1999: Areal rainfall estimation using conventional and polarimetric radar methods. Preprints, *29th Conf. on Radar Meteorology,* Montreal, Quebec, Canada, Amer. Meteor. Soc., 293–296.

Gorgucci, E., G. Scarchilli, and V. Chandrasekar, 1999: A procedure to calibrate multiparameter weather radar using properties of the rain medium. *IEEE Trans. Geosci. Remote Sens.,* **37,** 269–276.

Hubbert, J., V. Chandrasekar, V. N. Bringi, and P. Meischner, 1993: Processing and interpretation of coherent dual-polarized radar measurements. *J. Atmos. Oceanic Technol.,* **10,** 155–164.

Raghavan, R., and V. Chandrasekar, 1994: Multiparameter radar study of rainfall: Potential application to area–time integral studies. *J. Appl. Meteor.,* **33,** 1636–1645.

Ryzhkov, A., and D. Zrnić, 1995: Precipitation and attenuation measurements at a 10-cm wavelength. *J. Appl. Meteor.,* **34,** 2121–2134.

——, and ——, 1996: Assessment of rainfall measurement that uses specific differential phase. *J. Appl. Meteor.,* **35,** 2080–2090.

——, and ——, 1998: Beamwidth effects on the differential phase measurements of rain. *J. Atmos. Oceanic Technol.,* **15,** 624–634.

Sachidananda, M., and D. Zrnić, 1987: Rain rate estimated from differential polarization measurements. *J. Atmos. Oceanic Technol.,* **4,** 588–598.

Zahrai, A., and D. Zrnić, 1993: The 10-cm-wavelength polarimetric weather radar at NOAA's National Severe Storms Laboratory. *J. Atmos. Oceanic Technol.,* **10,** 649–662.

Zrnić, D., and A. Ryzhkov, 1996: Advantages of rain measurements using specific differential phase. *J. Atmos. Oceanic Technol.,* **13,** 454–464.

——, and ——, 1998: Observations of insects and birds with a polarimetric radar. *IEEE Trans. Geosci. Remote Sens.,* **36,** 661–668.