Stochastic Modeling of Hurricane Damage

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ABSTRACT

A compound Poisson process is proposed as a stochastic model for the total economic damage associated with hurricanes. This model consists of two components, one governing the occurrence of events and another specifying the damages associated with individual events. In this way, damage totals are represented as a ''random sum,'' with variations in total damage being decomposed into two sources, one attributable to variations in the frequency of events and another to variations in the damage from individual events. The model is applied to the economic damage, adjusted for societal vulnerability, caused by North Atlantic hurricanes making landfall in the continental United States. The total number of damaging storms per year is fitted reasonably well by a Poisson distribution, and the monetary damage for individual storms is fitted by the lognormal. The fraction of the variation in annual damage totals associated with fluctuations in the number of storms, although smaller than the corresponding fraction for individual storm damage, is nonnegligible. No evidence is present for a trend in the rate parameter of the Poisson process for the occurrence of storms, and only weak evidence for a trend in the mean of the log-transformed damage from individual storms is present. Stronger evidence exists for dependence of these parameters, both occurrence and storm damage, on the state of El Niño.

1. Introduction

Much concern has been expressed, especially within the insurance and reinsurance industry, about increases in total economic damage associated with extreme weather or climate events (e.g., floods or hurricanes) in recent decades. With concomitant increases in societal vulnerability to extremes (e.g., because of development in flood plains), questions remain about the extent to which these trends are attributable to changes in climate (Changnon et al. 2000; Kunkel et al. 1999). Total economic damage fluctuates both because of variations in the frequency of extreme events and in the damage from individual events. Thus, it would be helpful in analyzing these damages to make use of a statistical approach that explicitly takes into account these different sources of variation.

A case in point is the economic damage caused by North Atlantic hurricanes making landfall in the continental United States. In an attempt to control for changes in vulnerability, this damage dataset has been adjusted for inflation, wealth, and population at risk by Pielke and Landsea (1998). Once these adjustments are made, a marked increasing trend in annual total damage disappears. Also of interest is the El Niño signal, found not only in the frequency of occurrence of such hurricanes (Gray 1984), but in the adjusted total damage as well (Pielke and Landsea 1999). In the present paper, this dataset will be reanalyzed by making use of a formal stochastic model.

A compound Poisson process is proposed as a stochastic model for the total economic damage associated with hurricanes (e.g., storms that cause damage exceeding a certain threshold). This model consists of two components, one governing the occurrence of events and another specifying the damages associated with individual events. In this way, damage totals are represented as a ''random sum,'' with variations in total damage being decomposed into two sources, one attributable to variations in the frequency of events and another to variations in the damage from individual events. Although the representation of extreme event damage as a random sum is relatively novel in the climate literature, it has received much attention elsewhere, even being referred to as the ''bread and butter of insurance mathematics'' (Embrechts et al. 1997). In particular, the statistical approach employed in the present paper is quite close to that in a study of economic damage from extreme wind storms in Sweden by Rootzén and Tajvidi (1997).

The compound Poisson process for total economic damage is described in section 2, with the event oc-

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currence being modeled as a Poisson process and the event damage as a lognormal distribution. Then this model is applied to the hurricane damage data (section 3). Various extensions of the model are considered in section 4, including whether there are any trends in the parameters of the Poisson process for event occurrence and of the lognormal distribution for event damage and what are the effects of the El Niño phenomenon on these parameters. Section 5 consists of a discussion.

2. Random sum model

In this section, the compound Poisson process is formally defined, including the event occurrence and event damage components. Then the probabilistic properties of a random sum are reviewed, focusing on the decomposition of its variance. Last, techniques for the statistical estimation of the parameters of a compound Poisson process are treated.

a. Event occurrence

It is assumed that the occurrence of events follows a homogeneous Poisson process $N(t)$, $t \geq 0$, denoting the number of events occurring within the time interval $[0, t]$. This process has a single parameter $\lambda > 0$, specifying the instantaneous rate of event occurrence. A Poisson process is intended to represent events that occur at ''random,'' as it arises from certain axioms thought to correspond to randomness (e.g., Ross 1970, chapter 2). In more general terms, a stochastic model for a series of events is termed a point process (e.g., Guttorp 1995, chapter 5).

The random variable $N(t)$ has a Poisson distribution with parameter λt. That is, the probability of k events occurring within the interval $[0, t]$ can be expressed as

$$\Pr\{N(t) = k\} = e^{-\lambda t}(\lambda t)^k/k!, \qquad k = 0, 1, 2, \ldots . \quad (1)$$

In particular, the mean and variance of $N(t)$ are both given by the same parameter λt; that is,

$$E[N(t)] = \mathrm{Var}[N(t)] = \lambda t. \quad (2)$$

The Poisson distribution also arises as an approximation to the binomial distribution for the number of events over a sequence of independent trials, when the probability of occurrence on a single trial is small (e.g., Feller 1968, chapter VI). Given these features, it is a natural candidate for the stochastic modeling of the occurrence of extreme events.

An example of a climate application of the Poisson process is the work of Keim and Cruise (1998), who modeled extreme events such as U.S. ''nor'easters'' and heavy rainfall. The Poisson distribution has been fitted to the frequency of hurricanes by Bove et al. (1998), Elsner and Bossak (2001), and Elsner et al. (1999), including distributions conditional on the state of El Niño. Solow (1995a,b) employed a more general nonparametric approach to the stochastic modeling of El Niño, allowing for point processes that are not necessarily Poisson (also see Solow and Moore 2000).

b. Event damage

Let $X_k > 0$ denote the monetary damage associated with the kth event, $k = 1, 2, \ldots$, and assume that the X_ks are independent and identically distributed. Let the common cumulative distribution function be denoted by $F(x) = \Pr\{X_k \leq x\}$, with mean μ_X and variance σ_X^2. Further assume that the damage process $\{X_k\}$ is statistically independent of the occurrence process $\{N(t)\}$. The joint process, consisting of $\{X_k\}$ in combination with $\{N(t)\}$, is referred to as a ''marked'' Poisson process (i.e., the damage associated with an event is viewed as a mark; Guttorp 1995, chapter 5).

In practice, F would be some positively skewed distribution function on the interval $(0, \infty)$, such as the lognormal. In this case, the log-transformed damages would have a normal distribution; that is,

$$Y_k = \ln X_k \sim N(\mu_Y, \sigma_Y^2). \quad (3)$$

The untransformed parameters, μ_X and σ_X^2, are each functions of both the mean and variance, μ_Y and σ_Y^2, of the transformed variable Y (e.g., Johnson and Kotz 1970, chapter 14):

$$\mu_X = \exp(\mu_Y + \sigma_Y^2/2),$$
$$\sigma_X^2 = \exp[2(\mu_Y + \sigma_Y^2)] - \mu_X^2. \quad (4)$$

Because the standard statistical theory for normally distributed variables can be applied to the transformed data, it is straightforward to incorporate cycles, trends, or covariates into the damage process. Hogg and Klugman (1984, chapter 4) found that the lognormal distribution fit a set of insured hurricane damage data for the United States well (adjusted only for inflation, not societal vulnerability).

c. Total damage

For a compound Poisson process, the total damage over the time interval $[0, t]$ can be expressed as the sum

$$S(t) = X_1 + X_2 + \cdots + X_{N(t)} \quad (5)$$

[conditional on $N(t) \geq 1$; otherwise, $S(t) = 0$]. This representation is termed a random sum, because the number of terms $N(t)$ is not fixed a priori.

The mean and variance of a random sum can be expressed in terms of the parameters of the two component processes through conditioning on the number of events $N(t)$ (e.g., Feller 1968, chapter XII). Making use of the relationship between the unconditional mean and the conditional means and of the expression for the mean of the Poisson distribution (2), the mean total damage is given by

$$E[S(t)] = E\{E[S(t)|N(t)]\} = E[N(t)]E(X_k) = \lambda t \mu_X. \quad (6)$$

Similarly, making use of the relationship between the unconditional variance and the conditional means and conditional variances, as well as of the expressions for the mean and variance of the Poisson distribution (2), the variance of total damage is given by

$$\begin{aligned}\mathrm{Var}[S(t)] &= E\{\mathrm{Var}[S(t)|N(t)]\} + \mathrm{Var}\{E[S(t)|N(t)]\} \\ &= E[N(t)]\,\mathrm{Var}(X_k) + \mathrm{Var}[N(t)][E(X_k)]^2 \\ &= \lambda t E(X_k^2) = \lambda t(\mu_X^2 + \sigma_X^2). \quad (7)\end{aligned}$$

d. Parameter estimation

Suppose that we are given data on the event occurrence process of the form $\{n_i, i = 1, 2, \ldots, m\}$, where n_i denotes the number of storms occurring in the ith year, available for a record of length m years. In the case of a homogeneous Poisson process, the maximum likelihood estimate of the rate parameter λ, denoted by $\hat{\lambda}(\mathrm{yr}^{-1})$, is simply the relative frequency of occurrence of the event in the sample; that is,

$$\hat{\lambda} = n/m,$$

where

$$n = \sum_1^m n_i \quad (8)$$

(Johnson et al. 1992, chapter 4).

Further suppose that we are given data on the event damage process of the form $\{x_k(i), k = 1, 2, \ldots, n_i; i = 1, 2, \ldots, m\}$, where $x_k(i)$ denotes the monetary damage associated with the kth storm in the ith year (provided $n_i \geq 1$). In the case of the distribution of damage from individual storms being lognormal, the maximum likelihood estimates of the parameters μ_Y and σ_Y^2, denoted by $\hat{\mu}_Y$ and $\hat{\sigma}_Y^2$, are simply the sample mean and sample variance (with divisor n, not $n - 1$) of the log-transformed damage data. That is,

$$\hat{\mu}_Y = (1/n) \sum_1^m s_1(i),$$

$$\hat{\sigma}_Y^2 = (1/n) \sum_1^m s_2(i) - \hat{\mu}_Y^2, \quad (9)$$

where

$$y_k(i) = \ln x_k(i), \qquad s_1(i) = \sum y_k(i),$$

$$s_2(i) = \sum [y_k(i)]^2 \quad (10)$$

(Johnson and Kotz 1970, chapter 14). Here the sums for $s_1(i)$ and $s_2(i)$ in (10) range from $k = 1$ to n_i [provided $n_i \geq 1$; otherwise, $s_1(i) = s_2(i) = 0$].

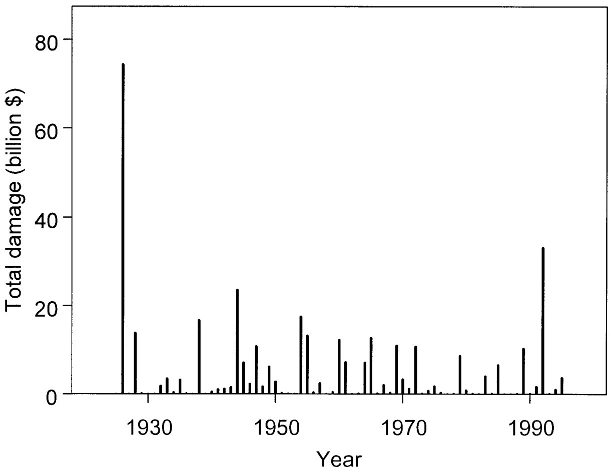

FIG. 1. Annual time series of adjusted total North Atlantic hurricane damage (1995 U.S. $ billion), 1925–95.

3. Application to hurricane damage

In this section, the results of fitting the compound Poisson process to a set of hurricane damage data (Pielke and Landsea 1998) are described. For more detailed background about hurricanes and their societal impact, the reader is referred to Pielke and Pielke (1997).

a. Dataset

Pielke and Landsea (1998) produced a set of damage estimates for individual North Atlantic tropical cyclones (primarily hurricanes) making landfall in the United States (data available at: http://sciencepolicy.colorado.edu/pielke/hp_roger/hurr_norm/data.html). Ranging over the time period 1925–95 (i.e., $m = 71$ yr), the damage data have been adjusted for inflation, wealth, and population at risk and expressed in 1995 U.S. $ billion. One potential problem with this dataset is that a disproportionate number of years with no storms were reported early in the record, suggesting a bias. To minimize any effects of the recording process and adjustment method and consistent with a focus on events with substantial economic impact, any individual storms whose adjusted damage fell below a threshold of $0.01 billion have been omitted in the present study (15 storms were eliminated, mostly late in the record).

Figure 1 shows the annual time series of total hurricane damage for this adjusted dataset, with no trend evident; instead a high degree of volatility from year to year predominates. Table 1 provides descriptive statistics for the hurricane damage data, with a total of $n = 129$ damaging storms occurring over the 71-yr time period.

b. Model fitting

1) EVENT OCCURRENCE

Figure 2 shows the annual time series of the number of damaging hurricanes, again with no obvious trend.

TABLE 1. Descriptive statistics for compound Poisson model of adjusted damage (1995 U.S. $ billion) from North Atlantic hurricanes, 1925–95.

	Sample size	Mean	Median	Std dev	Skewness
Occurrence (storms per year)	71	1.817	—	1.324	—
Damage ($ billion per storm)					
Untransformed	129	2.697	0.349	7.575	1.351
Log-transformed	129	−1.090	−1.082	2.307	−0.002
Total damage ($ billion yr^{-1})	71	4.901	1.031	10.349	0.617

The observed mean rate of hurricanes is slightly less than two per year [an estimate for the rate parameter λ of the Poisson process of 1.817, see (8)], and the sample variance of 1.752 is only slightly smaller (see Table 1). Recall from section 2a that equality of the mean and variance is a fundamental property of the Poisson distribution [see (2)].

As a formal test of whether the mean equals the variance, the chi-square test statistic (Johnson et al. 1992, chapter 4) is given by

$$\chi^2 = (m - 1)\hat{\sigma}_N^2/\hat{\lambda}, \quad (11)$$

where $\hat{\sigma}_N^2$ denotes the sample variance of the annual number of hurricanes (with divisor $m - 1$). This statistic has an approximate chi-squared distribution with $m - 1$ degrees of freedom (df), under the null hypothesis of equality of the mean and variance. For the hurricane occurrence data, the observed value of χ^2 in (11) is 67.49 with 70 df, yielding a P value of 0.874, or no indication whatsoever that the variance actually differs from the mean.

2) EVENT DAMAGE

Because the storms whose damage fell below a threshold of $c = \$0.01$ billion have been removed from the data, the logarithmic transformation is applied to the excess in damage over this threshold [i.e., $Y_k = \ln(X_k - c)$ instead of (3)]. Figure 3 shows the time series of the logarithm of the adjusted damage from individual hurricanes. Although any pattern is somewhat difficult to interpret because of the varying number of storms from year to year, no trend is evident. For the untransformed damages, the sample mean is much larger than the sample median ($2.697 vs $0.349 billion; see Table 1), indicating that the distribution is highly positively skewed. A quantile–quantile (Q–Q) plot (Fig. 4) of the log-transformed damages versus the normal distribution indicates an acceptable fit, except perhaps in the extreme upper tail.

As an index of symmetry, the measure

$$d = (\text{mean} - \text{median})/\text{IQ} \quad (12)$$

is employed (Hinkley 1977). Here IQ denotes the interquartile range (i.e., IQ = upper-quartile − lower-quartile), a robust measure of the spread of the distribution. For the untransformed damage, this index is $d = 1.351$, as compared with $d = -0.002$ for log-transformed damage whose sample mean and median are quite close (−1.090 vs −1.082) (see Table 1). Recall that the sample mean and variance of the transformed data provide the maximum likelihood estimates for the lognormal distribution as well [(9)–(10)]. Alternative transformations, such as the square, cube, or fourth roots, do not eliminate as much of the positive skewness.

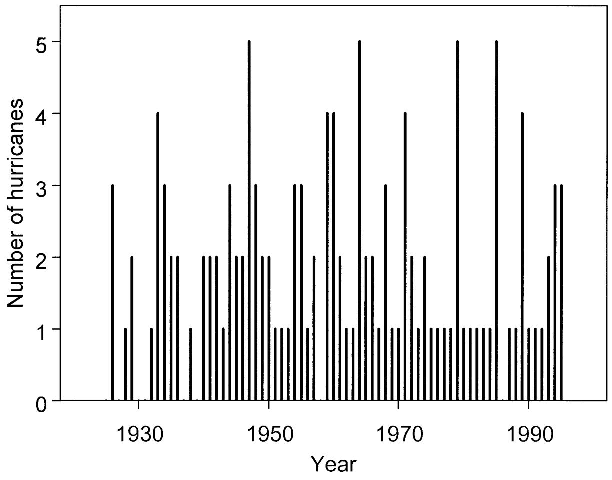

FIG. 2. Annual time series of number of damaging North Atlantic hurricanes, 1925–95.

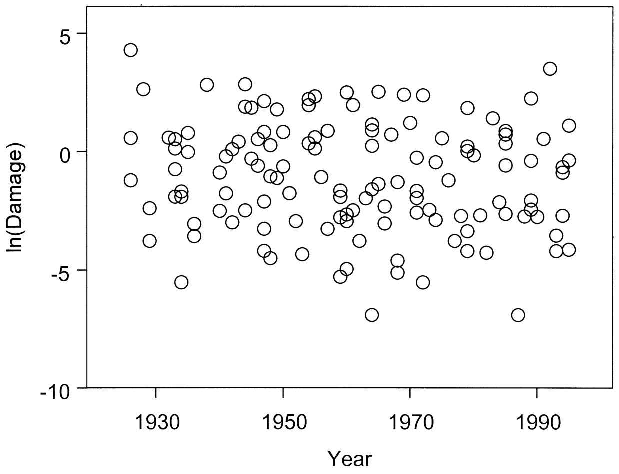

FIG. 3. Time series of logarithm (ln) of adjusted damage from individual North Atlantic hurricanes, 1925–95.

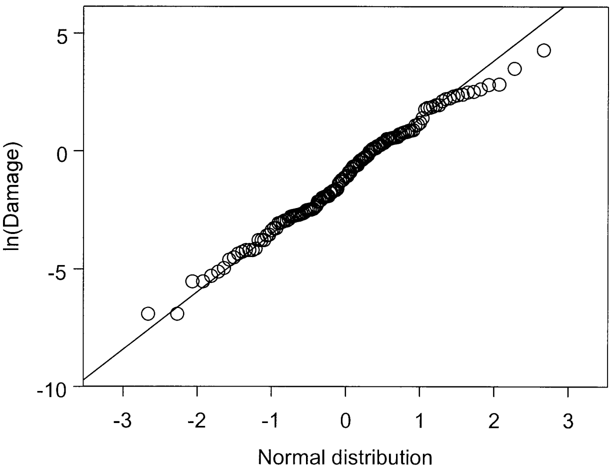

FIG. 4. Q–Q plot for logarithm (ln) of adjusted damage from individual North Atlantic hurricanes vs normal distribution.

Thus, there is some evidence to support the use of a lognormal distribution for individual storm damage.

It remains to check the assumption that, conditional on the number of hurricanes within a year, the individual damages are independent and identically distributed. The damage for the first and second storms within a year is compared (35 yr with 2 or more hurricanes). The sample means of the log-transformed damage are -1.483 and -1.227, respectively (P value $= 0.638$ for two sample t test of equality of means), or a lack of evidence of nonidentical distributions. The sample correlation coefficient between the log-transformed damage for the first and second storms is -0.036, or a lack of evidence of dependent distributions. The damage for the second and third storms within a year could be compared as well, but the sample size is small (18 yr with three or more hurricanes). If necessary, it would be possible to relax the assumption of independence and identical distribution while still retaining the random sum representation [e.g., as done for total precipitation in Katz and Parlange (1998)].

c. Total damage

The adjusted annual total damage from hurricanes has a skewed distribution, with the sample mean being much larger than the sample median ($4.901 vs $1.031 billion; see Table 1). But because of the effect of summing up the damage from an, albeit random, number of storms, the degree of skewness ($d = 0.617$) is somewhat less than that for individual storms. To more fully characterize the sources of variation in total damage, the random sum representation can be utilized. The most general approach to decomposing the variance of total damage is taken, imposing as few conditions as possible. Specifically, the assumptions that the number of storms be Poisson-distributed and that the damages from individual storms be independent and identically distributed (made in section 2) are both relaxed.

TABLE 2. Decomposition of variance of adjusted annual total damage from North Atlantic hurricanes ($ billion).

No. storms per year	No. yr	Conditional mean	Conditional variance
0	8	—	—
1	28	3.332	$(7.111)^2$
2	17	3.431	$(3.815)^2$
3	9	15.081	$(22.453)^2$
4	5	5.528	$(4.819)^2$
5	4	8.246	$(1.654)^2$
Expected value	—	4.901	$(9.440)^2$
Variance	—	$(4.241)^2$	—

Table 2 lists the sample estimates of the conditional means and variances necessary to perform such a decomposition. As not many cases occur with this stratification of the data (e.g., only 4 yr had exactly 5 storms), it is difficult to interpret any of the apparent patterns in the relationship between the conditional mean or the conditional variance and the number of damaging storms. Substituting these sample estimates into the first equation in (7) yields [in ($ billion)^2]:

$$(10.349)^2 = (9.440)^2 + (4.241)^2.$$

In other words, about 17% of the variation in annual damage totals is attributable to fluctuations in the annual number of storms. Despite this component being considerably smaller than the one for individual damage, it still makes a contribution that should not be neglected.

4. Extensions of model

The purpose of this section is to demonstrate ways in which the compound Poisson process for hurricane damage can be generalized. These extensions include trends in the event occurrence and damage processes and the incorporation of covariates such as El Niño.

a. Trends

1) EVENT OCCURRENCE

The possibility of a trend over time in the rate parameter of the Poisson process for the occurrence of storms (Fig. 2) is considered; specifically, an exponential curve.

$$\ln\lambda(i) = \alpha_\lambda + \beta_\lambda i, \qquad i = 1, 2, \ldots, m, \qquad (13)$$

where $\lambda(i)$ denotes the rate in the ith year. This form of trend model is convenient, in that it constrains the rate of storms to be positive. Table 3 includes the results of fitting (13), with the maximum likelihood estimate of the slope parameter β_λ of the trend curve corresponding to a proportionate increase in the mean number of storms of only about 0.003 yr^{-1}, for a P value of 0.492 (based on the likelihood ratio test; Stuart and Ord 1991, chapter 23), or no evidence of a trend. If the events whose damage fell below a threshold of $0.01 billion

TABLE 3. Fit and test of trends (yr^{-1}) in parameters of compound Poisson model for adjusted North Atlantic hurricane damage.

Model	Intercept	Slope (std error)	Test: Statistic	Test: *P* value
Poisson process for occurrence	0.4889	0.0030 (0.0043)	$\chi^2 = 0.47$	0.492
Lognormal distribution for damage	−0.5132	−0.0155 (0.0105)	$t = -1.48$	0.142

had not been eliminated (see section 3a), then this apparent increasing trend would obtain borderline statistical significance (perhaps indicative of a recording bias early in the record or an artifact of the adjustment method).

2) EVENT DAMAGE

The possibility of a trend in the mean of the normal distribution for the log-transformed damage from individual hurricanes (Fig. 3) is considered; specifically, a linear trend

$$\mu_Y(i) = \alpha_\mu + \beta_\mu i, \qquad i = 1, 2, \ldots, m, \qquad (14)$$

where $\mu_Y(i)$ denotes the mean in the ith year. Because the logarithmic transformation has been applied, this trend model also has the convenience of constraining the untransformed damage to be positive. Table 3 includes the results of fitting (14), with the least squares estimate of the slope β_μ corresponding to a proportionate decrease in the median damage of about −0.015 yr^{-1}, for a *P* value of 0.142 (based on the *t* test) or only weak evidence of a negative trend. If the low threshold had not been imposed, then a statistically significant decreasing trend would have been obtained (again, perhaps an artifact of the recording process or adjustment method).

TABLE 4. Fit and identification of conditional Poisson model for North Atlantic hurricane occurrence given El Niño state.

El Niño state	No. yr	No. storms	Rate	Variance
(a) Descriptive statistics				
La Niña	22	48	2.182	1.394
Neutral	28	54	1.929	2.661
El Niño	21	27	1.286	0.614
All states	71	129	1.817	1.752

(b) Model identification

Model	No. parameters	Estimated parameters $\hat{\lambda}_{-1}$	$\hat{\lambda}_0$	$\hat{\lambda}_1$	Deviance	BIC
$\lambda_{-1} = \lambda_0 = \lambda_1$	1	1.817	1.817	1.817	72.228	76.491
$\lambda_0 = \lambda_1$	2	2.182	1.653	1.653	69.967	78.493
$\lambda_{-1}/\lambda_0 = \lambda_0/\lambda_1$	2	2.278	1.777	1.387	67.489	76.015
$\lambda_{-1} = \lambda_0$	2	2.040	2.040	1.286	67.276	75.801*
No constraints	3	2.182	1.929	1.286	66.890	79.678

* Denotes minimum BIC value (*P* value = 0.029 for likelihood ratio test).

b. El Niño effects

As another extension, the dependence of hurricane damages on the El Niño phenomenon is explored. A connection between the frequency of hurricanes and El Niño has long been known (Gray 1984). More recently, Pielke and Landsea (1999) established a relationship between total hurricane damage and El Niño events. It remains to determine the extent to which this relationship is attributable to the connection between El Niño and the frequency of hurricanes or to any connection between El Niño and the damage associated with individual storms (i.e., through a dependence of the intensity of hurricanes on El Niño).

The classification of Trenberth (1997), as adapted by Pielke and Landsea (1999), is based on sea surface temperatures in the so-called Niño 3.4 region of the Pacific. Each year is classified into one of three possible states of El Niño ($j = -1, 0, 1$):

1) La Niña event ($j = -1$),
2) neutral event ($j = 0$), and
3) El Niño event ($j = 1$).

1) EVENT OCCURRENCE

The Poisson rate parameter, say λ_j, $j = -1, 0, 1$, of the occurrence of damaging hurricanes now possibly depends on the state of El Niño. Table 4a provides descriptive statistics classified according to the El Niño state, with the estimated rate being nearly one damaging hurricane per year higher during La Niña than El Niño events (a result consistent with many earlier analyses). The fit of five candidate models for the Poisson rate parameter, ranging from no dependence (i.e., $\lambda_{-1} = \lambda_0 = \lambda_1$) to complete dependence (i.e., three different rates) and including those with constraints imposed to reduce the number of parameters estimated, is compared in Table 4b. Here the "deviance" statistic listed in the table for each candidate model is a goodness-of-fit measure, essentially −2 times the maximized log likelihood function. The Bayesian information criterion (BIC), listed in the table for each model, involves penalizing the deviance for the number of parameters, with smaller values being preferable (Katz 1981; Schwarz 1978; Venables and Ripley 1999, chapters 6–7).

Two of the models involving dependence on the state of El Niño have lower BIC values than the case of no dependence; one varies the rate parameter between El Niño and non-El Niño years (i.e., $\lambda_{-1} = \lambda_0$; this model

TABLE 5. Fit and identification of conditional lognormal distribution for adjusted damage from individual North Atlantic hurricanes given El Niño state.

El Niño state	No. storms	Mean	Median	Std dev
(a) Descriptive statistics for untransformed damage ($ billion)				
La Niña	48	2.517	0.942	3.891
Neutral	54	3.427	0.199	10.781
El Niño	27	1.559	0.149	3.518
All states	129	2.697	0.349	7.575
(b) Descriptive statistics for log-transformed damage				
La Niña	48	−0.454	−0.087	2.047
Neutral	54	−1.237	−1.666	2.360
El Niño	27	−1.925	−1.973	2.322
All states	129	−1.090	−1.082	2.307

(c) Model identification

Model	No. parameters	Estimated parameters $\hat{\mu}_Y(-1)$	$\hat{\mu}_Y(0)$	$\hat{\mu}_Y(1)$	MSE	BIC
$\mu_Y(-1) = \mu_Y(0) = \mu_Y(1)$	1	−1.090	−1.090	−1.090	5.325	219.99
$\mu_Y(-1) = \mu_Y(0)$	2	−0.869	−0.869	−1.925	5.140	219.70
$\mu_Y(0) = \mu_Y(1)$	2	−0.454	−1.466	−1.466	5.085	218.32
$\mu_Y(-1) - \mu_Y(0) = \mu_Y(0) - \mu_Y(1)$	2	−0.469	−1.210	−1.952	5.020	216.65*
No constraints	3	−0.454	−1.237	−1.925	5.019	220.90

* Denotes minimum BIC value (P value = 0.006 for partial F test).

has minimum BIC; see Table 4b), the other constrains the effect to be proportionately the same magnitude for both El Niño and La Niña events (i.e., $\lambda_{-1}/\lambda_0 = \lambda_0/\lambda_1$). Comparing the no-effect model with the one constraining $\lambda_{-1}/\lambda_0 = \lambda_0/\lambda_1$ (although this model does not attain minimum BIC, it is more physically appealing by allowing both La Niña and El Niño events to have an effect), a likelihood ratio test also indicates statistical significance (P value = 0.029) despite the relatively small decrease in the value of BIC. For both La Niña and El Niño events, the sample variance is quite a bit less than the mean (Table 4a), suggesting that the Poisson may not necessarily be an ideal approximation for these conditional distributions. Bove et al. (1998) also noted a departure from the Poisson distribution for hurricane counts during El Niño events.

2) EVENT DAMAGE

The mean of the log-transformed adjusted damage from individual hurricanes, say $\mu_Y(j)$, $j = -1, 0, 1$, now possibly depends on the state of El Niño. Although the variance of the log-transformed damage σ_Y^2 is taken independent of the El Niño state, a shift in the mean μ_Y with El Niño would imply shifts in both the mean and variance of the untransformed damage, μ_X and σ_X^2, as well [see (4)]. Table 5a provides descriptive statistics for the untransformed storm damages classified according to the state of El Niño, with the sample median ranging from $0.149 to $0.942 billion for El Niño and La Niña events, respectively. Table 5b provides descriptive statistics for the corresponding log-transformed damages, with the sample mean being highest during La Niña events (consistent with the ordering of the sample medians for the untransformed data; Table 5a).

The fit of five candidate models for the mean of log-transformed damage, ranging from no dependence, [i.e., $\mu_Y(-1) = \mu_Y(0) = \mu_Y(1)$] to complete dependence (i.e., three different means) and including those with constraints imposed, is compared in Table 5c. Here the mean-square error (MSE) listed in the table for each candidate model is a goodness-of-fit measure, which the BIC penalizes for the number of parameters estimated. All three constrained models have lower BIC values than the case of no dependence, with the best one constraining the effects on the mean to be the same magnitude for both El Niño and La Niña events [i.e., $\mu_Y(-1) - \mu_Y(0) = \mu_Y(0) - \mu_Y(1)$]. Comparing the no-effect model with the one with minimum BIC, a partial F test also indicates statistical significance (P value = 0.006). This result is consistent with the finding of Landsea et al. (1999) that hurricane intensity is higher during La Niña than El Niño.

3) TOTAL DAMAGE

The sample median of adjusted annual total damage from hurricanes ranges from $0.149 to $3.072 billion for El Niño and La Niña events, respectively (Table 6). Using the first equation in (7), the variance of total damage can be decomposed into that attributable to the El Niño phenomenon and that due to other sources (i.e., conditioning on the state of El Niño instead of the number of events). For the statistics listed in Table 6, about 3%–4% of the variance in total damage is attributable to El Niño. In other words, although the El Niño phe-

TABLE 6. Descriptive statistics for conditional distribution of adjusted annual total damage ($ billion) from North Atlantic hurricanes given El Niño state.

El Niño state	No. yr	Mean	Median	Std dev
La Niña	22	5.491	3.072	6.373
Neutral	28	6.610	0.656	14.787
El Niño	21	2.004	0.149	3.925
All states	71	4.901	1.031	10.349

nomenon is a significant source of variation, much of the variation in total damage remains to be explained.

The relationship (6) expresses the mean of annual total damage in terms of the parameters of both storm occurrence and damage. It can be applied to determine how much of the effect of El Niño on total damage is attributable to the individual effect of El Niño on storm occurrence or on storm damage. If only the rate of storm occurrence is varied using its sample values listed in Table 4a, then the mean of annual total damage would range from $3.468 billion in El Niño to $5.885 billion in La Niña (a difference of $2.417 billion). If only the mean of individual storm damage is varied using its sample values listed in Table 5a, then the mean of annual total damage would range from $2.833 billion in El Niño to $4.573 billion in La Niña (a difference of $1.740 billion). So each component makes a roughly comparable contribution to the effect of the El Niño phenomenon on the mean of total damage.

5. Discussion

A stochastic modeling approach has been advocated to treat more explicitly different sources of variation in total economic damage from hurricanes. The model has been applied in a reanalysis of a set of adjusted damage data from hurricanes (Pielke and Landsea 1998). The conclusions reached are consistent with those of Pielke and Landsea (1998) concerning the lack of any trends, as well as with other trend analyses of damages from extreme weather or climate events (e.g., Changnon et al. 2000; Kunkel et al. 1999). They also are consistent with those of Pielke and Landsea (1999) concerning the existence of a relationship with the El Niño phenomenon (cf. Landsea et al. 1999).

By enabling the variations in total damage to be attributed to either variations in event occurrence or in event damage, the present modeling approach has an inherent advantage over previous analyses. For instance, it has been shown that the connection between annual total hurricane damage and the state of El Niño is due as much to the effects of El Niño on damage from individual hurricanes as to its effects on the frequency of occurrence of hurricanes. In this regard, it would be straightforward to incorporate other covariates, including continuous variables instead of discrete ones (e.g., a continuous index of the Southern Oscillation instead of the discrete El Niño state). Possible covariates for damage from hurricanes in the North Atlantic were discussed in Landsea et al. (1999).

Although the lognormal distribution has been fitted to the damage from individual hurricanes in the present paper, there is some evidence that the damage distribution has a heavier right-hand tail (Katz 2002). Rootzén and Tajvidi (1997) also found that the damage from windstorms has a distribution whose upper tail is heavier than the lognormal. The question of how extremely high damages are distributed is naturally of particular interest to the insurance and reinsurance industry.

The stochastic approach presented is general enough that it could be readily applied to other extreme weather or climate phenomena besides hurricanes. For example, economic damage from floods raises similar issues to those for hurricanes, in that there are marked increasing trends in damage unadjusted for shifts in societal vulnerability. Despite apparent increasing trends in extreme high precipitation events (e.g., Karl and Knight 1998), it is not necessarily the case that any of these trends in flood damage are actually attributable to climate change (Pielke and Downton 2000).

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