NOTES AND CORRESPONDENCE

Comments on “Estimating Soil Water Contents from Soil Temperature Measurements by Using an Adaptive Kalman Filter”

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ABSTRACT

A scheme was proposed by Zhang et al. to estimate soil water content from soil temperature measurements by using an adaptive Kalman filter. Their scheme is based on the fact that soil heat capacity and thermal conductivity are a monotonic function of soil water content. However, thermal diffusivity, a more critical thermal parameter in such an estimation, is not a monotonic function of soil water content in most cases. This could result in multiple solutions in some cases when deriving soil water content from soil temperatures.

1. Introduction

Recently, Zhang et al. (2004, henceforth Z04) proposed a method to inversely estimate soil water content from the soil temperature profile based on the relation between soil water content and soil thermal properties (heat capacity and thermal conductivity). Although their method was tested with in situ collected data at two experiment sites, this note will indicate that multiple solutions can exist in some cases.

2. Relation between thermal properties and soil water content

The heat transfer process in a soil can be described by the thermal diffusion equation

$$C(\theta)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[\lambda(\theta)\frac{\partial T}{\partial z}\right], \qquad (1)$$

where T is the soil temperature (K), θ is the soil water content (m^3 m^{-3}), $C(\theta)$ is the volumetric soil heat capacity (J m^{-3} K^{-1}), $\lambda(\theta)$ is the soil thermal conductivity (Wm^{-1} K^{-1}), and z is the vertical coordinate (distance from the surface, m).

If the soil thermal conductivity varies slowly with respect to the depth [e.g., in the case of wet soils, $\lambda(\theta)$ is no longer sensitive to θ, and, thus, $\partial\lambda(\theta)/\partial z \sim 0$], Eq. (1) may be rewritten as

$$\frac{\partial T}{\partial t} = k(\theta)\frac{\partial^2 T}{\partial z^2}. \qquad (2a)$$

If the soil heat capacity varies slowly with respect to the depth [e.g., in the case of dry soils, θ is relatively uniform, and, thus, $\partial C(\theta)/\partial z \sim 0$], Eq. (1) may be rewritten as

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[k(\theta)\frac{\partial T}{\partial z}\right], \qquad (2b)$$

where $k(\theta)=\lambda(\theta)/C(\theta)$ is the soil thermal diffusivity (m^2 s^{-1}).

Given soil temperatures as boundary conditions, as in Z04, the solution of Eq. (2) is the thermal diffusivity $k(\theta)$. Therefore, the necessary condition to uniquely derive soil water content from soil temperature is that $k(\theta)$ is a monotonic function of soil water content, rather than that $C(\theta)$ and/or $\lambda(\theta)$ are monotonic functions of soil water content. The following discusses the monotonicity of the thermal diffusivity function based on some recent investigations on soil heat capacity and thermal conductivity.

Soil heat capacity linearly increases with soil water content, and can be calculated from

$$C(\theta) = \rho_d c_d + \rho_w c_w \theta, \qquad (3)$$

where $\rho_w c_w$ is the water heat capacity, and $\rho_d c_d$ is the heat capacity of the dry soil. Here, $\rho_w c_w = 4.18 \times 10^6$

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J m^{-3} K^{-1}, and $\rho_d c_d = (0.076 + 0.748\rho_d) \times 10^6$ J m^{-3} K^{-1} (Global Soil Data Task 2000); ρ_d is the bulk density of the dry soil (g cm^{-3}).

The relation between the thermal conductivity and soil water content is quite complex. The following formula simply relates the conductivity with soil water potential, and is used in Z04 and some other studies (e.g., McCumber and Pielke 1981; Noilhan and Planton 1989):

$$\lambda(\theta) = \begin{cases} 418\exp(-\log_{10}|\psi| - 2.7) & \text{for} \quad \log_{10}|\psi| \le 5.1 \\ 0.17 & \text{for} \quad \log_{10}|\psi| > 5.1, \end{cases} \tag{4}$$

where ψ is the soil water potential (cm) and is defined by $\psi = \psi_s(\theta/\theta_s)^{-b}$. Here, θ_s is the saturated soil volumetric water content (m^3 m^{-3}), ψ_s is the "saturated" soil water potential (cm), and b is the pore-sized distribution index constant (Clapp and Hornberger 1978).

Johansen (1975) takes quartz content into account for the thermal conductivity. Farouki (1986) compares various empirical formulas and indicates that Johansen's method is generally the most accurate. Thus, the method of Johansen (1975) has been applied in a number of studies and has several variants (Peters-Lidard et al. 1998; Cherkauer and Lettenmaier 1999; Dai et al. 2001). We give the following variant and discuss its properties:

$$\lambda_s = \lambda_d + (\lambda_m - \lambda_d)\exp[0.36(1 - \theta_s/\theta)], \tag{5}$$

$$\lambda_d = (135\rho_d + 64.7)/(2700 - 947\rho_d), \quad \text{and} \tag{6}$$

$$\lambda_m = 0.5^{\theta_s}(7.7^{m_q}2.0^{1-m_q})^{1-\theta_s}, \tag{7}$$

where m_q is the quartz mass fraction in the soil material.

Another widely used empirical relation for the thermal conductivity is as follows (see Bristow 2002):

$$\lambda(\theta) = A + B\theta - (A - D)\exp[-(C\theta)^E], \tag{8}$$

$$A = (0.57 + 1.73\phi_q + 0.93\phi_m)/(1 - 0.74\phi_q - 0.49\phi_m) - 2.8\phi_s(1 - \phi_s), \tag{9}$$

$$B = 2.8\phi_s, \tag{10}$$

$$C = 1 + 2.6/m_c^{0.5}, \tag{11}$$

$$D = 0.03 + 0.7\phi_s^2, \quad \text{and} \tag{12}$$

$$E = 4, \tag{13}$$

where ϕ is the volume fraction of a particular component, subscripts "q," "m," and "s" indicate quartz, minerals other than quartz, and total solids, respectively, and m_c is the clay mass fraction in the soil material.

Figures 1a–c show the thermal conductivity calculated from Eqs. (4), (5), and (8) for a range of typical soil types and Figs. 2a–c show the correspondent thermal diffusivity. The relevant soil parameters in these equations are shown in Table 1; derived from the Global Soil Wetness Project (GSWP)-2 input data (information online at http://grads.iges.org/gswp/input.html). It is clear that the thermal conductivity used in Z04 increases rapidly when the soil water content approaches the saturation point (Fig. 1a), and results in the thermal diffusivity $k(\theta)$ being a monotonic function of soil water content (Fig. 2a). However, the thermal conductivity values are unrealistically high (>3 W m^{-1} K^{-1}), and, thus, the monotonic function $k(\theta)$ is unreliable. On the other hand, both Eqs. (5) and (8) give a similar variation of the thermal conductivity with soil water content and have realistic thermal conductivity values near saturation (<3 W m^{-1} K^{-1}), as shown in Figs. 1b and 1c. A distinguishing feature of the two figures is that the thermal conductivity increases rapidly between 20% and ~60% soil wetness, while it increases slowly in very dry or very wet conditions. This variation is also demonstrated in Global Soil Data Task (2000). Accordingly, the thermal diffusivity is not a monotonic function of soil water content. As shown in Figs. 2b and 2c, maximum thermal diffusivity does not occur at the saturation point but at a moderate water content; and the thermal diffusivity is not sensitive when the soil wetness is greater than 0.5. Garratt (1992) also gives a similar variation (e.g., for a sand soil, the thermal diffusivity is 2.3×10^{-7} m^2 s^{-1} at $\theta = 0$, 8.43×10^{-7} m^2 s^{-1} at $\theta = 0.2$, and 7.40×10^{-7} m^2 s^{-1} at $\theta = 0.4$; for a clay soil, the thermal diffusivity is 1.80×10^{-7} m^2 s^{-1} at $\theta = 0$, and 5.20×10^{-7} m^2 s^{-1} at $\theta = 0.2$ and $\theta = 0.4$).

In effect, both the soil heat capacity and the thermal conductivity monotonically increase with respect to soil water content, but the thermal diffusivity is not a monotonic function of soil water content. If a soil becomes quite dry or quite wet, deriving soil water content from soil temperature can in some cases result in multiple solutions.

3. Summary and suggestion

The close relationship between soil thermal properties and soil water content forms the physical basis of Z04. This note suggests that in most cases the thermal diffusivity is not a monotonic function of soil water content, which can result in multiple solutions of soil water content. Water content estimation is peculiarly difficult for wet soils because the thermal conductivity and diffusivity are insensitive to soil water content in this case. A robust algorithm for deriving soil water content should take both soil temperatures and soil heat fluxes into account. For example, an inverse method modeling both subsurface thermal diffusion and surface energy budget may be more appropriate.

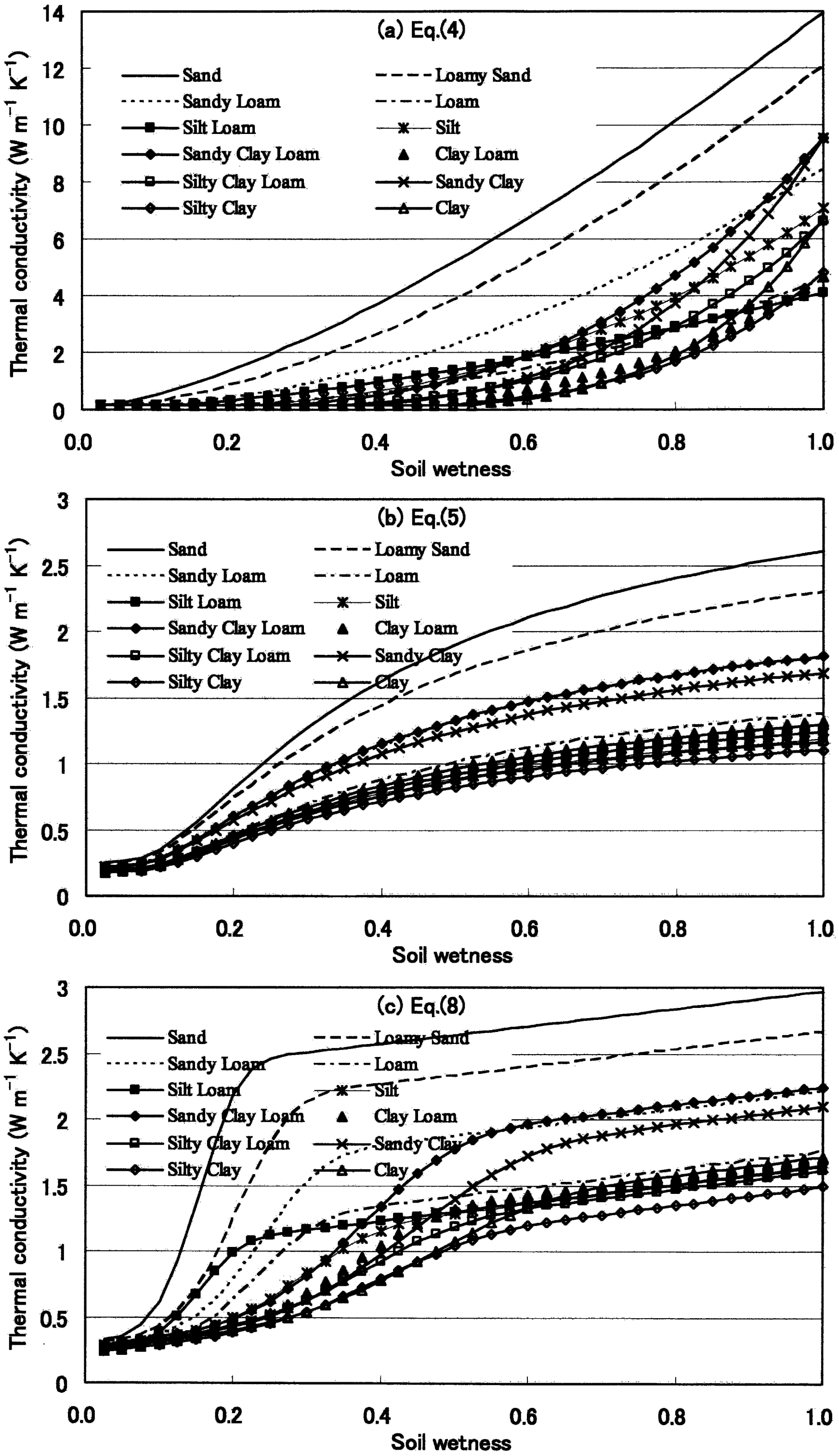

FIG. 1. Soil thermal conductivity calculated from (a) Eq. (4), (b) Eq. (5), and (c) Eq. (8), using the parameters in Table 1.

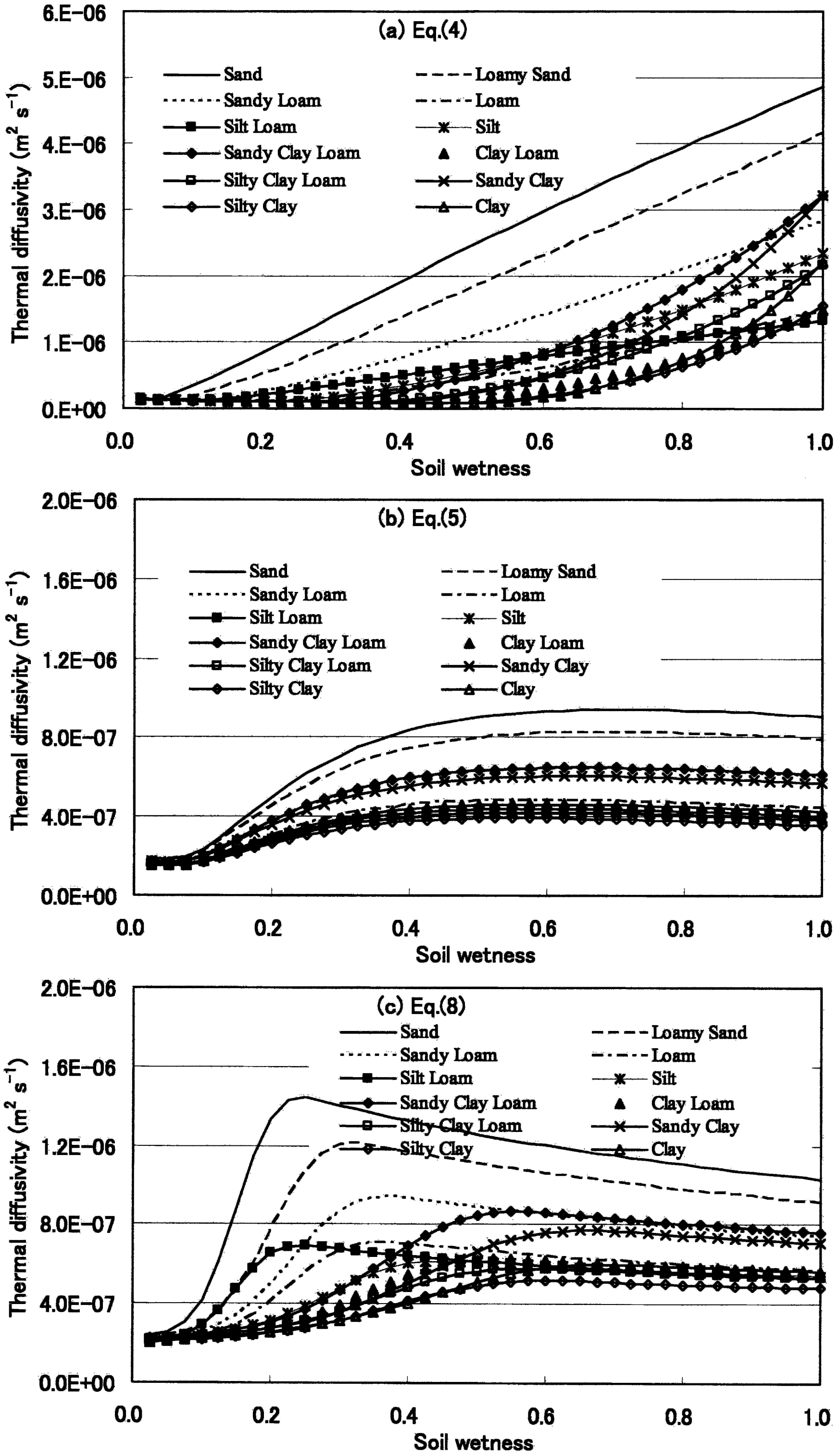

FIG. 2. Soil thermal diffusivity calculated from (a) Eq. (4), (b) Eq. (5), and (c) Eq. (8), using the parameters in Table 1.

TABLE 1. Relevant soil parameters for U.S. Geological Survey (USGS) soil texture classifications.

Soil type	θ_s (m^3 m^{-3})	b	ψ_s (cm)	ρ_b (g cm^{-3})	ϕ_q	ϕ_m	ϕ_s	m_c	m_q
Sand	0.373	3.3	−5	1.66	0.58	0.05	0.63	0.03	0.92
Loamy sand	0.386	3.8	−7	1.63	0.50	0.11	0.61	0.06	0.82
Sandy loam	0.419	4.34	−16	1.54	0.35	0.23	0.58	0.10	0.60
Loam	0.476	5.25	−65	1.39	0.21	0.31	0.52	0.13	0.40
Silt loam	0.471	3.63	−84	1.40	0.13	0.40	0.53	0.05	0.25
Silt	0.437	5.96	−24	1.49	0.06	0.51	0.56	0.18	0.10
Sandy clay loam	0.412	7.32	−12	1.56	0.35	0.24	0.59	0.27	0.60
Clay loam	0.478	8.41	−63	1.38	0.18	0.34	0.52	0.34	0.35
Silty clay loam	0.447	8.34	−28	1.47	0.06	0.50	0.55	0.34	0.10
Sandy clay	0.415	9.7	−12	1.55	0.30	0.28	0.59	0.42	0.52
Silty clay	0.478	10.78	−58	1.38	0.05	0.47	0.52	0.47	0.10
Clay	0.45	12.93	−27	1.46	0.14	0.41	0.55	0.58	0.25

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