Reply

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1. Introduction

In their comment, Yang and Koike (2005, referred to as YK05 henceforth) analyzed two recent versions of empirical formulations for soil thermal conductivity (which are considered to be more accurate) in comparison with the old version used in Eq. (4) of Zhang et al. (2004, referred to as Z04 henceforth). As shown in YK05's Fig. 2, the soil thermal diffusivities computed by these recent versions of formulations do not always increase monotonically with the soil water content. Based on this, YK05 discussed the possible existence of multiple solutions if the recent formulations are used by the method of Z04 to estimate the daily averaged soil water content from observed diurnal variations of soil temperatures. While we thank YK05 for drawing attention to this problem, we would like to add some comments of our own on the applicability and limitation of the adaptive Kalman filter method of Z04.

2. Applicability of the method

YK05 showed that the soil thermal conductivity computed by the empirical formulation in Eq. (4) of Z04 increases too rapidly and becomes unrealistically high as the soil water content approaches the saturation point. Because of this, the soil water content was underestimated for wet soil in Z04. This explains why the estimated soil water content could not reach the peak values that were measured in the shallow soil layer on the heavy rain days (see Figs. 1–3 of Z04). This problem was discussed qualitatively in the last paragraph of section 3 of Z04, which is consistent with the quantitative results in Fig. 1 of YK05. Here, we only need to discuss the applicability of the adaptive Kalman filter method with the improved formulations in Eqs. (5)–(13) of YK05.

As explained in the introduction of Z04, the work of Z04 was motivated by the previous study of Xu and Zhou (2003, referred to as XZ03 henceforth). Although the adaptive Kalman filter method is a significant improvement over the simple linear-regression method of XZ03, both methods depend on the variabilities of the soil heat capacity and thermal conductivity as functions of the soil water content w (or θ, as in YK05). Because the above-mentioned problems can be easily seen for the method of XZ03, they are discussed below in connection with XZ03 first and then Z04.

In the method of XZ03, the vertical variation of the soil thermal conductivity D (or λ, as in YK05) is neglected, so D can be combined with the soil heat capacity C into a single parameter, that is, the soil thermal diffusivity $k = D/C$. In this case, the thermal diffusion equation is simplified into $dT/dt = kd^2T/dz^2$. With this simplification, as shown in XZ03, the daily averaged soil thermal diffusivity $k = D/C$ can be estimated for a soil layer by the linear regression from observed diurnal variations of soil temperatures at three (or two) different depths. Then, the averaged soil water content w is estimated by inverting the function form of $k(w)$. When the improved formulations in Eqs. (5)–(7) or Eqs. (8)–(13) of YK05 are used, there are two obvious problems for the inversion—(a) the inversion becomes inaccurate and ill posed when the $k(w)$ curve becomes flat (see Figs. 2b,c of YK05) and, thus, k is insensitive to w; and (b) the inversion yields two estimates of w when the estimated value of k is intercepted twice by the $k(w)$ curve on the two sides of the maximum. The latter (b) is the problem mentioned by YK05 that causes multiple solutions.

The adaptive Kalman filter method of Z04 considers the random part of the equation error, so the required sensitivities of C and D to w should be significantly lower than those required by the method of XZ03. This means that the adaptive Kalman filter method should

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be less severely affected by the above-mentioned problem (a) than the method of XZ03. Furthermore, the adaptive Kalman filter method uses the original form of the soil thermal diffusion equation in which C and D cannot be generally combined into a single parameter, such as $k = D/C$. Because the method estimates the vertical profile (rather than a single value) of the daily averaged soil water content from observed diurnal variations of soil temperatures at different depths, it is unlikely to have the above-mentioned problem (b). To see this, we rewrite the original form of soil thermal diffusion equation into $dT/dt = kd^2T/dz^2 + [(dD/dz)/C]dT/dz$. The two terms on the right-hand side of this equation indicate that both k and $(dD/dz)/C$ need to be estimated to match the observed diurnal variations of soil temperatures at different depths. When the improved formulations in Eqs. (5)–(7) or Eqs. (8)–(13) of YK05 are used, the estimated vertical profile of k may correspond to two different profiles of w, denoted by $w_1(z)$ and $w_2(z)$. The estimated $(dD/dz)/C$, however, can match only one of the two profiles, in general. This implies that the adaptive Kalman filter method can have only one solution.

Based on the above discussion, a rigorous proof of the uniqueness of the solution for the adaptive Kalman filter method can be given as follows. Assume that there are two solutions w_1(z) and w_2(z), satisfying both

$$k[w_1(z)] = k[w_2(z)] \tag{1}$$

and

$$\{dD[w_1(z)]/dz\}/C[w_1(z)] = \{dD[w_2(z)]/dz\}/C[w_2(z)]. \tag{2}$$

Note that Eq. (1) can be satisfied only if $k[w_1(z)]$ and $k[w_2(z)]$ are on two sides of the maximum of a $k(w)$ curve in Fig. 2b or 2c of YK05. In this case, dk/dw_1 and dk/dw_2 must have opposite signs. Then, dw_1/dz and dw_2/dz must also have opposite signs, because $(dk/dw_1)(dw_1/dz) = (dk/dw_2)(dw_2/dz)$ as derived from the vertical derivative of Eq. (1). Note that dD/dw and C are always positive, and so the left-hand side of Eq. (2), that is, $\{dD[w_1(z)]/dz\}/C[w_1(z)] = (dD/dw_1)(dw_1/dz)/C[w_1(z)]$ has the same sign as dw_1/dz. Similarly, the right-hand side of Eq. (2) has the same sign as dw_2/dz. But, as required by Eq. (1), dw_1/dz and dw_2/dz must have opposite signs, as well as the two sides of Eq. (2). Thus, Eqs. (1) and (2) cannot be simultaneously satisfied unless $dw_1/dz = dw_2/dz = 0$ over the entire depth. Only one solution, $w_1(z)$ or $w_2(z)$, can be truly optimal.

3. Summary

The applicability and reliability of the adaptive Kalman filter method proposed by Z04 depend on the variabilities of the soil heat capacity and thermal conductivity as functions of the soil water content. Clearly, if the soil heat capacity and thermal conductivity, and, thus, the diurnal variations of soil temperatures, were not affected by the soil water content, there would be no way to estimate the daily averaged soil water contents from observed diurnal variations of soil temperatures. Fortunately, the soil heat capacity is a linear function of the soil water content, so the applicability and reliability of the adaptive Kalman filter method are only partially affected by the soil thermal conductivity. Nevertheless, the method may have some difficulties when the recent formulations in Eqs. (5)–(8) or Eqs. (9)–(13) of YK05 are used to replace the old version used in Eq. (4) of Z04. In particular, because these recent formulations show that the soil thermal conductivity should increase much more slowly than indicated by the old formulation as the soil water content approaches the saturation point, the adaptive Kalman filter method may become less reliable or even inapplicable when the soil is very wet. In this case, a reliable estimate of the soil water content may need additional information (from direct observations or a prior estimate). This problem is subject to further investigation, as suggested by YK05.

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