Parameterization of Inversion Breakup in Idealized Valleys. Part I: The Adjustable Model Parameters

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ABSTRACT

The factors that affect the atmospheric energy budget approach used in the thermodynamic valley inversion destruction model of Whiteman and McKee are investigated theoretically. The height at which the sinking inversion top meets the rising convective boundary layer to destroy valley inversions can be uniquely determined by the topographic characteristics of the valley and an adjustable model parameter, relating to the fraction of sensible heat flux going to convective boundary layer growth, through a simple parabolic relationship. The time required to break a temperature inversion can be expressed with very good approximation as a simple power-law function of the topographic parameters and the fraction of extraterrestrial solar flux that is partitioned to sensible heat flux in the valley atmosphere. The theoretical estimates compare very favorably to predictions from the bulk thermodynamic model of Whiteman and McKee. A new approach to handle time-dependent sensible heat fluxes is outlined. The paper ends with recommendations for future research.

1. Introduction

Complex terrain field studies have improved our understanding of physical processes leading to local thermally developed valley wind systems (e.g., Whiteman 1982; Whiteman et al. 1989a,b, 2003, 2004; Sakiyama 1990; Kuwagata and Kimura 1995; Triantafyllou et al. 1995). A number of analytical and numerical models have also been used in the past to study inversion breakup in valleys (e.g., Whiteman and McKee 1982; Müller and Whiteman 1988; Bader and McKee 1983, 1985; Kuwagata and Kimura 1997; Allwine et al. 1997; Anquetin et al. 1998; Colette et al. 2003).

The effect of circulations on a cross-valley section during breakup was investigated by Whiteman (1982), who classified inversion breakup patterns in deep mountain valleys. A hypothesis was developed to explain the observed temperature structure evolution (e.g., Whiteman 1982, 1990). Initially, just before sunrise, a deep temperature inversion fills the valley. After sunrise, solar radiation received on the valley floor and sidewalls is partly converted to sensible heat flux, which causes a convective boundary layer (CBL) to grow over the valley surfaces. Mass is entrained into the CBLs from the stable core and is carried from the valley in the upslope flows that are developed in the CBLs over the sidewalls. This removal of mass from the base and sides of the stable core causes (through mass continuity) the elevated inversion to sink into the valley and to warm adiabatically as a result of subsidence. This subsidence in the stable valley atmosphere produces warming and provides a very effective mechanism for transporting the energy received at the valley sidewalls throughout the entire valley atmosphere. Temperature inversions in the Colorado valleys are typically developed to the full depths of the valleys and are destroyed after sunrise following one of the three idealized patterns of temperature structure evolution (Whiteman 1982). The first pattern, often observed in very wide or shallow valleys, approximates inversion destruction over flat terrain, in which the nocturnal inversion is destroyed after sunrise by the upward growth from the ground of a warming CBL. The inversion destruction is thus characterized by a near-linear growth of the CBL with time and a slow descent of the inversion top. In the second pattern, often observed in steep and narrow valleys or when snow cover prevents surface heating, the CBL

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growth is limited, and the inversion is then destroyed as the top of the nocturnal inversion descends into the valley. The third pattern is the most common pattern of temperature inversion destruction in valleys, in which the inversion is destroyed by a combination of two continuous processes—upward growth of a CBL into the base of the valley inversion and descent of the inversion top.

To explain the gross features of the breakup of nocturnal temperature inversions in several deep mountain valleys of western Colorado, Whiteman and McKee (1982) formulated a bulk thermodynamic model under the following assumptions: 1) The valley topography can be adequately represented by a trapezoidal cross section having sidewalls of inclination i_1 and i_2 and valley floor width l (see their Fig. 1). 2) The temperature structure is horizontally homogeneous across the valley. 3) The initial inversion at sunrise time t_i can be adequately represented by a constant potential temperature gradient layer of arbitrary depth h_i. The mass is removed from the valley in such a way that the potential temperature gradient γ in the stable core is constant during the period of inversion breakup. 4) Convective boundary layers can be adequately represented as constant potential temperature layers of arbitrary height. 5) Using the first law of thermodynamics, the energy required to change the valley potential temperature profile at time t_i to the profile at time t can be obtained by an integration over the valley volume below the height of the inversion top. Because the sloping valley sidewalls enclose less volume to be heated by the same energy input, the valley atmosphere warms more rapidly than that over the plain. This situation means that the valley geometry, for example, expressed by the ratio of the area at the top of the valley to the volume of the valley, is a key factor in any thermal energy budget consideration. This ratio, when divided by the area-to-volume ratio for a same-depth volume over a plain, has been termed the topographic amplification factor (e.g., Steinacker 1984; Müller and Whiteman 1988; Whiteman 1990, 2000); it is a geometrical factor that plays an important role in determining the relative rate of warming of the valley atmosphere for a given energy input.

Following Whiteman's hypothesis, the total energy required to break up a temperature inversion is composed of two parts: the energy increment Q_3 that causes a CBL to grow and the energy increment Q_2 that removes mass from the valley and allows the top of the inversion to sink. By differentiating the individual energies Q_3 and Q_2 with respect to time, the rates of change of the height of the CBL H and the height of the top of the inversion h are obtained, such that

$$\frac{dQ_3}{dt} = \rho c_P \frac{T}{\theta}\gamma\left[H\frac{dH}{dt}\left(l + \frac{HC}{2}\right)\right] \quad \text{and} \tag{1}$$

$$\frac{dQ_2}{dt} = \rho c_P \frac{T}{\theta}\gamma\left[-h\frac{dh}{dt}\left(l + \frac{hC}{2}\right)\right], \tag{2}$$

where $C = (1/\tan i_1) + (1/\tan i_2)$, c_P is the specific heat at constant pressure, and ρ is the density of air. 6) The total rate of energy input into the valley atmosphere is a fraction A_0 of the solar energy flux F_{SR} coming across the horizontal upper area L of the top of the inversion that is converted to sensible heat flux. The extraterrestrial solar energy flux is assumed to be a sinusoidal function of time from sunrise to sunset, so that the total rate of energy input becomes

$$\frac{dQ_1}{dt} = A_0 L F_{SR} = A_0(l + hC)A_1 \sin\left[\frac{\pi}{\tau}(t - t_i)\right], \tag{3}$$

where the solar irradiance parameters A_1 and τ are the amplitude and the period of the sinusoidal function, respectively. 7) The model has, as its basis, a simplified energy budget equation in which the sensible heat flux is the driving force. An energy balance for the valley atmosphere is obtained by equating Eq. (3) to the sum of Eqs. (1) and (2). The fraction of the energy input used to drive the growth of the CBL is assumed to be of the form

$$K = k\left(\frac{l + HC}{l + hC}\right); \tag{4}$$

the remainder of the incoming energy $(1 - K)$ is used to remove mass from the valley and results in the descent of the top of the inversion. Both the parameters A_0 and k (relating to the surface energy budget and energy partitioning) are bounded fractions between 0 and 1. If the rate of warming in the neutral layer above the inversion β_w is nearly zero, the fraction of the rate of energy input used to drive the growth of the CBL is equated to Eq. (1) and the remaining fraction is equated to Eq. (2). These equations can then be solved, respectively, for the rate of ascent of the CBL dH/dt and the rate of descent of the inversion top dh/dt, resulting in the general model equations

$$\frac{dh}{dt} = -\frac{\theta}{T}\frac{1}{\rho c_P}\left[\frac{l + hC - k(l + HC)}{l + 0.5hC}\right]\frac{A_0 A_1}{\gamma h} \times \sin\left[\frac{\pi}{\tau}(t - t_i)\right] \quad \text{and} \tag{5}$$

$$\frac{dH}{dt} = \frac{\theta}{T}\frac{k}{\rho c_P}\left(\frac{l + HC}{l + 0.5HC}\right)\frac{A_0 A_1}{\gamma H}\sin\left[\frac{\pi}{\tau}(t - t_i)\right]. \tag{6}$$

In the case of $\beta_w \neq 0$ (when significant warming occurs in the neutral layer above), the resultant equations require minor modifications (see Whiteman and McKee

1982, p. 297). An integration of the model equations allows the simulation of the time-dependent height of the inversion top, the height of a convective boundary layer that grows upward from the valley floor after sunrise, and the vertical potential temperature profiles of the valley atmosphere. The topographic amplification factor in the general model equations (depending on the idealized trapezoidal valley cross section) accounts for the reduced volume of air within the mountain valley relative to that over the plains for the same energy flux on a horizontal surface. Note finally that because of the bulk nature of the thermodynamic model, it does not differentiate between CBL growth over the major topographic surfaces (i.e., valley floor, two sidewalls, and a ridgetop). Thus, all energy going into the CBL growth is attributed to growth of the valley-floor CBL.

In this contribution, we investigate theoretically the parameters k and A_0 that affect the surface energy budget and the partitioning of energy and thus affect the evolution of vertical temperature structure in deep mountain valleys.

2. A theoretical investigation of parameters k and A_0

Temperature inversions in an idealized valley are typically developed to the full depth of the valley and are destroyed after sunrise following one of the three patterns of inversion breakup. The thermodynamic model of Whiteman and McKee (1982) was applied to simulate the three patterns of temperature structure evolution observed in Colorado's Eagle and Yampa Valleys in different seasons. An approximation to pattern-1 inversion destruction was obtained from the model equations by setting $k = 1$. To simulate a pattern-2 inversion destruction, the parameter k was set to 0. When k was between 0 and 1, the model provided a simulation of pattern-3 inversion destruction and the stable layer was destroyed at the time t_D at which the ascending CBL met the descending inversion top at h_D (the height of inversion destruction). The fraction of sensible heat flux K that drives the growth of the CBL must be determined. It is obvious that the parameter K is a function of time (and probably of sensible heat flux) and depends on the topographic characteristics of the valley. Because the functional dependencies of K and A_0 are not yet known, arbitrary values for k and A_0 were chosen until the best simulation of the actual data was obtained with the model [see Eqs. (3) and (4)]. As the authors pointed out, further research is necessary to gain a better understanding of the physical interpretation of the parameters that affect the partitioning of energy and thus affect the evolution of vertical temperature structure during the breakup of nocturnal temperature inversions in actual valleys. In the next sections, we will discuss the role of k and A_0 in more detail.

a. Dependence of h_D on k

The accuracy of the estimates of the inversion characteristics through Eqs. (1)–(4) reflects the ability of the model to determine the partitioning of energy in the valley atmosphere. By adopting the atmospheric energy budget approach used by Whiteman and McKee (1982) and requiring that the energy partitioning be controlled by the empirical parameter K, an energy balance for the valley atmosphere is obtained by equating the fraction of the rate of energy input used to drive the growth of the CBL to Eq. (1) and the remainder to Eq. (2):

$$\frac{dQ_3}{dt} = KQ \quad \text{and} \quad \frac{dQ_2}{dt} = (1 - K)Q, \tag{7}$$

where $Q = dQ_1/dt$ is the total rate of energy input. Combining Eqs. (1), (2), (4), and (7) finally yields

$$\frac{dh}{dH} = -\frac{H}{h}\frac{l + 0.5HC}{l + 0.5hC}\left[\frac{l + hC}{k(l + HC)} - 1\right], \tag{8}$$

where k is between 0 and 1 (with $k \neq 0$). A numerical integration of Eq. (8) gives the dependence of the height h of the inversion top on the depth H of the CBL. Thus, h can be expressed as a universal function of H, k, h_i, l, and C. Based on Eq. (8), for different values of k, the plot in Fig. 1a was constructed; it expresses the dependence of the variable $\omega = h - H$ on the depth H of the CBL, for the Eagle Valley topography. The inversion is destroyed when $\omega = 0$, that is, when the ascending CBL meets the descending inversion top at $H = h_D$. It is obvious then, from Fig. 1a, that the fraction k uniquely determines the height h_D of inversion destruction. Figure 1b shows the relation between h_D and k, for the Eagle ($l = 1450$ m, $i_1 = 21°$, $i_2 = 10°$, and $C = 8.28$) and Yampa ($l = 2580$ m, $i_1 = 9°$, $i_2 = 16°$, and $C = 9.80$) Valley topographic parameters, respectively. It follows from Fig. 1b that the height of inversion destruction can be expressed with excellent approximation by the simple parabolic form

$$h_D = H_m k^{1/2}, \tag{9}$$

where H_m is a scaling inversion height obtained from Eq. (8) by setting $k = 1$ (i.e., $H_m = h_D$). The curves computed with the analytical formula in Eq. (9) are shown in Fig. 1b as dashed lines. For example, $H_m = 568$ m and $H_m = 470$ m for the Eagle and Yampa Valley topographic parameters, respectively [e.g., see also the plots presented in Fig. 4 of Whiteman and McKee (1982)]. Last, it can be easily proved that H_m is a geometrical factor that depends on the topographic characteristics of the valley; that is, it follows from Eq. (8)

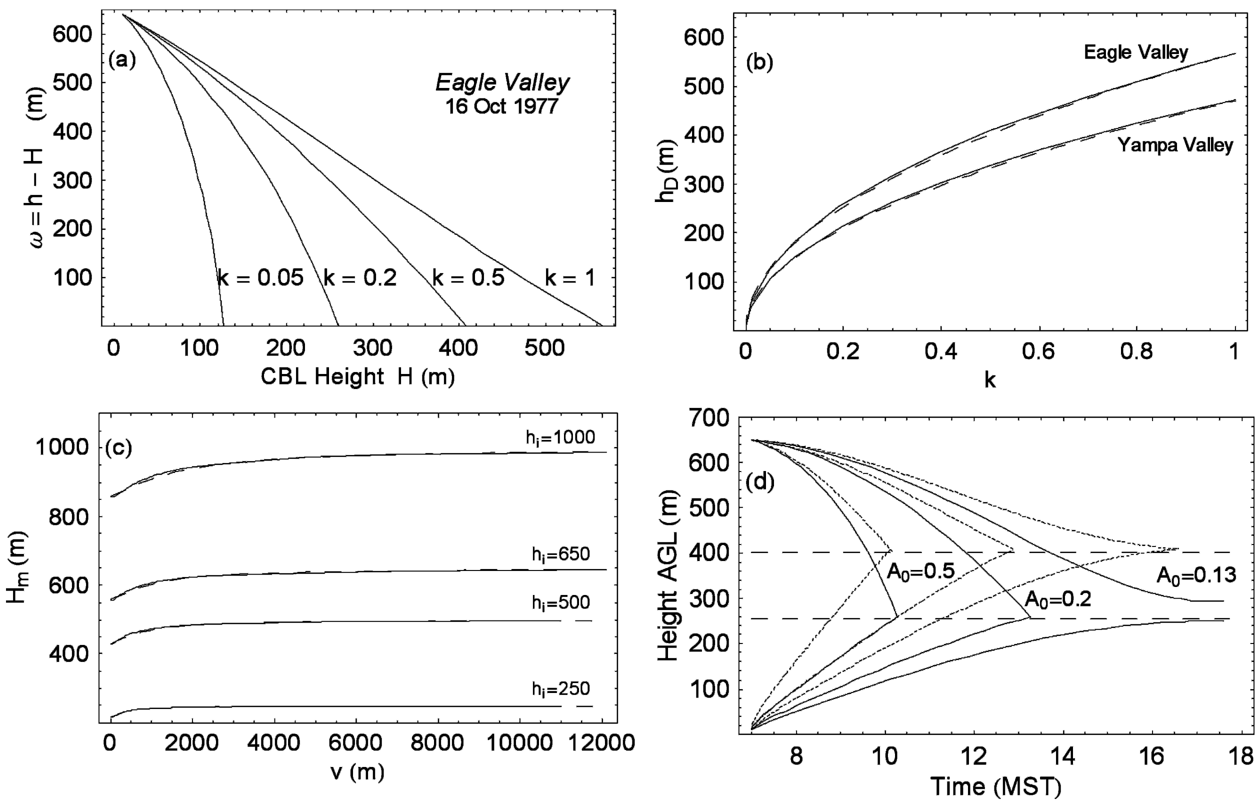

FIG. 1. (a) The dependence of $\omega = h - H$ on the depth H of the CBL, for the Eagle Valley topographic parameters, for different values of k. (b) The height of inversion destruction h_D as a function of k for the Eagle and Yampa Valley topographic parameters, respectively. The curves computed with the analytical formula in Eq. (9) are shown as dashed lines. (c) The topographic parameter H_m as a function of $\nu = l/C$ for different values of h_i. The curves computed with the analytical formula in Eq. (13) are shown as dashed lines. (d) Comparison of model predictions of h_D for $k = 0.2$ (solid lines) and $k = 0.5$ (dotted lines) and different values of A_0 with the height of inversion destruction (dashed lines) computed with the analytical formula in Eq. (17).

that H_m can be expressed as a universal function of h_i and l/C.

In the case of $k \neq 0$, application of Eq. (8) to a valley that is very wide (i.e., $l \to \infty$) results in the relation

$$\lim_{l\to\infty} H_m = h_i. \tag{10}$$

In the case of $k = 1$, application of Eq. (8) when the width of the valley is small ($l \to 0$) results in the relation

$$\lim_{l\to 0} H_m = h_i \exp(-\xi_*), \tag{11}$$

where

$$\xi_* = \int_0^1 \frac{\xi(1-\xi)}{\xi^2(1-\xi)+1}\, d\xi, \tag{12}$$

and from Eq. (12) yields $\xi_* \cong 0.151\,761\,3$. Based on Eq. (8), for different values of h_i, the plot in Fig. 1c (solid lines) was constructed and expresses the dependence of the topographic parameter H_m on the characteristic length $\nu = l/C$ (m), with $C \neq 0$. It follows from Eqs. (10) and (11) and Fig. 1c that the height of inversion destruction in Whiteman's idealized pattern 1 (i.e., $k = 1$) can be approximated by the simple relationship

$$H_m = A_m + B_m \tan^{-1}(C_m \nu), \tag{13}$$

where

$$A_m = h_i \exp(-\xi_*), \tag{14}$$

$$B_m = \frac{2}{\pi}(h_i - A_m), \quad \text{and} \tag{15}$$

$$C_m = \frac{2}{\pi} h_i^{-1}. \tag{16}$$

The curves computed with the analytical formula in Eq. (13) are shown in Fig. 1c as dashed lines. The important point to be mentioned here is that the valley geometry uniquely determines the height of inversion destruction in Whiteman's idealized pattern 1 of stable-layer breakup. (i.e., $h_D = H_m$).

Furthermore, combining Eqs. (9) and (13)–(16) yields

$$h_D = h_i\left\{a_D + b_D \tan^{-1}\left[\frac{2}{\pi}\left(\frac{l}{Ch_i}\right)\right]\right\}k^{1/2}, \tag{17}$$

where $a_D = \exp(-\xi_*) \cong 0.859\,193\,345$ and $b_D = 2(1-a_D)/\pi \cong 0.089\,640\,3$. Comparison of model predictions of h_D for $k = 0.2$ (solid lines) and $k = 0.5$ (dotted lines) and different values of A_0 with the height of inversion destruction (dashed lines) computed with the analytical formula in Eq. (17) is illustrated in Fig. 1d. In the special case of A_0 generally less than 0.13 (for $k = 0.2$), the rate of input of sensible heat flux is low; thus, the fraction of the extraterrestrial solar radiation that is converted to sensible heat flux within the valley (during the inversion breakup period) fails to destroy the temperature inversion.

Last, it is concluded that the height h_D of inversion destruction in Whiteman's idealized patterns 1, 2, and 3 of stable-layer breakup can be uniquely determined by k, h_i, and the (nondimensional) topographic parameter $V_* = l/h_iC$ through the simple parabolic relationship in Eq. (17).

b. An investigation of the effect of k on the partitioning of energy

Following Whiteman's hypothesis, the morning temperature inversion represents an energy deficit that is overcome by continued input of energy into the valley atmosphere (Whiteman 1982). The total energy E_{TOTAL} required to destroy a nocturnal valley temperature inversion is composed of two parts: the energy E_{CBL} that causes a CBL to grow and the energy E_{INV} that removes mass from the valley and allows the top of the inversion to sink (Whiteman and McKee 1982). Combining Eqs. (1), (2), and (7) and integrating from the initial conditions $H = H_0$ and $h = h_i$ at $t = t_i$ to the final conditions $H = h = h_D$ at $t = t_D$ finally yields

$$E_{\mathrm{TOTAL}} = E_{\mathrm{CBL}} + E_{\mathrm{INV}} = \int_{t_i}^{t_D} Q\,dt = \rho c_P \frac{T}{\theta}\gamma\left[\frac{l}{2}(h_i^2 - H_0^2) + \frac{C}{6}(h_i^3 - H_0^3)\right], \quad (18)$$

$$E_{\mathrm{CBL}} = \int_{t_i}^{t_D} KQ\,dt = \rho c_P \frac{T}{\theta}\gamma\left[\frac{l}{2}(h_D^2 - H_0^2) + \frac{C}{6}(h_D^3 - H_0^3)\right], \quad (19)$$

$$E_{\mathrm{INV}} = \int_{t_i}^{t_D} (1-K)Q\,dt = \rho c_P \frac{T}{\theta}\gamma\left[\frac{l}{2}(h_i^2 - h_D^2) + \frac{C}{6}(h_i^3 - h_D^3)\right], \quad \text{and} \quad (20)$$

$$E_{\mathrm{TOTAL}} = E_{\mathrm{CBL}} + E_{\mathrm{INV}}. \quad (21)$$

Therefore, E_{TOTAL} can be expressed as an analytical function of the initial inversion characteristics and valley topographic parameters. Further, the fraction λ of the total sensible heat flux that is used for CBL growth is given by

$$\lambda = \frac{E_{\mathrm{CBL}}}{E_{\mathrm{TOTAL}}}, \quad (22)$$

with the remaining fraction ($\mu = 1 - \lambda$) used for transporting mass up the sidewall and allowing the top of the inversion to sink:

$$\mu = \frac{E_{\mathrm{INV}}}{E_{\mathrm{TOTAL}}}. \quad (23)$$

Furthermore, combining Eqs. (18), (19), (20), (22), and (23) yields

$$\lambda = \frac{\frac{l}{2}(h_D^2 - H_0^2) + \frac{C}{6}(h_D^3 - H_0^3)}{\frac{l}{2}(h_i^2 - H_0^2) + \frac{C}{6}(h_i^3 - H_0^3)} \quad \text{and} \quad (24)$$

$$\mu = \frac{\frac{l}{2}(h_i^2 - h_D^2) + \frac{C}{6}(h_i^3 - h_D^3)}{\frac{l}{2}(h_i^2 - H_0^2) + \frac{C}{6}(h_i^3 - H_0^3)}, \quad (25)$$

where h_D is an analytical function of k [see Eqs. (9) and (17)]. Based on Eqs. (9), (24), and (25), Fig. 2 shows λ and μ as functions of k for the Eagle Valley topographic parameters. In conclusion, the important point to be mentioned here is that the fraction k uniquely determines the parameters λ and μ that affect the partitioning of E_{TOTAL} in the valley atmosphere.

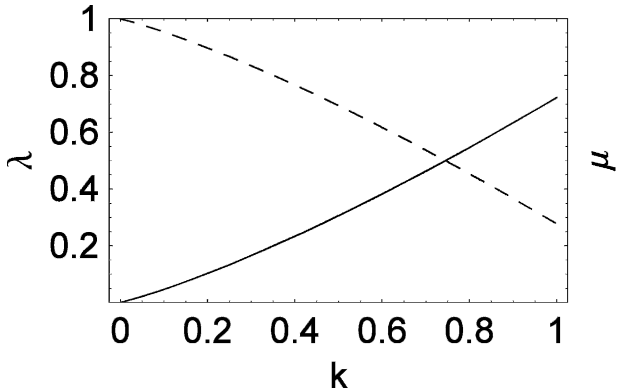

FIG. 2. The relation from Eqs. (9), (24), and (25) between λ and k (solid line) and between μ and k (dashed line) for the Eagle Valley topographic parameters.

c. Dependence of t_D on A_0 and k

The rate of energy input into the valley atmosphere is assumed to be a constant fraction A_0 of the theoretical incoming (i.e., extraterrestrial) solar energy flux at the inversion top that is then converted to sensible heat flux. Integrating Eq. (3) yields

$$\int_{t_i}^{t_D} A_0(l + hC)A_1 \sin\left[\frac{\pi}{\tau}(t - t_i)\right] dt = \int_{t_i}^{t_D} Q\, dt. \quad (26)$$

By combining Eqs. (3), (4), and (7), it follows that

$$\int_{t_i}^{t_D} A_0 k(l + HC)A_1 \sin\left[\frac{\pi}{\tau}(t - t_i)\right] dt = \int_{t_i}^{t_D} KQ\, dt. \quad (27)$$

By assuming that the growth of the CBL is nearly linear for all values of k (see Whiteman and McKee 1982, p. 296), such that

$$H \approx H_0 + \left[\frac{h_D - H_0}{t_D - t_i}(t - t_i)\right], \quad (28)$$

an approximate analytical expression for t_D can be obtained from Eqs. (19), (27), and (28):

$$E_{\mathrm{CBL}} \approx \frac{A_0 A_1 k\tau}{\pi^2 t_{DS}}\left[(l + H_0 C)\pi t_{DS} - (l + h_D C)\pi t_{DS}\cos\left(\frac{\pi}{\tau}t_{DS}\right) + C(h_D - H_0)\tau \sin\left(\frac{\pi}{\tau}t_{DS}\right)\right], \quad (29)$$

where k determines h_D through the relation in Eq. (9) and $t_{DS} = (t_D - t_i)$. Thus, a numerical solution of Eq. (29) gives the dependence of the inversion breakup time t_D on the parameters A_0 and k that affect the partitioning of energy. Based on Eqs. (19) and (29), the dashed lines in Fig. 3 represent the dependence of t_D on k (for different values of A_0) for the Eagle Valley topographic parameters and initial inversion characteristics [the solid lines in Fig. 3 were obtained by a numerical integration of the general model equations in Eqs. (5) and (6)]. It follows from Fig. 3 (for a given k) that t_D decreases with increasing A_0. With the assumption of a constant fraction A_0, Fig. 3 illustrates the expected decrease in t_D with increasing k. Thus, as the parameter k approaches 1, the inversion destruction more nearly follows pattern 1, resulting in an earlier inversion breakup. However, for large values of A_0, the effect of k on the inversion breakup time t_D is relatively small, because the most sensitive parameters affecting t_D are the initial inversion characteristics and the amount of energy available to destroy the stable layer (see Whiteman and McKee 1982, 292–296).

Similar analytical expressions to Eq. (29) can be obtained for the breakup time $(t_D - t_i)_{\mathrm{I}}$ for pattern-1 inversion ($k = 1$) by an integration of Eq. (6) from the initial condition $H = 0$ at $t = t_i$ to the final condition of $h_D = H_m$ at $t = t_D$,

$$(t_D - t_i)_{\mathrm{I}} = \frac{\tau}{\pi}\cos^{-1}\left\{1 - \frac{T}{\theta}\frac{\rho c_P \gamma}{A_0 A_1}\frac{\pi}{\tau}\left[\frac{H_m^2}{4} + \frac{lH_m}{2C} + \frac{l^2}{2C^2}\ln\left(\frac{l}{l + H_m C}\right)\right]\right\}, \quad (30)$$

and for pattern-2 inversion destruction (i.e., for $k = 0$ and $h_D = 0$) by an integration of Eq. (5),

$$(t_D - t_i)_{\mathrm{II}} = \frac{\tau}{\pi}\cos^{-1}\left\{1 - \frac{T}{\theta}\frac{\rho c_P \gamma}{A_0 A_1}\frac{\pi}{\tau}\left[\frac{h_i^2}{4} + \frac{lh_i}{2C} + \frac{l^2}{2C^2}\ln\left(\frac{l}{l + h_i C}\right)\right]\right\}. \quad (31)$$

Last, with the assumption of a constant fraction A_0, it follows from Fig. 3 that t_{DS} can be expressed with very good first approximation by a simple exponential form [i.e., $t_{DS} = A\exp(-Bk)$]. Furthermore, as the parameter k approaches 1, the inversion destruction more nearly follows pattern 1 and the inversion breakup time t_{DS} approaches $(t_D - t_i)_{\mathrm{I}}$ from Eq. (30). In addition, as the parameter k approaches zero, the inversion destruction more nearly follows pattern 2 and the inversion breakup time t_{DS} approaches $(t_D - t_i)_{\mathrm{II}}$ from Eq. (31). Following the above considerations, it can be easily proven that the breakup time t_{DS} for pattern-3 inversion destruction can be expressed, with very good approximation, by a simple power law:

$$t_D - t_i = (t_D - t_i)_{\mathrm{II}}\left[\frac{(t_D - t_i)_{\mathrm{I}}}{(t_D - t_i)_{\mathrm{II}}}\right]^k \quad (32)$$

or, equivalent,

$$t_{DS} = T_m \psi^k, \quad (33)$$

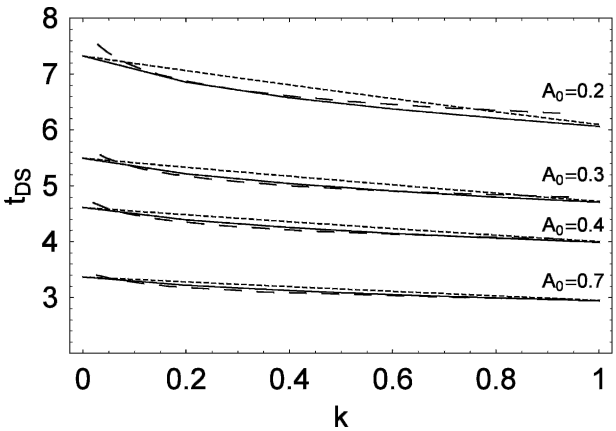

FIG. 3. The inversion breakup time t_{DS} as a function of k for different values of A_0 for the Eagle Valley topographic parameters and initial inversion characteristics. The solid lines were obtained by a numerical integration of the general model equations in Eqs. (5) and (6). The curves computed with the analytical expressions in Eqs. (29) and (33) are shown as dashed and dotted lines, respectively.

where $T_m = (t_D - t_i)_{\mathrm{II}}$ and $\psi = (t_D - t_i)_{\mathrm{I}}/(t_D - t_i)_{\mathrm{II}}$ are analytical functions of A_0. Furthermore, for pattern-1 inversion destruction (i.e., $k = 1$), Eq. (32) yields

$$t_{DS} = (t_D - t_i)_{\mathrm{I}}; \tag{34}$$

for pattern 2 ($k = 0$), it yields

$$t_{DS} = (t_D - t_i)_{\mathrm{II}}, \tag{35}$$

where $(t_D - t_i)_{\mathrm{I}}$ and $(t_D - t_i)_{\mathrm{II}}$ are given from the analytical expressions in Eqs. (30) and (31), respectively. The curves computed with the analytical formula in Eq. (33) are shown in Fig. 3 as dotted lines. In the special case of $k = 1$, it follows from Eqs. (33) and (34) that A_0 and valley topography uniquely determines the inversion breakup time t_{DS} in Whiteman's idealized pattern 1.

In conclusion, the theoretical analysis led to the simple power law in Eq. (33), in which the breakup time t_{DS} (in Whiteman's idealized patterns 1, 2, and 3 of inversion destruction) can be expressed with very good approximation as an analytical function of the valley topographic parameters and A_0 (where the parameter k is the power-law exponent).

As a first approximation, the rate of energy input into the valley atmosphere is assumed to be a constant fraction A_0 of the solar energy flux coming across the horizontal upper surface of the inversion. However, as stated by Whiteman et al. (2003, 2004) the fraction A_0 is not constant with time, as hypothesized in the thermodynamic model. Figure 4 shows that the fraction of the theoretical incoming (i.e., extraterrestrial) solar radiation at the inversion top that must be converted to sensible heat flux to explain recent observations in small Rocky Mountain and Alpine basins is not constant with time. This fraction decreases with time for both dolines, but is initially much higher in the drier Peter Sinks doline than in the Gruenloch doline (see Whiteman et al. 2003, 2004). The solid lines in Fig. 4 represent simple exponential regression curves. Assuming, as a working hypothesis, that the actual functional form of A_0 may be approximated by a simple exponential law,

$$A_0 = A_W \exp[-B_W(t - t_i)], \tag{36}$$

where A_w (nondimensional) and B_w (s^{-1}) are regression coefficients, an approximate analytical expression similar to Eq. (29) for the breakup time can be obtained from Eqs. (19), (27), (28), and (36):

$$E_{\mathrm{CBL}} \approx \frac{A_1 A_W k \tau \left\{ -\pi \Phi_1 + \left[\pi \Phi_2 \cos\left(\frac{\pi}{\tau} t_{DS}\right) + \tau \Phi_3 \sin\left(\frac{\pi}{\tau} t_{DS}\right) \right] \exp(-B_W t_{DS}) \right\}}{t_{DS}(\pi^2 + B_W^2 \tau^2)^2}, \tag{37}$$

where

$$\Phi_1 = -l t_{DS}(\pi^2 + B_W^2\tau^2) + C\{-2B_W h_D \tau^2 + H_0[-\pi^2 t_{DS} + B_W(2 - B_W t_{DS})\tau^2]\},$$

$$\Phi_2 = -l t_{DS}(\pi^2 + B_W^2\tau^2) + C\{2\,B_W H_0 \tau^2 - h_D[\pi^2 t_{DS} + B_W(2 + B_W t_{DS})\tau^2]\}, \quad \text{and}$$

$$\Phi_3 = -B_W l t_{DS}(\pi^2 + B_W^2\tau^2) + C\{H_0(-\pi^2 + B_W^2\tau^2) + h_D[\pi^2(1 - B_W t_{DS}) - B_W^2(1 + B_W t_{DS})\tau^2]\}.$$

Therefore, instead of using a constant fraction A_0, it may be possible, by incorporating a more appropriate analytical expression for A_0 into the set of the model equations, to simulate the time-dependent behavior of the heights of the CBL and inversion top. It is realized that this idea needs further verification, however (see also Whiteman et al. 2003, 2004).

As stated by Whiteman (1990), Allwine et al. (1997), Savov et al. (2002), and so on, the only adjustable model parameter in the above general model in Eqs.

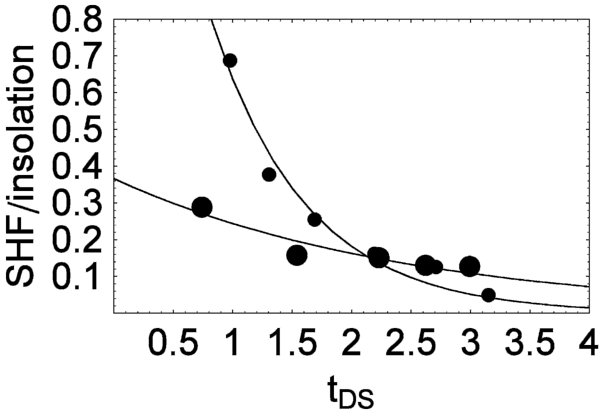

FIG. 4. Ratio of the rate of heat gain in the basin accumulated to a height of 120 m (above the basin floor) and the theoretical solar radiation as a function of time after sunrise. The observations are indicated by the special symbols [small circles (9 Sep 1999; Peter Sink) and large circles (3 Jun 2002; Gruenloch)], and the exponential regression curves are the solid lines. The data used in this investigation were obtained from Whiteman et al. (2003).

(5) and (6) is the fraction k. In principle, a rough estimation of the fraction A_0 may be obtained from measurements and some simple assumptions (e.g., see Whiteman et al. 2004). It is obvious that appropriate complex terrain field experiments are needed to make such measurements in actual valley atmospheres. Moreover, a bulk thermodynamic model for predicting the valley temperature structure evolution must have simple input requirements. Because the fraction A_0 is not normally available from routine meteorological observations, it remains an adjustable model parameter.

3. Conclusions and future research

In this study, a theoretical investigation is developed on the parameters k and A_0 that affect the atmospheric energy budget approach and partitioning of energy used in the thermodynamic inversion destruction model of Whiteman and McKee (1982). The following five conclusions result.

1) The height h_D of inversion destruction can be uniquely determined by the topographic characteristics of the valley and the parameter k through a simple parabolic relationship [Eqs. (9) and (17)].
2) The valley geometry uniquely determines the height of inversion destruction in Whiteman's idealized pattern 1 of stable-layer breakup [Eq. (13)].
3) The fraction k and the valley topography uniquely determine the parameters λ and μ that affect the partitioning of energy in the valley atmosphere [Eqs. (24) and (25) and Fig. 2].
4) The theoretical analysis led to a simple power law in which breakup time t_{DS} in Whiteman's idealized patterns 1, 2, and 3 of inversion destruction can be expressed with very good approximation as an analytical function of the fraction A_0 and valley topographic parameters, where the parameter k is the power-law exponent [Eqs. (32) and (33)].
5) Instead of using a constant fraction A_0, it may be possible to incorporate a more appropriate analytical expression for a time-dependent A_0 into the set of the model equations and thus to simulate the time-dependent behavior of the heights of the CBL and inversion top [Eqs. (36) and (37)]. It is realized that this idea needs further verification.

It is obvious that a regulatory meteorological model for predicting the valley temperature structure evolution must have simple input requirements, such as solar irradiance, initial inversion characteristics, and valley topography. The primary inputs to the thermodynamic model are the valley floor width, sidewall inclination angles, characteristics of the valley inversion at sunrise, and an estimate of sensible heat flux obtained from solar radiation calculations. The thermodynamic results (e.g., the time t_D required to destroy an initial stable layer) agree favorably with dynamical model predictions (e.g., Bader and McKee 1983, 1985; Colette et al. 2003). Therefore, because of its simplicity, the bulk thermodynamic model is a straightforward approach for obtaining quick but reliable preliminary estimates of inversion breakup in a wide range of valley topography (e.g., Whiteman and McKee 1982; Müller and Whiteman 1988; Zoumakis et al. 1992; Allwine et al. 1997; Savov et al. 2002; Whiteman et al. 2003). In conclusion, this research has improved understanding of the physical interpretation of the parameters that affect the atmospheric energy budget approach and partitioning of energy used in the thermodynamic inversion destruction model of Whiteman and McKee (1982).

To use the general model equations, the fraction A_0 of solar irradiance F_{SR} coming across the top of the inversion that is converted to sensible heat and the fraction K of sensible heat flux that drives the growth of the CBL must be determined. Because two of the input parameters to the thermodynamic model (i.e., A_0 and k) are not available from routine meteorological observations, the general model equations cannot be applied directly. Instead, arbitrary values for the adjustable parameters k and A_0 are chosen until the best simulation of the actual data is obtained with the model. Further investigation is currently under way to determine the actual functional form of K [see Eq. (4)] so that a better understanding of the energy-partitioning phenomenon in the valley atmosphere can be obtained, resulting in more accurate simulations (see also Whiteman and McKee 1982, p. 295). In addition, because energy gain and loss from the valley volume occur primarily at the valley surfaces, further research is also necessary to investigate simple schemes of radiation and surface energy budgets at the valley floor and sidewalls and partitioning of energy in the valley atmosphere, so as to eliminate the need for using arbitrary values for the adjustable model parameters. These topics are discussed further in the accompanying article (Zoumakis and Efstathiou 2006).

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