Sensitivity Studies of the Models of Radar-Rainfall Uncertainties

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ABSTRACT

It is well acknowledged that there are large uncertainties associated with the operational quantitative precipitation estimates produced by the U.S. national network of the Weather Surveillance Radar-1988 Doppler (WSR-88D). These errors result from the measurement principles, parameter estimation, and the not fully understood physical processes. Even though comprehensive quantitative evaluation of the total radar-rainfall uncertainties has been the object of earlier studies, an open question remains concerning how the error model results are affected by parameter values and correction setups in the radar-rainfall algorithms. This study focuses on the effects of different exponents in the reflectivity–rainfall (Z–R) relation [Marshall–Palmer, default Next Generation Weather Radar (NEXRAD), and tropical] and the impact of an anomalous propagation removal algorithm. To address this issue, the authors apply an empirically based model in which the relation between true rainfall and radar rainfall could be described as the product of a systematic distortion function and a random component. Additionally, they extend the error model to describe the radar-rainfall uncertainties in an additive form. This approach is fully empirically based, and rain gauge measurements are considered as an approximation of the true rainfall. The proposed results are based on a large sample (6 yr) of data from the Oklahoma City radar (KTLX) and processed through the Hydro-NEXRAD software system. The radar data are complemented with the corresponding rain gauge observations from the Oklahoma Mesonet and the Agricultural Research Service Micronet.

1. Introduction

Rain gauges represent the most direct and accurate way of measuring rainfall. Unfortunately, this strength is counterbalanced by the fact that these measurements are representative of a small area around the instrument. The capability of capturing the high spatial variability of the precipitation systems is unavoidably related to the availability of dense networks. Currently, dense rain gauge networks are available almost exclusively in a research context and cover only limited areas [e.g., Hydrological Radar Experiment (HYREX; Moore et al. 2000); Picomet (Ciach and Krajewski 2006)]. A viable solution to this problem is represented by radar-based estimates of rainfall. Weather radars can provide rainfall estimates over large areas (on the order of $10^8$ km$^2$) with high spatial and temporal resolutions (1 km$^2$ and 5 min). However, radar does not measure rainfall directly but measures the energy scattered back by hydrometeors in the atmosphere. This quantity called reflectivity $Z$ (mm m$^{-3}$) is then converted into the rainfall rate $R$ (mm h$^{-1}$) using empirical relations in the form of a power law ($Z$–$R$ relation).

It is well acknowledged that radar data are affected by large uncertainties, both systematic and random in nature [e.g., Krajewski and Smith 2002; Germann et al. 2006; Ciach et al. 2007; Berenguer and Zawadzki 2008; Habib et al. 2008; Germann et al. 2009; see Villarini and Krajewski (2010) for a recent extensive review]. Even though this problem was first recognized more than three decades ago [e.g., Harrold et al. 1974; Wilson and Brandes 1979; Zawadzki 1984; Collier 1986; Austin 1987], radar-rainfall estimates are still generated in a deterministic way, without quantitative information about
the associated uncertainties. If neglected, these uncertainties may have a large impact on all of those applications in which radar data are used as input or as a reference value (e.g., Winchell et al. 1998; Sharif et al. 2002, 2004; Borga et al. 2006; Habib et al. 2008a; Collier 2009; Villarini et al. 2009b; Villarini and Krajewski 2009b).

Recently, Ciach et al. (2007) proposed a model in which the relation between true ground rainfall and radar rainfall could be described as the product of a systematic function and a random component. The former accounts for systematic errors and was parameterized by a power-law function. The random component describes the remaining random errors and was parameterized with a Gaussian distribution with a mean equal to 1, a standard deviation that is a hyperbolic function of the radar-rainfall estimates, and significant spatiotemporal dependencies. The authors applied this model to a 4-km spatial scale and hourly (and larger) accumulation times in Oklahoma. Building on the results of Ciach et al. (2007), Villarini et al. (2009a) developed a generator of ensembles of probable true rainfall fields, conditioned on given radar-rainfall maps. For a given hourly accumulation radar-rainfall map, the generator in Villarini et al. (2009a) accounts for the systematic errors by correcting for conditional and unconditional biases. The Cholesky decomposition method is used to generate Gaussian-distributed random fields, with prescribed spatial correlation and with a variance that changes from pixel to pixel depending on the radar-rainfall value [as discussed in Villarini et al. (2009a), at the present stage the generator cannot account for temporal dependencies]. This error model (and generator) has already been applied toward improving statistical validation of satellite precipitation estimates (Villarini et al. 2009b), to investigate the impact of radar-rainfall uncertainties on the scaling properties of rainfall (Villarini and Krajewski 2009b), and toward a probabilistic flash flood guidance system (Villarini et al. 2009c, manuscript submitted to *J. Hydrol.*).

The model by Ciach et al. (2007) was then applied by Villarini and Krajewski (2009a) to a C-band radar in Great Britain (2-km pixel; 5-, 15-, 60-, 180-min time scales). Because the data from a dense rain gauge network were available, the authors studied the sensitivity of the model to the spatial sampling errors [errors due to the approximation of an areal estimate by a point measurement (e.g., Kitchen and Blackall 1992; Villarini et al. 2008a; Villarini and Krajewski 2008)]. Villarini and Krajewski (2009a) found that the systematic function was not sensitive to the representativeness errors, while this additional source of uncertainties inflated the standard deviation of the random component at the subhourly scale.

In the literature, several studies considered the different sources of uncertainties associated with radar-rainfall estimates [e.g., radar miscalibration, radar signal attenuation, ground clutter and anomalous propagation, beam blockage, drop size distribution and fall speed variability, range-dependent effects (e.g., beam broadening, elevation of the radar beam), and vertical variability of the precipitation system; for a review see Villarini and Krajewski (2010)]. However, our quantitative knowledge of these different sources of errors and their interdependencies is very limited. For this reason, in this study we follow the product-error-driven approach proposed by Ciach et al. (2007), in which we focus on the combined effects of all of these different sources of uncertainties and we model them following an empirically based (or data driven) approach.

Ciach et al. (2007) applied their model to the digital precipitation array (DPA; Fulton et al. 1998), which is operationally used by the National Weather Service (NWS). However, the results in Ciach et al. (2007) apply to a particular product with a particular set of parameters, which is also indicated by the name of the approach the authors followed (product-error driven). Questions about the impact of different parameter values and correction setups on the error model components require additional studies. In particular, the impact of different Z–R relations is still an open question. Studies investigating the effects of different parameterizations in the algorithms generating the radar-rainfall products will incrementally improve our understanding of the radar-rainfall uncertainties.

Moreover, the multiplicative model by Ciach et al. (2007) is just one among several different possible models (e.g., Germann et al. 2009). In this study, we expand the model by Ciach et al. (2007) to the additive formulation in which the relation between true rainfall and radar rainfall can be described as the sum of a systematic function and a random component.

Therefore, the aim of this study is twofold:

1) The investigation of the sensitivity of the empirically based error model proposed by Ciach et al. (2007) to three of the most widely used Z–R relations: Marshall–Palmer \((Z = 200R^{1.6})\); Marshall and Palmer 1948; Marshall et al. 1955), Next Generation Weather Radar (NEXRAD; \(Z = 300R^{1.4}\); Fulton et al. 1998), and tropical \((Z = 250R^{1.2}\); Rosenfeld et al. 1993). Additionally, the effects of the algorithm developed by Steiner and Smith (2002) to discriminate between meteorological and nonmeteorological returns on the radar-rainfall uncertainties are evaluated;
2) The extension of the error model formulation from multiplicative to additive.

The paper is organized in the following way: in the next section we describe the radar-rainfall error model, followed by a description of the data in section 3; section 4 presents the results of our modeling effort for different setups and model formulations; and section 5 summarizes the main points and concludes the paper.

2. Radar-rainfall error model

Ciach et al. (2007) proposed an error model in which the relation between true rainfall $R_{\text{true}}$ and radar rainfall $R_R$ was described as the product of a systematic distortion function $h(\cdot)$ and a random component $\varepsilon(\cdot)$, both dependent upon the radar-rainfall values. The former accounts for conditional biases, whereas the latter is for the remaining random errors. Notice that $R_{\text{true}}$ is approximated

![Fig. 1. Map of the rain gauge network and of the radar location.](image)

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<td>0.95</td>
<td>1.02</td>
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</table>

TABLE 1. Summary of the values of the overall bias estimated at the hourly time scale for different zones, seasons, and radar setups.

The values from Ciach et al. (2007) are in the column NWS.
by rain gauge measurements $R_a$. In this study, we extend the model formulation to an additive form in which the relation between $R_{true}$ and $R_R$ is described as the sum of $h(\cdot)$ and $\epsilon(\cdot)$. Therefore, we can write

$$ R_{true} = h(R_R)e_m(R_R) \quad \text{and} \quad (1) $$
$$ R_{true} = h(R_R) + e_a(R_R), \quad (2) $$

where $e_m(\cdot)$ and $e_a(\cdot)$ refer to the random component in the multiplicative and additive forms, respectively.

As a preliminary step, we compute and remove the overall bias $B_0$ (defined as the ratio between rain gauge and radar sample means). Then we estimate the systematic distortion function as a conditional expectation function:

$$ h(r_R) = E[R_{true}|R_R = r_R], \quad (3) $$

where $r_R$ and $r_{true}$ represent a specific value for the random variables $R_R$ and $R_{true}$ and $[\cdot]$ indicates expectation. The estimation of $h(\cdot)$ is performed using the nonparametric estimator described in Ciach et al. (2007) [see Villarini et al. (2008b) for a comparison between nonparametric and parametric (copula based) approaches].

After estimating unconditional and conditional biases, we can obtain the random component in the multiplicative and additive forms as

$$ e_m(R_R) = \frac{R_{true}}{h(R_R)} \quad \text{and} \quad (4) $$
$$ e_a(R_R) = R_{true} - h(R_R). \quad (5) $$

To fully characterize the random component, we need information about its mean, variance, probability distribution, and spatiotemporal dependencies. Because unconditional and conditional biases are accounted for by $B_0$ and $h(\cdot)$, we can assume that the mean of the random error in the multiplicative and additive forms is respectively equal to

$$ E[e_m(R_R)|R_R = r_R] = 1 \quad \text{and} \quad (6) $$

---

**FIG. 2.** Systematic component as a function of the radar-rainfall values at the hourly scale for the five different zones, the three seasons, and the entire dataset. NEXRAD Z-R relation was used to convert reflectivity into rainfall rate.
The variance of \( \varepsilon_m(R_R) \) and \( \varepsilon_a(R_R) \) can be defined as

\[
E[\varepsilon_m(R_R)|R_R = r_R] = 0. \tag{7}
\]

The variance of \( \varepsilon_m(R_R) \) and \( \varepsilon_a(R_R) \) can be defined as

\[
\sigma^2_{\varepsilon_m}(r_R) = E\left[ \left( \frac{R_{\text{true}}}{h(r_R)} - 1 \right)^2 | R_R = r_R \right] \quad \text{and} \tag{8}
\]

\[
\sigma^2_{\varepsilon_a}(r_R) = E\left[ (R_{\text{true}} - h(r_R))^2 | R_R = r_R \right]. \tag{9}
\]

The variance of \( \varepsilon_m \) can be estimated using the non-parametric estimator presented in Ciach et al. (2007), whereas in the case of the additive formulation we can write the following:

\[
\sigma^2_{\varepsilon,a}(r_R) = \frac{\sum_w \{w[R_a - h(r_R)]^2\}}{\sum_w} \bigg| \frac{r_R}{k} \leq R_R \leq kr_R, \tag{10}
\]

where \( k \) is the bandwidth parameter controlling the size of the averaging window, \( w \) is the weighting factor, and \( R_a \) is an approximation of the true rainfall \( R_{\text{true}} \) based on rain gauge measurements. For more details about the kernel regression procedure, refer to Ciach et al. (2007).

The first and second moments provide only very limited information about the distribution of the random component that is conditioned on radar-rainfall estimates. For certain levels of probability of exceedance \( p \), we can estimate the conditional quantiles \( q_p \) of the random component in the additive and multiplicative forms as

\[
\Pr[\varepsilon_m(R_R) \leq q_p | R_R = r_R] = p \quad \text{and} \tag{11}
\]

\[
\Pr[\varepsilon_a(R_R) \leq q_p | R_R = r_R] = p, \tag{12}
\]

where \( \Pr[\cdot|\cdot] \) is the operator of conditional probability. The estimation of the conditional quantiles has been performed using the “weighted-point-counting” procedure discussed in Ciach et al. (2007).

Once we have obtained the nonparametric estimates of the systematic function, as well as that of the conditional standard deviation and conditional quantiles of the

Fig. 3. Comparison among the systematic functions at the hourly scale for different radar setups, the three seasons, and the entire dataset. These results refer to Zone II. The results by Ciach et al. (2007) are labeled as NWS.
random components, we can attempt to approximate these using simple parametric representations. This approximation would provide a more concise, though less accurate, description of the components of the model. For a more detailed description of this error model, the reader is pointed to Ciach et al. (2007).

3. Data

Given the empirical nature of this approach, a large sample of radar and rain gauge data is necessary. We use a 6-yr sample (from January 1998 to December 2003) of radar-rainfall estimates from the Oklahoma City Weather Surveillance Radar-1988 Doppler (WSR-88D) site (KTLX). The WSR-88D are S-band (wavelength ranging from 10 to 11.1 cm) radars with Doppler capability and linear horizontal polarization [0.95° beamwidth; volumetric observations of reflectivity with a range from 5–6 to 10–12 min depending on the scanning strategy; for more information about the WSR-88D radars, among others, see Crum and Alberty (1993) and Crum et al. (1998)]. Several different algorithms are available to convert the base data products (reflectivity, mean radial velocity, and spectrum width) into hydrometeorological products (e.g., Klazura and Imy 1993). The precipitation processing system (PPS; Fulton et al. 1998) is a suite of algorithms that is used to generate radar-rainfall estimates, such as the DPA product used by Ciach et al. (2007).

As shown in Fig. 1, the Oklahoma City radar is located in the central part of the state of Oklahoma and is not affected by significant orography. The rain gauge measurements come from two well-maintained networks: the Oklahoma Mesonet (e.g., Elliott et al. 1994; Brock et al. 1995) and the U.S. Department of Agriculture (USDA) Agricultural Research Service Micronet (e.g., Allen and Naney 1991). The rain gauges belonging to the mesonet are almost uniformly distributed within the state of Oklahoma with an intergauge distance of about 50 km. The micronet is much denser and consists of 42 rain gauges with an intergauge distance of about 5 km.

In this study, we consider hourly scales averaged over the areas of the hydrologic rainfall analysis project (HRAP) grid (about 4-km pixel; e.g., Reed and Maidment 1999). Unlike Ciach et al. (2007), the generation of these accumulation maps is performed by processing Level II radar data through the Hydro-NEXRAD software system (Krajewski et al. 2007b; Krajewski et al. 2008) rather than the NWS PPS (Fulton et al. 1998; as in Ciach et al. 2007). Hydro-NEXRAD is a Web-based system that allows for the generation of radar-rainfall

### Table 2

<table>
<thead>
<tr>
<th>Zone</th>
<th>NEXRAD Z–R</th>
<th>NEXRAD Z–R (AP)</th>
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<tr>
<td></td>
<td>Cold season</td>
<td>Warm season</td>
</tr>
<tr>
<td>I</td>
<td>1.48</td>
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<tr>
<td>II</td>
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<td>III</td>
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<td>1.36</td>
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<tr>
<td>IV</td>
<td>1.41</td>
<td>1.44</td>
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<tr>
<td>V</td>
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<td></td>
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<table>
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<th>Zone</th>
<th>Marshall–Palmer Z–R</th>
<th>Tropical Z–R</th>
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<td>Warm season</td>
</tr>
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<td>II</td>
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<td>III</td>
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products (derived from NEXRAD data from about 40 sites in the United States) tailored to the specific needs of each researcher. Numerous customized modular radar-rainfall algorithms are available in Hydro-NEXRAD (Quick Look, High Res, Pseudo NWS PPS, and Custom; Krajewski et al. 2007a). In this study, we use the so-called Pseudo PPS, which is a suite of algorithms that tries to mimic the NWS PPS as closely as possible. Although the evaluation of these products in comparison with the “official” NWS ones is outside the scope of this study, we include the results from Ciach et al. (2007) for comparison. Besides the generation of user-specified radar products (in terms of spatial and temporal scales, as well as processing algorithms), a clear advantage for using the Hydro-NEXRAD system is represented by the processing time: for a 6-yr sample, processing radar data through NWS PPS took Ciach et al. (2007) more than 3 months, whereas in Hydro-NEXRAD, the processing time decreased to about a week. It is clear that this is a key feature for a sensitivity study like the one performed in this article. Because Ciach et al. (2007) included in the analysis hourly accumulation maps that were separated by at least 12 min, whereas the Hydro-NEXRAD system generates hourly maps every full hour, we have repeated the analysis on the NWS PPS data considering only maps that were generated approximately every full hour. We will not show these results because they were very similar to those in Ciach et al. (2007).
We converted radar reflectivity $Z$ to rainfall intensity $\mathcal{R}$ with three of the most widely used $Z-R$ relations: Marshall–Palmer ($Z = 200\mathcal{R}^{1.6}$), NEXRAD ($Z = 300\mathcal{R}^{1.4}$), and tropical ($Z = 250\mathcal{R}^{1.2}$). Additionally, in the Hydro-NEXRAD system the algorithm proposed by Steiner and Smith (2002) to discriminate between the nonmeteorological [anomalous propagation (AP) and ground clutter] and meteorological returns is implemented (Krajewski et al. 2007a)—we have used it to investigate its impact on this error model (only for the NEXRAD $Z-R$ relation). Following Ciach et al. (2007), we have accounted for range effects in the radar-rainfall errors by dividing the radar umbrella into five zones (Fig. 1). Additionally, the impact of different synoptic conditions on our modeling effort is investigated by stratifying the data into three seasons: cold (from November to March), warm (April, May, and October), and hot (from June to September). Though this choice is largely arbitrary, we expect more stratiform precipitation during the cold season and more convective activity during the hot season. For more details about seasonal stratification and range effects the reader is pointed to Ciach et al. (2007).

4. Results

As discussed above, we compute the error model components at the hourly scale, averaged over the HRAP grid

![Fig. 5. Comparison among the standard deviations of the random component (multiplicative error model) at the hourly scale for different radar setups, the three seasons, and the entire dataset. These results refer to Zone II. The results by Ciach et al. (2007) are labeled as NWS.](image)
Table 3. Summary of the values of coefficients $\sigma_{a,m}$ (upper value), $a_{a,m}$ (middle value), and $b_{a,m}$ (lower value) from the parameterization of the standard deviation of the random component for the multiplicative error at the hourly time scale for different zones, seasons, and radar setups.

<table>
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<tr>
<th>Zone</th>
<th>NEXRAD Z–R</th>
<th>NEXRAD Z–R (AP)</th>
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<tr>
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<td>Cold season</td>
<td>Warm season</td>
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<tr>
<td>I</td>
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(a) Conditional and unconditional bias

The first step is the computation of the overall bias $B_0$. We summarize its values for the different zones, seasons, and radar setups in Table 1. Because the overall bias is defined as the ratio between rain gauge and radar sample means, a value of $B_0$ larger (smaller) than 1 corresponds to underestimation (overestimation) by the radar with respect to the rain gauges. In most of the cases, the radar tends to overestimate rainfall, which is in agreement with the results of Ciach et al. (2007). In the cold season, we notice a decrease in the bias up to Zone III and then an increase with range. It is likely that between 100 and 140 km the radar beams intersect the bright band, resulting in an increase in the reflectivity values and, consequently, of the estimated rainfall (e.g., Smith 1986; Joss and Waldvogel 1990; Fabry et al. 1992). At farther distances, the radar beam height over the ground increases, and the radar-rainfall estimates are affected by partial beam filling and overshooting of the precipitation systems (e.g., Kitchen and Jackson 1993; Smith et al. 1996). These features are more evident during the cold season because it is generally characterized by low-level stratiform events. During the hot season, which is mostly characterized by stronger convective activity, we do not notice a strong dependence of the bias on the distance from the radar. The warm season can be considered as an intermediate case between the cold and the hot seasons. As expected, the use of the tropical $Z–R$ relation results in the lowest values of bias (more significant overestimation by the radar) while there are no large differences between the NEXRAD and Marshall–Palmer relationships. Overall, the results

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in Ciach et al. (2007) tend to be less unconditionally biased (values of $B_0$ are closer to 1) relative to those using the Pseudo PPS. Finally, the use of the AP algorithm generally improves the comparison between radar and rain gauge data (bias is close to 1). This improvement is more significant for Zone I because ground clutter is primarily found close to the radar site. The only case in which the use of this algorithm would result in a worse agreement between radar and the rain gauges is for the cold season at far range. A possible explanation for this could be found in a check that is performed in the algorithm to separate meteorological and non-meteorological returns. This AP algorithm looks for vertical continuity of the returned signal; in low-level stratiform events, it is likely that at far range the radar beam intersects only the top of the rainfall systems. Therefore, these echoes are erroneously flagged as non-meteorological and removed, resulting in a more significant underestimation by the radar with respect to the rain gauges. Nevertheless, the use of the AP algorithm improves the overall comparison between the radar and rain gauges.

Once the overall bias is computed and removed, we estimate $h(\cdot)$. Figure 2 shows the results for the NEXRAD $Z-R$ relation (similar statements are valid for the other setups) for the five zones, the three seasons, and the entire dataset. If these lines were on the $Y = X$ line, it would mean that there is a lack of conditional bias; any departure from this line would represent conditional underestimation or overestimation. In general, during the cold season we have a significant overestimation by the radar for the radar-rainfall values larger than

![Fig. 6](image_url)
5–10 mm. These patterns are similar to those in Ciach et al. (2007), even though these results tend to show a slightly smaller conditional bias associated with the products generated from the Hydro-NEXRAD system. In Fig. 3, we focus on Zone II (which has the highest number of rain gauges) and compare the results for the different radar setups. In general, we have a rather regular pattern: at large radar-rainfall values the use of Marshall–Palmer $Z$–$R$ results in rainfall underestimation, whereas the opposite behavior is shown by the tropical $Z$–$R$ relationship (the NEXRAD setup results in an intermediate case between these two). This behavior was expected because of the different exponents in these power-law relationships. The use of the AP removal algorithm tends to affect more significantly the results for large radar-rainfall values. Overall, the use of the NEXRAD $Z$–$R$ relation provides the least conditionally biased radar-rainfall estimates.

As in Ciach et al. (2007), we parameterized these results using a power-law function:

\[ h(r_R) = a_R r_R^{b_R}, \]  

obtaining good visual agreement between fitted curve and data points, especially in the warm and hot seasons. We have summarized the results from the parameter estimation in Table 2.

b. Random component

To provide a comprehensive statistical description of the random component, we need to characterize it in terms of its mean, variance, probability distribution, and
spatial and temporal dependencies. Here, we present
the results for both additive and multiplicative forms
of the error model. From Eqs. (6) and (7), we specify
that the mean is not a function of the radar-rainfall
estimates. On the other hand, the standard deviation
depends on the radar-rainfall values. In Fig. 4, we show
the results for $\sigma_{e,m}$ and the NEXRAD $Z$–$R$
relation. The standard deviation for the multiplicative error
model tends to decrease for increasing radar-rainfall
values. With the sole exception of the cold season, we
observe a regular pattern for a given radar-rainfall
value, with the smallest values of $\sigma_{e,m}$ for Zone I
and then increasing with increasing distance from the radar.
One feature that is common to all of the seasons and
other studies applying this error model (Ciach et al. 2007; Villarini and Krajewski 2009a) is the presence of
a quick decrease in $\sigma_{e,m}$ for radar-rainfall values ap-
proximately smaller than 5 mm. This behavior could be
because of the impact of the discretization of the radar-
rainfall estimates (Fulton et al. 1998), more significant
for smaller accumulation values, or because of larger
uncertainties associated with rain gauge measurements of
small rainfall accumulations (e.g., Habib et al. 2001b;
Ciach 2003). Moreover, we also observe an undulating
behavior in some of the standard deviations of the
random error (e.g., Zone I in the cold season), which
could be due to sample-size issues or physical reasons.

Overall, these results are similar both quantitatively
and qualitatively to those in Ciach et al. (2007). A very
similar behavior was observed for the other radar set-
ups as well. In Fig. 5, we focus our attention on Zone II
and compare the results from the different $Z$–$R$
relations and the AP algorithm. In general, we observe
the same hyperbolic behavior as in Fig. 4, with a quick
decrease for small radar-rainfall accumulations. In the
cold season, the smallest values of $\sigma_{e,m}$ are associated
with the AP removal algorithm setup, likely resulting
from the correction of brightband effects during strat-
iform events. Disregarding the cold season, the results
in the warm and hot seasons, as well as for the entire
dataset, tend to suggest that the standard deviation of
the random component in the multiplicative form is not
sensitive to the radar parameterization (only for the
case of tropical $Z$–$R$, $\sigma_{e,m}$ tends to be larger than the
other setups).

Given the observed regular behavior, we have param-
eterized the standard deviation of the random component
in the multiplicative form using the same hyperbolic
function as in Ciach et al. (2007):

$$
\sigma_{e,m}(r_r) = \sigma_{0e,m} + a_{e,m} r_r^{b_{e,m}},
$$

obtaining good agreement between data points and fit-
ted functions, especially in the warm and hot seasons.

<table>
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<th>Warm season</th>
<th>Hot season</th>
<th>Entire dataset</th>
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We have summarized the results from the parameter estimation in Table 3. Figure 6 shows the results for the standard deviation of the random component for the additive case and the NEXRAD $Z$–$R$ relation. Unlike in the multiplicative error model, it tends to increase for increasing radar-rainfall values. In general, for a given radar-rainfall value, $s_{e,a}$ tends to be smaller during the cold season, followed by the warm and hot seasons with differences larger than those observed in Fig. 4 for $s_{e,m}$. There is also no clear and systematic arrangement across zones, making it difficult to separate range effects, and there is also a more erratic behavior, which can complicate any attempt to parameterize the observed results. Moreover, we do not observe the counterpart of the sharp decrease at the small accumulation values observed for $s_{e,m}$ in Fig. 4.

Another difference with respect to $s_{e,m}$ is the sensitivity with respect to the different radar setups. As shown in Fig. 7, $s_{e,a}$ varies depending on the selected $Z$–$R$ relation. More precisely, for a given radar-rainfall value, it tends to be larger for increasing values of the exponent in the $Z$–$R$ relationships. Based on these results for the error model in the multiplicative form, accounting for unconditional and conditional biases seems to be sufficient to remove the effects of different radar parameterizations. This statement is not valid for the error model in the additive form.
Similarly to the systematic distortion function, we have parameterized $\sigma_{e,a}$ using a power-law function:

$$\sigma_{e,a}(r) = a_{e,a} r^{b_{e,a}}. \quad (15)$$

We have summarized the results from the parameter estimation in Table 4. The visual agreement between the fitted function and data points was not as good as for $\sigma_{e,m}$, and we were not able to capture the sharp changes and variability observed in Figs. 6 and 7.

Once we obtain information about the mean and the standard deviation of the random component, we can investigate its distribution, starting from the multiplicative error model. Ciach et al. (2007) found that the random component at the hourly scale could be described by a Gaussian distribution with a mean equal to 1 and a standard deviation of $\sigma_{e,m}(R_R)$; this is the distribution used in this study as well. We have compared the empirical and Gaussian quantiles for different probability levels $p$ using the four radar setups (figures not shown). In general, there is good visual agreement between theoretical and empirical quantiles, particularly for radar-rainfall values larger than 5 mm h$^{-1}$. These results tend to support the fact that the random component can be described by a Gaussian distribution, and we can generalize this statement to any of the $Z$–$R$ relations considered in this study. The removal of overall and conditional biases and the conditional nature of the error model could explain the normality of the random component. Moreover, as discussed by Villarini et al. (2009a), the fact that the random component can be described by a Gaussian

![Fig. 9. As in Fig. 8, but for the warm season.](image-url)
distribution is instrumental in the development of a generator of probable true rainfall fields conditioned on given radar-rainfall maps.

Although we had some information about the possible distribution of $\varepsilon_m()$, the distribution of the random component in the additive form is still unexplored. We have summarized the results of our analysis in Figs. 8–11 for the different seasons and zones, using the NEXRAD $Z$–$R$ to convert reflectivity to rainfall rate (similar results were obtained for the other cases considered in this study). From Eq. (7), we know that the mean of the random component is equal to 0 for all radar-rainfall values. Figures 8–11 illustrate that the median (the 50th percentile) tends to be close to 0 independently of the radar-rainfall values, providing evidence that a symmetric distribution could be a good candidate. Therefore, we have computed the theoretical quantiles for a Gaussian distribution with a mean equal to 0 and a standard deviation equal to $\sigma_{\varepsilon,m}(R_R)$. Based on the visual inspection of the results (Figs. 8–11), we notice a good agreement between empirical and theoretical quantiles, independently of the season, zone, and $Z$–$R$ relation. The fact that a Gaussian distribution could be used to model the random component in the additive form is an important finding from the point of view of the generation of ensembles of probable true rainfall fields that are conditioned on radar-rainfall maps. The approach described in Villarini et al. (2009a) for the multiplicative model can be directly extended to this case.

To fully describe the random component, we use Pearson’s product-moment correlation coefficient $\rho$ to estimate the temporal and spatial correlations as

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**Fig. 10.** As in Fig. 8, but for the hot season.
where $X, Y$ represent a bivariate random sample. In this study we have assumed that the spatial correlation is isotropic (see also Germann et al. 2009).

Because the random component can be described by a Gaussian distribution, this estimator should not be affected by the limitations discussed in the literature (e.g., Kowalski 1972; Habib et al. 2001a; Serinaldi 2008). Ciach et al. (2007) and Villarini and Krajewski (2009a) showed how both the spatial and temporal correlations are significant for the multiplicative error model. Similar to Villarini et al. (2009a), we focus on Zone II to discuss the spatial and temporal dependencies of a random component.

\[
\]

(16)

Figure 12 shows the temporal correlation of $\varepsilon_m(\cdot)$ for different seasons and different radar setups, which is significantly different from zero independently of the season or radar setup. These results are similar to those in Ciach et al. (2007). We notice that the random component shows the highest values of temporal correlation in the cold season, mimicking the larger degree of spatial and temporal homogeneity associated with stratiform systems. On the other hand, the temporal correlation of $\varepsilon_m(\cdot)$ is the lowest in the hot season, which can be explained by the higher convective activity that occurs during the summer months in Oklahoma. Besides noticing that the temporal correlations from the tropical $Z-R$ relation are the highest independently of the season, it is not possible to make a general statement about the impact of the other radar setups. Similar to Villarini
et al. (2009a), we have parameterized these results with a two-parameter exponential correlation function in the form of

\[ r(D_t) = \exp\left\{ \frac{-b_t}{a_t} \right\} \]

where \( a_t \) is correlation distance, \( b_t \) is the shape parameter, and \( D_t \) is the separation lag in time. We have summarized the values of these coefficients in Table 5. In general, the observed pattern is captured well by the selected two-parameter exponential correlation function.

In Fig. 13, we have plotted the temporal correlation of the random component for the additive error model. In this case, the temporal correlation tends to be significant only for the first few lags, and it tends to be less correlated in time relative to the multiplicative model. Recalling from Eq. (16) that in Pearson’s estimator the standard deviation appears at the denominator, it is possible that the observed behavior occurs because \( \sigma_{e,a} \) is much larger than \( \sigma_{e,m} \). Additionally, the impact of the different radar parameterization on the temporal correlation of the random component tends to be larger in the multiplicative formulation compared to the additive one. We have fitted the empirical results using Eq. (17) and summarized the values of the obtained coefficients in Table 5. Even in this case, the selected parametric correlation function captures the observed behavior well.

The final element we need to investigate is the spatial correlation of the random component. The results for
the different radar parameterizations in the additive and multiplicative forms are similar to those in Ciach et al. (2007, figures not shown). Both \( e_m(\cdot) \) and \( e_a(\cdot) \) are correlated in space, in agreement with the findings in Ciach et al. (2007) and Villarini and Krajewski (2009a). Similar to what we observed for the temporal dependence, the cold season presents the highest correlation values, mimicking the higher degree of homogeneity that characterizes the stratiform events. Comparing the results of the multiplicative and additive models, \( e_a(\cdot) \) is more correlated than \( e_m(\cdot) \). This behavior could again be explained by the higher values of standard deviation associated with the random errors in the additive form. Similar to Villarini et al. (2009a), we have parameterized the results for both \( e_a(\cdot) \) and \( e_m(\cdot) \) using the following two-parameter exponential function:

\[
\rho(\Delta s) = \exp \left( - \frac{\Delta s}{a_s} \right)^{b_s}, \quad a_s \geq 0; \quad b_s \in [0; 2],
\]

where \( a_s \) is correlation distance, \( b_s \) is the shape parameter of the fitted exponential function, and \( \Delta s \) is the separation lag in space. We have summarized the values of these coefficients in Table 5. Overall, the selected two-parameter exponential function captures the observed pattern well.

5. Discussion and conclusions

It is well acknowledged that radar-based estimates of rainfall are affected by several sources of uncertainties that are systematic and random in nature. In this study, we provided more insight into the impact of different radar setups on the modeling results. In particular, we investigated the effects of different \( Z-R \) relationships and the use of an AP removal algorithm (Steiner and Smith 2002). Moreover, we compared the results from the modeling of radar-rainfall uncertainties in both additive and multiplicative form. Our findings can be summarized as follows:

1) According to the overall bias \( B_0 \), the radar tends to overestimate rainfall with respect to the rain gauges. NEXRAD and Marshall–Palmer \( Z-R \) relations tend to provide similar values of \( B_0 \), whereas the tropical one results in a much larger overestimation. The use of the AP removal algorithm tends to improve the radar–rain gauge comparison in terms of \( B_0 \), especially close to the radar site.

2) After removing the overall bias, there is still a considerable conditional bias. The systematic distortion function is sensitive to the radar parameterization.

3) The standard deviation of the random component in the multiplicative form \( \sigma_{e,m} \) is a decreasing function of the radar-rainfall values and does not seem to be very sensitive to the selected radar setup. On the other hand, the standard deviation of the random component in the additive form \( \sigma_{e,a} \) is an increasing function of radar-rainfall values and is more sensitive to the selected \( Z-R \) relation or to the use of the AP removal algorithm.

4) A Gaussian distribution with mean equal to 1 (0) and standard deviation equal to \( \sigma_{e,m}(R_R) \) [\( \sigma_{e,a}(R_R) \)] can...
be used to describe the random component in the multiplicative (additive) form.

5) The random component presents a nonzero correlation in time for both the additive and multiplicative forms. These correlations are higher during the cold season and do not seem to be very sensitive to the radar setup.

6) The random component, both in the additive and multiplicative forms, is significantly correlated in space and presents the same seasonal dependence as that for the temporal correlation. The radar setup does not seem to significantly affect the results.

Based on these results, at least for Oklahoma, it seems that NEXRAD $Z$–$R$ should be preferred over Marshall–Palmer or tropical $Z$–$R$ relationships. However, it is worth clarifying that this statement is valid over a long period, and it is likely that different $Z$–$R$ relationships would be more appropriate on a storm or even on a scan basis (e.g., Steiner and Smith 2000; Uijlenhoet et al. 2003; Lee and Zawadzki 2005; Habib et al. 2008b).

Moreover, the use of the AP removal algorithm developed by Steiner and Smith (2002) improves the radar–rain gauge comparison and is particularly effective close to the radar site. However, at far range and for the case of low-level stratiform events, we found a decrease in the overall performance. Our results may provide some indications about the areas of the algorithm that may be improved.

While the results in Ciach et al. (2007) are based on products generated through the PPS algorithm by the NWS, the products in this study are generated through the Hydro-NEXRAD system. In general, we notice that the results from this study compare favorably to the ones

![Figure 13](image-url)
in Ciach et al. (2007). The NWS PPS products tend to have a smaller overall bias, especially close to the radar site, whereas the Hydro-NEXRAD results seem to have a slightly smaller conditional bias. In terms of the random component, we did not observe significant differences. An evaluation of the Hydro-NEXRAD system is outside the scope of this work and is currently the object of separate studies. Nevertheless, we want to emphasize the significant decrease in processing time as a result of the use of Hydro-NEXRAD.

Transferability of these results to other regions still requires further investigation. At present, any statement would be at the level of speculation, because these results not only depend on the rainfall regime in the area but also on the radar characteristics (e.g., calibration) and product. Therefore, the application of this error model to other radars and radar-rainfall products is necessary to provide better insight into the transferability issue. The statistical robustness of both our parametric and nonparametric estimates of the model’s component should also be investigated, providing valuable information about the sampling errors associated with them. Another issue that should be addressed in future studies concerns the stratification of the dataset. Following Ciach et al. (2007), we have partitioned the data according to different seasons, which we considered as a proxy for different precipitation systems. Future studies should investigate the issue of stratifying the data on an event or scan basis according to more physically meaningful factors (e.g., height of the 0°C isotherm). Moreover, rather than accounting for range effects by dividing the radar umbrella into zones, it would be useful to model the different radar-rainfall components as a smooth function of radar range.

As specified in the text, the rain gauge measurements were used to approximate the true ground rainfall. Based on previous studies about rain gauge sampling errors and the rainfall process in the area (e.g., Ciach and Krajewski 2006; Villarini et al. 2008a; Villarini and Krajewski 2008), we believe that the rain gauge–radar sampling mismatch should not have significantly affected our results at the hourly scale.

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**REFERENCES**


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