Application of Slice Theory to Account for the Horizontal Extent of Convective Precipitation Radar Echoes[1]

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ABSTRACT

Classical slice theory modified to take account of mixing and entrainment is used to explain the areal extent of 10-cm convective radar echoes in nonconvergent, barotropic situations in the subtropics.

Relative humidity in the layer of convective cloudiness, which is taken as a measure of the ability of mixing and entrainment to destroy buoyancy and evaporate clouds, correlates very significantly with the ratio of observed scope coverage to slice-indicated maximum updraft area. Furthermore, the computed slice updraft area exceeds the observed scope coverage in all cases where surface convergence was absent.

Incorporation of saturated air descent into the slice equations is performed in Appendix 1. The results show that the presence of saturated downdrafts increases the updraft area by an amount that is directly proportional to the area and velocity of these downdrafts.

1. Introduction

A survey of the appropriate literature reveals that little work has been performed to explain the mesoscale statistical properties of cloud and shower populations in terms of synoptic parameters. Not even the most crude objective thermodynamic model is available to relate the areal extent of convective clouds and showers to conventional data. A previous investigation (Myers, 1964) established that a small but significant part of the variation in the horizontal extent of convective precipitation is correlated to basic synoptic parameters. In order to improve upon these statistical results it may prove beneficial to build upon convection theory. This study represents an attempt to apply the slice theory of convection to account for the areal extent of convective precipitation radar echoes.

2. Review of convection theory

Investigations of atmospheric convection over the past half century have resulted in the development of two general pseudo-adiabatic thermodynamic convection models, the parcel method and the slice method. The parcel method results from consideration of the buoyancy forces acting on a parcel of air that is displaced adiabatically in an undisturbed environment without mixing with its surroundings. Recently, various alternatives have been proposed to modify the parcel method in order to include effects of mixing, drag, and release of latent heat of fusion (Squires and Turner, 1962; Levine, 1959; Malkus, 1960). The slice method, which was introduced by Bjerknes (1938) and extended by Petterssen (1939), Berry *et al.* (1945) and Cressman (1946), broadens the simple parcel method through the equation of continuity to take account of sinking and resultant adiabatic heating in the area not occupied by upward moving parcels. Clouds are assumed to delineate the updrafts, the velocity of which are uniform for the whole horizontal cross section. The compensating downdrafts, which are also assumed to be uniform, occur in the cloudless gaps.

Each of the two basic theories has its own criteria for stability and the amount of available energy and each generally predicts different cloud top levels. Although the slice method is a theoretical improvement over the simple parcel method and the amounts of energy, vertical accelerations and vertical velocities computed through its use should be somewhat more realistic, it has not been employed on a practical basis. Apparently, this has been due in part to the lack of knowledge about the nature of the descending currents and also to the dependence of the stability criteria and the amounts of energy on the ratio of the ascending mass to the descending mass, quantities that are not easily observable.

Also, Schmidt (1947) pointed out that strict application of the theory allows dry downdrafts to become warmer than cloudy updrafts in the lower part of the slice. In reality, however, this compressional heating of the downdrafts may be offset by evaporation of cloud elements.

A very significant feature of the slice theory when applied to saturated-adiabatically ascending air surrounded by a dry-adiabatically descending environ-

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ment is that the horizontal area over which air is ascending is limited by the hydrostatic stability. Specifically, when there is no net vertical mass transport, the maximum areal extent of upward moving air that is theoretically possible for a given environment lapse rate in a cloud layer is given by

$$\frac{A_0}{A'}=\frac{L-L_s}{L_d-L}, \tag{1}$$

where A_0 is the maximum possible mean fraction of total area occupied by updrafts, A' the minimum possible mean fraction of total area occupied by downdrafts, L the mean environment lapse rate through the layer, L_d the dry adiabatic lapse rate and L_s the mean saturated adiabatic lapse rate through the layer. This equation shows that the mean horizontal area of updrafts in the layer of convective cloudiness is limited by the mean excess of the environment lapse rate over the mean saturated adiabatic lapse rate.

When the area of updrafts is below this limit and impulses are present, cumulus convection converts latent heat into kinetic energy. Since there is only a small diurnal variation in the lapse rate above the condensation level, the observed increase in areal coverage of tall cumulus clouds and showers during midday probably occurs because an abundant supply of thermals and impulses capable of accomplishing this conversion of energy becomes available.

3. Basis for the application of slice theory to radar data

When convection is active through a deep layer, clouds cannot exist in appreciable amounts nor survive for any length of time in the region of downdrafts and it is reasonable to expect precipitation to develop in the organized updrafts. Thus, the amount of deep clouds present during active convection is not likely to exceed by much the amount specified by the slice theory, and 10-cm radar echoes may be found within the columns of upward motion. If this is the case, a practical means may exist for investigating the slice method and applying it to account for the areal coverage of convective precipitation. Note that it is not a requirement that the echoes delineate the actual regions of updrafts since the area occupied by precipitation downdrafts in dissipating cells may be of the approximate order of magnitude of the area occupied by developing showers that have not yet reached the rain stage and, therefore, give little or no 10-cm radar echoes.

It should be realized that the slice method of Bjerknes and Petterssen takes no account of mixing, entrainment, saturated downdrafts, the weight of liquid water and glaciation. Also not accounted for are synoptic effects, such as vertical wind shear and net vertical motion over the area, which have been shown to influence the areal coverage of convective radar echoes (Myers, 1964) in middle latitudes.

The success of this study—which hypothesizes that the slice theory is capable of relating the areal coverage of convective precipitation echoes to the static stability during active convection—hinges most strongly on the validity of the assumptions that the areal extent of radar echoes will bear some relationship to the area of updrafts and that one sounding is representative of a large area. To approach these conditions, the complicating effects of organized vertical motion fields, saturated downdrafts, vertical wind shear, mixing and entrainment must be minimized or accounted for systematically.

Vertical motion. As was pointed out by Cressman (1946) and as shown in Appendix 2, consideration of net vertical motion in an area leads to results that differ from those of Eq. (1) by a term that is a function of the ratio of net vertical motion to cloud updraft velocities. The latter, of course, cannot be measured routinely and the former must be estimated from large scale data.

Localized vertical motion fields resulting from topographic features may also influence scope coverage, with the net effect of such influence depending upon the position of showers relative to the organized currents.

Saturated downdrafts. Allowance for the effects of saturated descent in slice theory is developed in Appendix 1. The results show that the presence of saturated downdrafts increases the updraft area limit; furthermore, this increase is directly related to the area and velocity of these downdrafts. Byers and Braham (1949) reported that thunderstorm downdrafts are less numerous, slower and of smaller horizontal and vertical extents than saturated updrafts, which implies that the mass transport in saturated downdrafts is only a small fraction of the transport in updrafts. Thus, it is seen from Fig. 4 that the increase in maximum updraft area is probably of the order of 20 or 30 per cent at most. Since saturated downdrafts are always present to some degree, the range of increased updraft area may be less than 20 per cent and possibly under 10 per cent. Consequently, the effects of saturated downdrafts should not distort the results of this study.

Wind shear. The positive correlation found between vertical wind shear and the areal coverage of convective precipitation echoes in Pennsylvania (Myers, 1964) may be a result of many factors:

1) Frontal situations and their accompanying large scope coverage are often associated with strong shear.

2) Strong shear frequently enhances the development of large cumulonimbus and squall lines.

3) Radars with wide vertical beams cause echoes at moderate and long ranges to appear much larger than the horizontal cross section of the rain core when shear is present.

Entrainment and mixing. Since mixing and entrainment cause the in-cloud lapse rate to approach the environment lapse rate as the environment relative humidity decreases, the maximum area of updrafts supportable by a given lapse rate should vary with the humidity. Furthermore, the ratio of precipitation area to updraft area may decrease with humidity because of increasing evaporation of cloud and precipitation elements at the cloud boundaries. Consequently, if the baroclinic effects such as vertical wind shear and organized vertical motion fields can be minimized, the ratio of scope coverage to the updraft area computed by Eq. (1) may prove to be mainly a function of relative humidity. To approach these conditions, it is thus necessary to select testing grounds situated in a relatively flat area where summertime air masses are uniform and baroclinicity and convergence are minimal. Lake Charles, La. (LCH), was chosen since it meets these requirements and is the site of a WSR-57 10-cm radar and regular radiosonde observations.

4. Data analysis

All films of the PPI scope of the 10-cm WSR-57 radar at LCH of reasonable quality between 17 June and 17 August 1964 were analyzed to determine the areal coverage of echoes within 100 mi of the antenna each day at 1700, 1800 and 1900 CST. The mean echo coverage of these three separate analyses was used to represent the areal coverage corresponding to the 1800 CST sounding.

Scope coverage analyses were performed with the aid of a grid consisting of square meshes whose dimensions of one fourth inch are approximately equivalent to 10 mi. The coverage was estimated to the nearest quarter of a mesh which corresponds to 0.1 per cent of the overall scope coverage. The analyses of scope coverages are probably accurate to within a fraction of 1 per cent except in a few instances when errors of 1–2 per cent are possible because many echoes were concentrated near and in the areas of ground clutter.

For each day when at least two of the three scope analyses could be performed, the 1800 Lake Charles sounding was plotted and the maximum possible horizontal area of ascending air as specified by Eq. (1) was computed in the manner described below (and illustrated in Fig. 1).

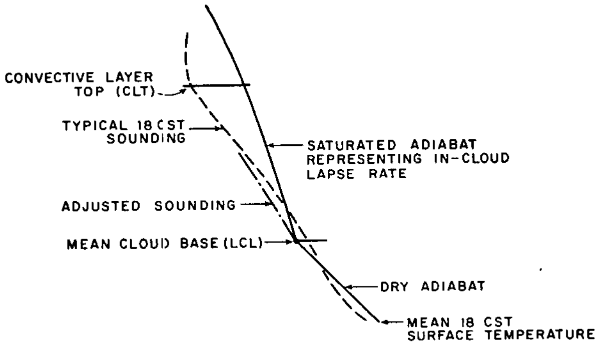

FIG. 1. Schematic of a Skew T, Log P diagram showing how in-cloud lapse rate and adjusted sounding are obtained.

Since the in-cloud temperature profile in the absence of mixing and entrainment depends very critically on the surface temperature and the height of cloud base, great care was used in obtaining representative values for each. Airways data from Alexandria, La., Lake Charles, La., New Iberia, La., Lafayette, La., and Port Arthur, Tex., were examined and the mean 1800 CST surface temperature of the stations that had not experienced cooling rain within the past 10 hours was computed. This temperature was plotted on a Skew T, Log P diagram and a line was constructed from this point along the dry adiabat to the height of the average cloud base reported by the stations which figured in the temperature computation. This second point was designated the LCL and the moist adiabat defined by it was assumed to represent the in-cloud temperature lapse rate.

According to slice theory, cloud tops will be found only slightly above the level where the environment lapse rate first becomes less than the saturated adiabatic rate rather than above the top of the positive area as specified by parcel theory. In this regard Petterssen *et al.* (1946) made aircraft cloud top observations that seemed to favor the slice height over the parcel height. More recent observations by Ludlam and Mason (1956) have shown that at least the best developed cumulonimbus tend to build beyond the slice-indicated top and approach the parcel-indicated level. This is not surprising because mixing and entrainment have a relatively small effect on giant thunderheads and the horizontal area occupied by these storms at their highest level is so small that the two theories become identical.

Because loss of buoyancy resulting from mixing, entrainment and weight of condensed water products restrict shower growth, use of the slice criterion, in general, is preferred in order to obtain a layer that is most representative of the one in which most of the showers' volume is embedded. In any event, slice theory criteria must be used because the parcel method, by definition, does not specify updraft area.

Frequently, there are several points in the sounding where the environmental lapse rate equals the saturated adiabatic rate and moderately buoyant air will easily penetrate beyond the lowest "inversion" to the second "inversion," and possibly to the third or fourth; consequently, if a strong inversion was not present, the top of the convective layer (CLT) was assumed to be the height at which the environment temperature corresponded to the coldest saturated adiabat.

Because of local variations, the lower part of radiosonde observations may not always be representative of average conditions in the radar coverage area. In order to ensure that the environment lapse rate used is representative, the sounding was adjusted to pass through the LCL, which, as was mentioned previously, is based on observed cloud bases.

Since both the in-cloud and adjusted environment lapse rates pass through the LCL, the numerator of Eq. (1) can be obtained simply by dividing the temperature difference between in-cloud air and environment at the CLT by the depth of the cloud layer.

5. Results

As has been suggested previously, if the slice theory can be used to indicate the areal coverage of air mass convective echoes in summertime barotropic situations at LCH when thermals are abundant and net vertical motion is absent, the relation should be best when the environment relative humidity in the cloud layer is high enough to discourage loss of buoyancy and evaporation of cloud elements through entrainment and mixing. Furthermore, one would expect the ratio of observed scope coverage to the maximum possible area of updrafts [Eq. (1)] to decrease with relative humidity.

Fig. 2 shows that, in fact, this is what occurs and the relation appears to be almost linear except for the four cases with relative humidities below 39 per cent and seemingly higher scope coverages than conditions justify (22 June, 7, 15 and 24 July). These (see squares in Fig. 2) are among the seven highest cloud top cases and are also included in the five cases with the most rapid decrease of humidity with height. These data suggest the following reasons may be important in explaining the deviations of these cases from the regression line:

1) For a given relative humidity the chilling effect and loss of buoyancy due to mixing and entrainment of environment air into clouds lessens with elevation.

2) The virtual temperature lapse rate is significantly steeper than the temperature lapse rate.

3) Convective (potential or layer) instability is present.

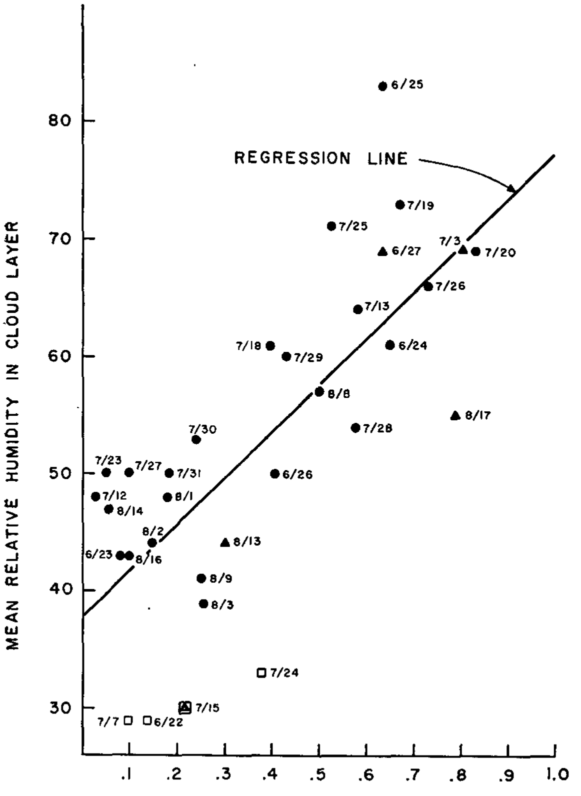

Fig. 2. Scatter diagram showing relation between ratio of scope coverage to slice updraft area and mean relative humidity in cloud layer using PPI scope films from Lake Charles, La., for the period between 17 June and 17 August 1964. See text for further details.

Fig. 2 also shows the very significant fact that the areal extent of convective precipitation echoes in the absence of noticeable surface convergence does not exceed the slice-indicated area of updrafts. Four cases (28, 29, 30 June, 1 July) have not been included in this diagram because during that period a strong easterly wave and the low level convergence field associated with it covered Louisiana. Nonetheless, it is interesting to note that the ratio of scope coverage to computed updraft area during these four days ranged from 1.40 to 4.02. U. S. Weather Bureau surface analyses showed that weak fronts lay across the radar coverage area on 15 July, 13 and 17 August, an easterly wave was in the area on 27 June and a "low" featured by convergence of surface winds was noted on 3 July. The points representing these cases (see triangles in Fig. 2) tend to be located to the right of the regression line because the atmosphere can support a greater area of upward motion when convergence exists as is illustrated in Fig. 5.

The linear correlation coefficient between the ratio of scope coverage to computed updraft area and relative humidity for all the cases plotted in Fig. 2 is 0.742, which is significant at the 1 per cent level even if cases are not considered independent unless they occur at least three days apart. If the cases of 22 June, 7, 15 and 24 July are not included, the correlation coefficient increases to 0.776. The multiple correlation coefficient between scope coverage and slice layer humidity and slice updraft area is 0.890, and the same coefficient with the trivial substitution of slice layer lapse rate for computed updraft area is 0.874. In contrast, the correlation coefficient between scope coverage and the mean relative humidity in the 850–500 mb layer is 0.541, and the multiple correlation coefficient with the influence of environment lapse rate in the 850–500 mb layer also included is only 0.652. These results suggest that for prediction purposes it might be better to express scope coverage as a linear function of slice layer humidity and either slice lapse rate or computed updraft area rather than as depicted in Fig. 2.

The advantage in choosing a representative convective layer is obvious, and there appears to be a significant advantage in approaching the problem of forecasting scope coverage thermodynamically rather than statistically.

It should be mentioned that the results reported herein should not be applied to other periods of the day without further research. It is quite possible, for example, that diurnal variations in the size and number of thermals influence the atmosphere's ability to convert heat into kinetic energy.

6. Summary and conclusions

Observational evidence has been advanced to demonstrate that the slice theory of convection is capable of accounting for the areal extent of air mass showers when there is no surface convergence and the atmosphere is barotropic. But more significantly, steps have been taken to quantify the relationship between the vertical profiles of temperature and water vapor and the areal coverage of radar echoes, something that lies at the very base of more reliable summertime precipitation forecasts, particularly those expressing probability.

In order to go a step further and predict the fraction of a territory to be affected by showers during a specified time period, it would be necessary to have additional information about the texture, movement and temporal variations of the echo patterns. For example, a squall line that occupies only a small fraction of the PPI scope may sweep out the entire area, whereas slow moving showers may only affect a small multiple of their instantaneous area. There is a definite need to relate the size of showers and the nature of shower patterns to synoptic parameters. There is also a need to quantify the influence of net vertical motion on scope coverage.

This study has ignored the dependence of cloud buoyancy on cloud diameter (Malkus, 1960). Other things being the same, total scope coverage should be greater with a few large cells than with many small ones. This is the subject of further research.

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APPENDIX 1

According to classical slice theory the maximum area of updrafts occurs when the mean temperature of saturated ascending air is equal to the mean temperature of its environment, which is composed of dry adiabatically descending air. We now consider the effect of saturated adiabatically descending air and assume that the mean updraft environment temperature depends on the relative areas of the two types of downdrafts.

Considering a horizontal slice of unit thickness, let A be the fraction of the total area covered by saturated updrafts, D' the fractional area of dry downdrafts and D the fractional area of saturated downdrafts. Then require

$$A+D+D'=1. \tag{1.1}$$

Also, let w be the updraft velocity and w' be the dry downdraft velocity. First, we will choose the saturated downdraft velocity as numerically equal to the updraft velocity, which is a reasonable assumption. If the slice is large so that the transport of mass through the vertical walls is negligible compared with the transport through the horizontal faces it follows that

$$Dw+D'w'=Aw, \tag{1.2}$$

which in unit time becomes

$$D\Delta z+D'\Delta z'=A\Delta z, \tag{1.3}$$

where the Δz's are distances that correspond to the velocities in (1.2).

If, after unit time, the temperature of saturated air that ascends to level P_x (see Fig. 3) is T_1, the temperature of air that descends dry adiabatically to P_x is T_d and the temperature of air that descends saturated adiabatically to P_x is T_w, and L, L_d and L_s are as defined in the text, then the area of updrafts is at a maximum when the mean temperature of the downdrafts equals the updraft temperature, or

$$T_1=\frac{DT_w+D'T_d}{D+D'}. \tag{1.4}$$

But since (see Fig. 3)

$$T_w=T_0-(L-L_s)\Delta z, \tag{1.5}$$

$$T_1=T_0+(L-L_s)\Delta z, \tag{1.6}$$

$$T_d=T_0+(L_d-L)\Delta z', \tag{1.7}$$

Eq. (1.4) becomes

$$\frac{(1-A+D)(L-L_s)}{D'(L_d-L)}=\frac{\Delta z'}{\Delta z}. \tag{1.8}$$

Substituting from (1.3) we have

$$\frac{L-L_s}{L_d-L}=\frac{A-D}{1-A+D}. \tag{1.9}$$

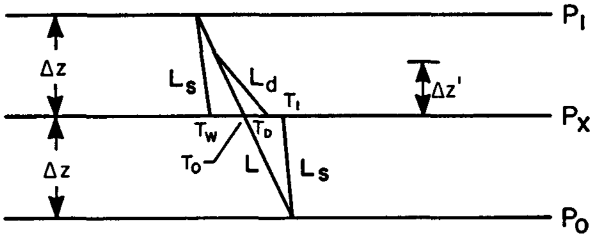

FIG. 3. Schematic section of a Skew T, Log P diagram. L, temperature curve; L_s, adiabatic of the ascending air and saturated descending air; L_d, adiabatic of dry descending air; T_0, initial temperature at P_x.

From classical slice theory (see Eq. (1) of text), however,

$$\frac{A_0}{1-A_0}=\frac{L-L_s}{L_d-L}, \tag{1.10}$$

where A_0 is the updraft area when $D=0$. Thus (1.9) becomes

$$A=A_0+D. \tag{1.11}$$

Next we will treat the less realistic case in which saturated downdraft velocities equal dry downdraft velocities. Thus,

$$A\Delta z=D\Delta z'+D'\Delta z'. \tag{1.12}$$

Following a procedure similar to that employed in deriving (1.11) we obtain

$$A_0=\frac{AD'}{1-A}. \tag{1.13}$$

The essence of both Eq. (1.11) (rapid saturated downdrafts) and Eq. (1.13) (slow saturated downdrafts) appears in Fig. 4. It is seen that saturated descent increases the possible area of updrafts by an amount that is directly related to the speed and area of these downdrafts.

If the mean downdraft temperature is assumed to depend on the mass transport within the two types of downdrafts rather than on their respective areas, an increase in the area of saturated downdrafts would effect only in the case of rapid downdrafts a somewhat greater increase in updraft areas than those which appear in Fig. 4.

In the above calculations, the initial temperature of the saturated downdrafts is equal to the environment temperature which seems more reasonable than using the cloud temperature. In any event, not very different results are obtained when the cloud temperature is used in the rapid downdraft case.

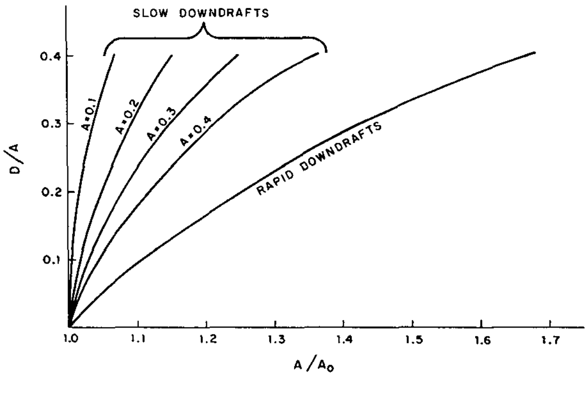

FIG. 4. Influence of saturated downdrafts on the area of updrafts. D represents the area of downdrafts, A_0, the area of updrafts when $D=0$ and A the area of updrafts when $D>0$.

APPENDIX 2

Cressman (1946) incorporated the effects of horizontal divergence into slice theory by using a net vertical velocity that is a function of cloud updraft area. It is possible to obtain a less complicated solution for the maximum updraft area by employing a net vertical velocity w'' which is uniform throughout the area in question. The net vertical transport is

$$(A+A')w''=Aw-A'w', \tag{2.1}$$

where $A+A'=1$. In unit time this becomes

$$\Delta z''=A\Delta z-A'\Delta z'. \tag{2.2}$$

When the area of updrafts is at a maximum,

$$\frac{L-L_s}{L_d-L}=\frac{\Delta z'}{\Delta z}. \tag{2.3}$$

Substituting from (2.2) we have

$$\frac{\Delta z''}{\Delta z}=\frac{(L-L_s)}{(L_d-L)}(A-1)+A. \tag{2.4}$$

But from classical slice theory,

$$\frac{A_0}{1-A_0}=\frac{L-L_s}{L_d-L}, \tag{2.5}$$

where A_0 is the maximum updraft area when $w''=0$. Thus, (2.4) becomes

$$A=A_0+\frac{\Delta z''}{\Delta z}(1-A_0). \tag{2.6}$$

This relation is diagramed in Fig. 5.

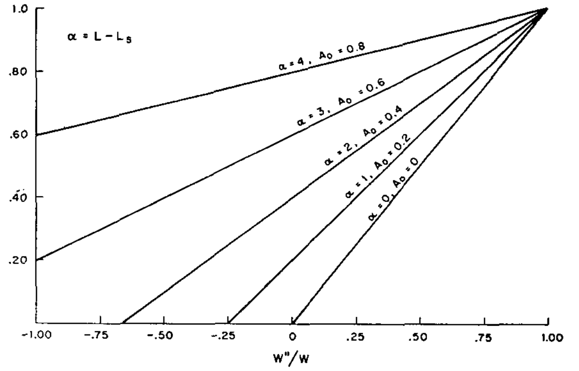

FIG. 5. Influence of net upward (right) and downward (left) motion on the area of updrafts. A_0 is the area of updrafts when $w''=0$, and A is the area of updrafts for all w''. The saturated adiabatic lapse rate is assumed to be 5C km^{-1}. α is in units of (°C) km^{-1}.

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