# CORRESPONDENCE

## Comments on "A Relationship between Reflectivity and Snow Rate for a High-Altitude S-Band Radar"

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Wolfe and Snider (2012) seek to determine a relationship between the equivalent radar reflectivity factor $Z_e$ and the liquid-equivalent snow precipitation rate $S$ that is of the customary power-law form $Z_e = \alpha S^\beta$. They follow the approach of Rasmussen et al. (2003) in attempting to use a mathematical analysis to determine the value of the exponent $\beta$. Examination of (2) and (3) of Wolfe and Snider (2012) (hereinafter these two equations will be referred to as WS2 and WS3; equations that are introduced in this comment will be labeled with a C) indicates that something is amiss:

$$Z_e = \left(22.2\frac{|K_i|^2}{|K_w|^2}\frac{\rho_w^{5/3}}{\rho_i^2}\frac{\Omega^{1/3}}{V_t^{5/3}n_o^{2/3}}\right)S^{5/3}\,(\mathrm{m^6\,m^{-3}}) \quad \text{and} \qquad \text{(WS2)}$$

$$Z_e = \left(21.9\frac{|K_i|^2}{|K_w|^2}\frac{\rho_w^2}{\rho_i^2}\frac{1}{V_t^2 N}\right)S^2\,(\mathrm{m^6\,m^{-3}}). \qquad \text{(WS3)}$$

Here $|K_i|^2$ and $|K_w|^2$ are the dielectric factors for ice and water, respectively; $\rho_w$ and $\rho_i$ are the densities of water and ice, respectively; $\Omega$ is a constant related to the density of a snowflake, assumed to be a spherical air–ice mixture; $V_t$ is the fall speed of the snowflakes, assumed in their analyses to be independent of the particle size; and $n_o$ and $N$ are parameters (discussed below) of the snowflake size distribution, assumed to be of exponential form. The form (WS2) was derived by Rasmussen et al. while Wolfe and Snider arrived at (WS3) via a slightly different approach focusing on $N$ instead of $n_o$.

Now $Z_e$ cannot simultaneously be proportional to both $S^{5/3}$ and $S^2$. Thus some quantity, or quantities, inside the parentheses in (WS2) and/or (WS3)—their coefficient $\alpha$—must also be a function of $S$. In that case one, or perhaps both, of the equations would not indicate the proper value of the exponent $\beta$. The likely candidates are easy enough to identify: $n_o$ in (WS2) and/or $N$ in (WS3).

That points back to the initial basic assumption of these authors that the snowflake size distribution can be represented as an exponential function when the particle size is expressed in terms of an *unmelted* diameter $D_U$ (in what follows the subscript "$U$" denotes quantities related to the unmelted-diameter formulation and subscript "$M$" denotes ones related to a similar *melted*-diameter formulation):

$$n(D_U) = n_{0U}\exp(-\lambda_U D_U) = N\lambda_U \exp(-\lambda_U D_U). \qquad \text{(C1)}$$

Here $n(D_U)$ is the particle number concentration per unit size interval, $n_{0U}$ and $\lambda_U$ are parameters of the distribution function, and $N$ is the total snowflake number concentration. In this assumption Wolfe and Snider follow Rasmussen et al. (2003). Looking back at that article reveals that those authors, citing Sekhon and Srivastava (1970), took the latter's exponential form for the snowflake size distribution when expressed in terms of the *melted* diameter $D_M$:

$$n(D_M) = n_{0M}\exp(-\lambda_M D_M) = N\lambda_M \exp(-\lambda_M D_M), \qquad \text{(C2)}$$

and assumed (with no apparent supporting evidence) that the same form would apply when the particle size is expressed in terms of $D_U$. Sekhon and Srivastava show that $n_{0M}$ and (by inference) $N$ do vary with $S$.

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The use of melted snowflake diameters has fallen out of fashion with the advent of optical particle sensors. More recent limited observations with instruments of that type (e.g., Brandes et al. 2007; Woods et al. 2008; Newman et al. 2009) present a mixed view of the applicability of (C1), confounded by varying definitions (for particles that in the vast majority of cases are not really spherical) of what measurement determines $D_U$. In addition, the following expression for the density $\rho_s$ of a snowflake is key to the analyses in both Rasmussen et al. (2003) and Wolfe and Snider (2012):

$$\rho_S = \Omega/D_U. \tag{C3}$$

Support cited by Rasmussen et al. for that formulation, with $\Omega$ being a constant related to the snowflake type, is buried in an unpublished report. Wolfe and Snider (2012) cite Brown and Francis (1995), which is reasonably consistent with (C3) [see also the summary of related literature in Hogan et al. (2012)], and Pokharel and Vali (2011), which is not. The latter is of special concern because the data came from the same area as the observations in Wolfe and Snider.

Nevertheless, if one accepts the expression (C3) then the exponential form (C1) *cannot* be the correct form for $n(D_U)$, at least for the aggregate snowflakes considered by Sekhon and Srivastava. To see this, two expressions for the mass of a snowflake are set equal:

$$\frac{\pi\rho_S D_U^3}{6} = \frac{\pi\rho_w D_M^3}{6}. \tag{C4}$$

Solving for $D_M$ yields

$$D_M = (\Omega/\rho_w)^{1/3} D_U^{2/3}. \tag{C5}$$

Using

$$n(D_U) = n(D_M)\frac{dD_M}{dD_U} \tag{C6}$$

along with (C5) and the well-established (C2) leads to

$$n(D_U) = \frac{2}{3}\left(\frac{\Omega}{\rho_w}\right)^{1/3} N\lambda_M D_U^{-1/3} \exp\left[-\left(\frac{\Omega}{\rho_w}\right)^{1/3}\lambda_M D_U^{2/3}\right]. \tag{C7}$$

In other words, the proper form for $n(D_U)$ would be a Weibull distribution [or a "modified gamma distribution" in the terminology used by Petty and Huang (2011)], and not the exponential distribution assumed by Rasmussen et al. and followed by Wolfe and Snider.

In summary, the mathematical analysis in Wolfe and Snider (2012), as well as that in Rasmussen et al. (2003), is based on a questionable assumption about the form of the snowflake size distribution. Considering that along with the fact that neither analysis leads to a clearly established value for the exponent $\beta$, the mathematical analyses in both papers should be disregarded.

That being said, one can derive the key aspects of (WS3) through a much simpler argument. The value of $Z_e$ corresponding to the well-established form (C2) for the snowflake size distribution using melted diameters is

$$Z_e = \frac{|K_i|^2}{|K_w|^2}\left(\frac{\rho_w}{\rho_i}\right)^2\frac{720N}{\lambda_M^6}. \tag{C8}$$

Under the assumption of a snowflake fall speed independent of size, the snowfall rate $S$ would be

$$S = \frac{\pi V_t N}{\lambda_M^3}. \tag{C9}$$

Eliminating $\lambda_M$ between those two expressions leads to the relationship

$$Z_e = \left[\frac{|K_i|^2}{|K_w|^2}\left(\frac{\rho_w}{\rho_i}\right)^2\frac{720}{\pi^2 V_t^2 N}\right]S^2. \tag{C10}$$

Apart from the constant, this has the same form as (WS3). That, however, does not remove the difficulty that none of the expressions [(WS2), (WS3), or (C10)] is a valid $Z_e$–$S$ relationship, because $N$ itself (and likewise $n_o$ in WS2) is a function of $S$ (Sekhon and Srivastava 1970). Consequently any discussion of how the coefficient of $S^x$ ("$\alpha$") in those equations behaves is moot, because the coefficient itself is not independent of $S$.

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