

Automatic and Probabilistic Foehn Diagnosis with a Statistical Mixture Model

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ABSTRACT

Diagnosing foehn winds from weather station data downwind of topographic obstacles requires distinguishing them from other downslope winds, particularly nocturnal ones driven by radiative cooling. An automatic classification scheme to obtain reproducible results that include information about the (un)certainty of the diagnosis is presented. A statistical mixture model separates foehn and no-foehn winds in a measured time series of wind. In addition to wind speed and direction, it accommodates other physically meaningful classifiers such as the (potential) temperature difference to an upwind station (e.g., near the crest) or relative humidity. The algorithm was tested for Wipp Valley in the central Alps against human expert classification and a previous objective method (Drechsel and Mayr 2008), which the new method outperforms. Climatologically, using only wind information gives nearly identical foehn frequencies as when using additional covariables. A data record length of at least one year is required for satisfactory results. The suitability of mixture models for objective classification of foehn at other locations will have to be tested in further studies.

1. Introduction

Foehn is “a wind warmed and dried by descent, in general on the lee side of a mountain” (WMO 1992). As such it is as ubiquitous as the mountain ranges of the earth. Diagnosing when foehn blows requires distinguishing it from other downslope flows, which can be nontrivial. An accurate diagnosis is a prerequisite for studies dealing with mechanisms, climatologies, and effects of foehn (e.g., on air quality). This paper describes a method to objectively diagnose both foehn occurrence and the (un)certainty of the diagnosis.

The conceptual model of foehn that fits best the results of the latest large field campaigns [e.g., the Mesoscale Alpine Programme (MAP; Mayr and Armi 2008) and Terrain-Induced Rotor Experiment (TREX; Armi and Mayr 2011)] is depicted in Fig. 1a. The descent of upstream air is possible when the virtual potential temperature of the descending upstream air mass is equal to

or lower than the air in the downstream valley. Since absolute humidity normally decreases away from the surface, relative humidity in the descended and compressed foehn air is lower than in the air it replaces downstream. Exceptions occur, for instance in the Appalachians (Gaffin 2002, 2007).

Other wind systems that flow down topography and might be mistaken for foehn are downslope/downvalley flows from nocturnal radiative cooling (Defant 1949; Whiteman 2000), depicted in Fig. 1b, and shorter-lived events such as frontal passages (Prandtl 1944, 326–330) or convective outflows.

The traditional method for detecting foehn (Conrad 1936) from observation data at a single (valley) station (location V in Fig. 1) is to analyze temporal changes of temperature, relative humidity, and wind. At onset wind speed must increase and direction must be down the local terrain, and for many locations temperature should increase and relative humidity decrease. This method normally requires a human expert, making results nonreproducible and very time consuming. And even a human expert has difficulty distinguishing weaker foehn flows from radiatively driven downslope flows.

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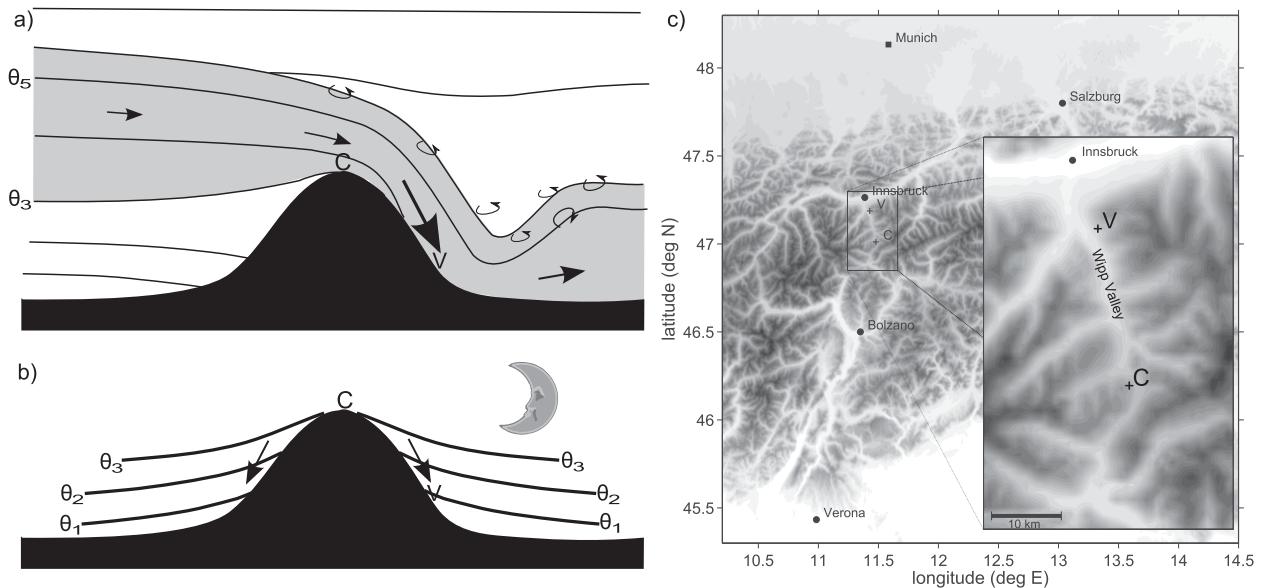


FIG. 1. Schematic representation of (a) foehn and (b) radiatively driven downslope flow, respectively. In (a) gray shading indicates the foehn layer and the arrows indicate qualitatively the flow direction and speed. Turbulent mixing at the upper edge of the foehn flow is indicated by curved arrows. The solid lines in both (a) and (b) are isentropes. (c) Topographic map of the eastern Alps with detailed insert of the Wipp Valley test region. The letter C depicts the crest station at Sattelberg (2.1 km MSL; 47.01143°N, 11.47926°E), and V is the valley station at Ellbögen (1.1 km MSL; 47.18802°N, 11.42961°E).

Gutermann (1970) pioneered the (semi)automated and objective foehn diagnosis. He used a wind index that combines speed and directional information together with anomalies of temperature and relative humidity. The statistical method was Fisher's (linear) discriminant analysis in combination with a long dataset with (manually) classified foehn.

Diagnostic accuracy increases with the availability of a second station farther upslope at (or near) the crest (location C in Fig. 1) with which one can exploit the physical differences between foehn and downslope winds. During foehn (Fig. 1a) the upstream air mass descends so that potential temperatures at stations V (valley) and C (crest) will almost¹ be identical ($\Delta\theta = \theta_{\text{crest}} - \theta_{\text{valley}} \approx 0$). In contrast to this, radiative cooling during a nocturnal downslope wind situation (Fig. 1b) leads to stable stratification ($\Delta\theta > 0$; Whiteman 2000). This difference was exploited to separate foehn from nocturnal downslope winds in an objective foehn classification method (OFC), first developed during MAP. It is described in detail in Vergeiner (2004) and summarized in Drechsel and Mayr (2008). The method classifies wind of at least 2 m s^{-1} from

a downslope sector as foehn when $\Delta\theta$ is below a station-dependent threshold, which has to be semimanually determined. Having to determine the hard threshold individually for each location is the main drawback of the method. Dürr (2008) developed another automated physically based classification that additionally includes relative humidity and gusts for Swiss stations by refining a manual procedure. This method was used to classify the most recent period of a unique 145-yr foehn data series for the Swiss station at Altdorf (Gutermann et al. 2012).

Even a human expert using a second station will encounter events that are not clearly distinguishable and that are classified by the OFC without giving an indication of the high uncertainty. An improved objective classification method should consequently avoid both drawbacks by making it unnecessary to determine anything except possibly the topographic downslope direction by hand and by moving from a deterministic to a probabilistic diagnosis.

2. Foehn diagnosis with a statistical mixture model

Separation of foehn from radiatively driven downslope winds is a typical classification problem, for which statistical science provides several methods. Since we want the method to be applicable to any location, we chose unsupervised classification. A mixture model allows us

¹ Small differences may occur because of, for example, imperfect sensor calibration, mixing in of nocturnally cooled air, or evaporation of precipitation into the downstream foehn layer. They are discussed at length in Vergeiner (2004).

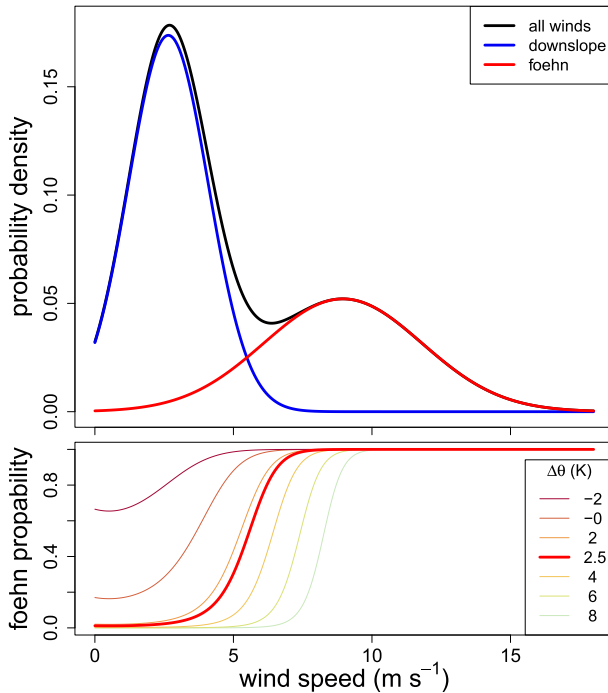


FIG. 2. (top) Distribution of wind speed (black) for winds from the foehn sector at the Wipp Valley station with distributions of foehn (red) and no-foehn (blue) fitted by the mixture model M2 for wind speed with the difference in potential temperature between crest and valley station as concomitant. (bottom) Foehn probability for model M2 as a function of wind speed and potential temperature difference $\Delta\theta$ (isolines). The red line represents the mean state for the whole distribution in the top part of the figure.

to estimate the *unknown* density distribution of foehn and no-foehn cases from the observed and thus *known* density of all cases and the probability that observation i belongs to one of the two classes (McLachlan and Peel 2000; Hastie et al. 2009). This assumes that the two densities (wind regimes) are statistically distinguishable as in Fig. 2.

In our case, the mixture model for the wind speed distribution $f(s)$ consists of two normally distributed components, downslope wind and foehn:

$$f(s_i) = \underbrace{(\pi - 1)\phi(s_i | \mu_1, \sigma_1^2)}_{\text{(i) downslope wind}} + \underbrace{\pi\phi(s_i | \mu_2, \sigma_2^2)}_{\text{(ii) foehn}}, \quad (1)$$

where

$$\phi(s_i | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(s_i - \mu)^2}{2\sigma^2}\right\} \quad (2)$$

denotes the Gaussian density function with mean μ and variance σ^2 . Also, π is the prior probability for foehn class and the complementary probability $\pi - 1$ is the

prior for downslope wind class. We can properly label these two classes since foehn is stronger than nocturnal downslope wind (i.e., $\mu_2 > \mu_1$). Note that in general any component density can be used in place of the Gaussian or that wind speed can be transformed (e.g., taking the square root) before being used in the mixture model; however, this did not improve our model.

The probability that one measurement of wind speed s_i belongs to the foehn cluster is given by the proportion of the probability density function for foehn to the total wind speed distribution and can be calculated by the ratio of (ii)/[(i) + (ii)] from the components of Eq. (1).

Additional physically meaningful classifiers such as the potential temperature difference between valley and crest or relative humidity called “concomitant” variables (Dayton and Macready 1988) may be used to improve the estimated distributions. Then the prior probability π changes from being constant to (in our two-component case) a binomial logit model. The prior for the second component (foehn) is then

$$\pi(\mathbf{x}) = \text{logit}^{-1}(\mathbf{x}^T \boldsymbol{\beta}), \quad (3)$$

where the vector

$$\mathbf{x} = \begin{pmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \end{pmatrix} \quad (4)$$

contains the used concomitant variables x_i after its first component, which is 1. The corresponding coefficients are written in the vector $\boldsymbol{\beta}$.

We used the FlexMix package (Leisch 2004; Grün and Leisch 2008) in the programming language R (<http://www.R-project.org>) to fit the mixture model. The only preprocessing was the application of a wind direction filter: only wind from a 180° sector centered along the topographic downslope direction could be classified as foehn. The downslope direction can either be determined manually or automatically (Pelletier 2013).

3. Application

This automatic classification method was tested for the Wipp Valley in the central Alps (see Fig. 1c). It is a typical foehn valley, oriented perpendicular to a gap in the main crest. The indentation in the crest line increases foehn occurrence (e.g., Jackson et al. 2013). It was a study area of MAP and has since then been instrumented to allow testing of different foehn classification schemes. We use one crest/gap station at 2.1 km MSL and one valley

TABLE 1. Summary of mixture models M1–M3 with variables used for fitting [wind speed s and direction d , relative humidity RH, and difference in potential temperature ($\Delta\theta = \theta_{\text{crest}} - \theta_{\text{valley}}$)] and relative frequency of no-foehn and foehn classification with two levels of certainty and sum of both levels, each for the 14-yr dataset at the valley station. Note that wind direction was only used to preprocess data but is not included in the statistical models. Additionally, results for the previous nonprobabilistic objective algorithm OFC are shown.

Model	Variables	No foehn (%)			Foehn (%)		
		≥99	50–99	Sum	Sum	50–99	≥99
M1	s	49.40	30.95	80.35	19.65	5.27	14.38
M2	s and $\Delta\theta$	71.78	9.61	81.39	18.61	4.30	14.32
M3	s , $\Delta\theta$, and RH	71.67	8.64	80.31	19.69	4.10	15.59
OFC	s , $\Delta\theta$, and d			84.26	15.74		

station at 1.1 km MSL, which is 21 km downstream. 14 years of measurements of wind speed and direction, temperature, relative humidity, and pressure averaged over 10-min intervals are available. Joint data availability is 91%.

Three different mixture models exploiting an increasing amount of measurement information were applied to probabilistically diagnose foehn occurrence at the valley station. The first model (M1) uses only wind speed at the valley station itself. The second model (M2) uses the measurements at the crest station to include the potential temperature difference $\Delta\theta$ between crest and

valley as the first concomitant variable. Finally, the third model (M3) adds relative humidity at the valley station as a second concomitant (cf. Table 1).

The behavior of all three models will be first explored in a case study and then in a foehn climatology over the whole 14-yr dataset.

a. Case study: Shallow foehn on 27–28 October 2005

For a subjective (human expert) verification of these foehn models more than 50 case studies were examined. Most of them, especially the stronger foehn events, are well captured by all models with only minor differences. For illustration, we present one in which the differences among models M1, M2, and M3 are especially pronounced and where the ending of the foehn period is difficult to analyze even by a human expert.

On 27 and 28 October 2005 the Alps separated a cold air mass to the south from warmer air to the north, which caused (shallow) south foehn through the lower alpine passes. At the Wipp Valley station, the foehn event started at 1400 UTC 27 October 2005 in our subjective classification (gray shading), shortly after the direction shifted from northwest (upvalley) to southeast (downvalley) with a strong increase in temperature and a decline in relative humidity (Fig. 3). It ended the following night at 0150 UTC with reverse but less pronounced signals in temperature and humidity. Notable is the continuous increase of $\Delta\theta$ to 4 K resulting from the mixing in of radiatively cooled air since 1700 UTC,

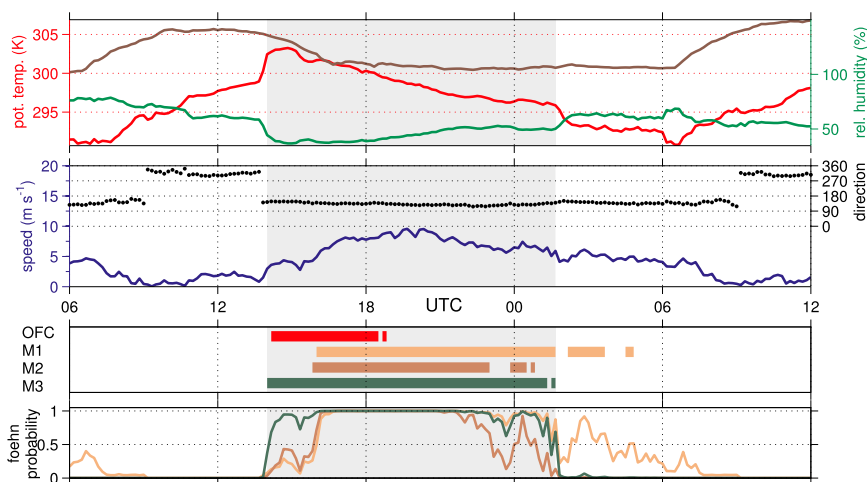


FIG. 3. Time series from 0600 UTC 27 Oct to 1200 UTC 28 Oct 2005 for the valley station with (top) potential temperature (red) and relative humidity (green) and the potential temperature of the crest station (brown); (middle) wind direction (black dotted line) and wind speed (blue line) for valley station; and (bottom) periods of foehn diagnosed by each model M1–M3 (cf. Table 1) having a foehn probability $\geq 50\%$ and the OFC, and foehn probabilities from mixture models M1–M3 with same colors as in the top half of the bottom panel. Manually classified foehn period is shaded light gray in the background of all panels.

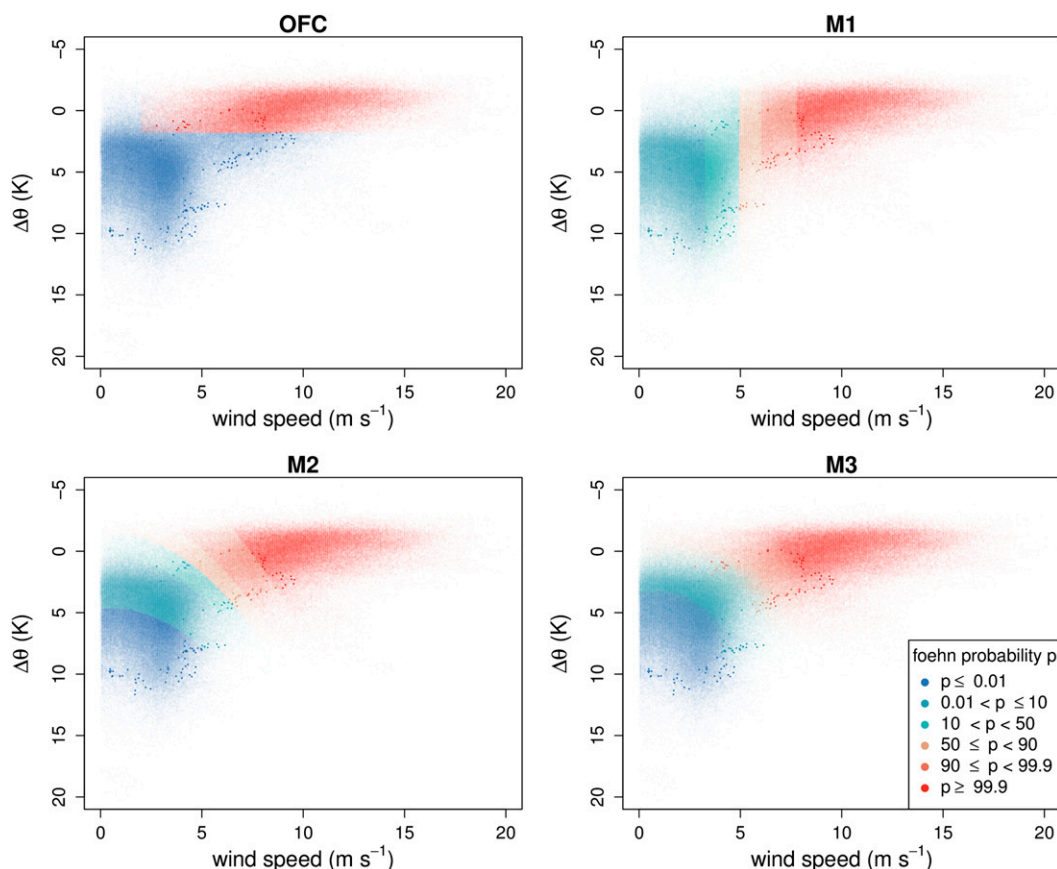


FIG. 4. Likelihood of foehn (colors) for wind speed and potential temperature difference ($\Delta\theta = \theta_{\text{crest}} - \theta_{\text{valley}}$) combinations for wind from the 180° wide foehn wind sector. Each point represents a 10-min mean. Results are shown for the previous objective method OFC (which is deterministic and only has yes/no), and the mixture models M1, M2, and M3 (see also Table 1). The points from foehn wind sector corresponding to the case study shown in Fig. 3 are drawn without transparency.

shortly before this foehn event ended. Afterward, the flow continued to be downvalley and of similar strength but no longer caused by foehn but rather by radiative cooling.

Overall all models captured the core part of this foehn event (cf. M1–M3 indicated by the bars and graph in Fig. 3, bottom). By using only wind speed at the valley station, the mixture model M1 misses the onset by a few hours because of low foehn wind speeds and erroneously postpones the ending caused by high radiatively driven speeds. However, by including the information about the uncertainty of the classification, foehn beginning and ending are indicated correctly in the graph at the bottom of Fig. 3. Adding the difference in potential temperature between mountain and valley station (M2) avoids the misclassification at the end of the foehn period but shortens the foehn period and still misses the first foehn hours—although again the foehn probability correctly increases. Adding the relative humidity at the valley station as further concomitant variable in M3 finally nearly coincides with our subjective classification.

b. Climatological and statistical aspects

Climatologically, foehn at the valley station is frequent and occurs for about one-fifth of the time. All three mixture models analyze the overall foehn frequency² for the investigated 14-yr period to within 1% (Table 1); using only wind speed gives 19.7%. Adding potential temperature difference reduces the frequency by about 1% to 18.6% (as in the case study), while adding relative humidity in M3 brings the frequency back to the speed-only value of 19.7%. The previous objective method (OFC) on the other hand, found only 15.7% of foehn events.

Figure 4 demonstrates the workings of the mixture models M1–M3 in comparison to the OFC. The point clouds have two maxima: one at low wind speeds and large potential temperature differences $\Delta\theta$ (high static stability), which indicates the downvalley winds, and

² Using a threshold of 50% foehn probability.

another with $\Delta\theta$ near zero and high wind speeds, which indicates foehn. The OFC uses hard thresholds for $\Delta\theta$ and minimum wind speed and misses events of moderate-to-low speeds and moderate stability. Mixture model M1 with wind speed only, on the other hand, includes too many of these cases but misses some low speed cases with low stability. If one were to use 50% probability for a yes/no classification, a line of 4.9 m s^{-1} would separate the classes. M2 divides the classes more appropriately by a curved cut, because now foehn probability also depends on $\Delta\theta$. This can also be seen in the lower part of Fig. 2. At low values of $\Delta\theta$, low wind speeds are sufficient for high foehn probabilities, while for a more stably stratified atmosphere (high $\Delta\theta$) much higher speeds are necessary for reaching the deciding probability of 50%. By adding relative humidity in M3 the separation conforms even more closely to the one a human would draw.

4. Discussion and conclusions

The statistical mixture model is a method to diagnose foehn automatically and probabilistically. It eliminates having to select threshold values individually for each location as required by previous automatic methods (Drechsel and Mayr 2008; Dürr 2008) and includes information about how certain the diagnosis is. A dataset with already analyzed foehn periods as in the statistical classification method of Gutermann (1970) is also not needed. Another advantage is the possibility to diagnose foehn for locations with well-established foehn winds, objectively without any mountain station (M1) at the expense of some misses and false positives. For our location, this modified total foehn time by only 0.04% due to about 7% temporal misclassifications of either foehn beginning or ending times, compared to the best model M3 (see Table 1 and compare Fig. 4 and case study in Fig. 3). OFC found about one-fifth less foehn (Table 1), mostly by ignoring weak nocturnal foehn cases that have a substantial amount of radiatively cooled air mixed in from the slopes and side valleys, thus exceeding the hard threshold for potential temperature difference between crest and valley (Fig. 4a).

At farther downstream locations (e.g., wide valleys or plains at edge of mountains) foehn winds might be too weak to be classified by speed (M1) alone. Additional concomitants as in M2 and M3 might help up to a location where the foehn airstream has become too diluted by the surrounding air to be distinguishable (Armi and Mayr 2011; Mayr and Armi 2010).

Prior to the fully automatic classification, one additional piece of information is still required: the selection

of the sector of foehn wind direction. Doing this manually only requires having to determine the valley axis direction and selecting an appropriate sector to each side. In our case this was the widest possible choice of $\pm 90^\circ$. This step could be automated using a digital elevation model and a routing algorithm (e.g., Pelletier 2013). For particular topographies (e.g., edge of mountain), when elimination of other wind systems is required, or when the distinction between different foehn regimes (e.g., south foehn and west foehn) is needed, smaller sectors can be chosen.

Incidentally, the simplest classification setup proved also to be the best when compared against the subjective human expert classification of the 50+ events and overall foehn frequency. Appropriate downslope wind direction, speed, and $\Delta\theta$ are the most important inputs for a foehn classification with a last refinement by relative humidity. Similar variables were also found in previous automated methods (Gutermann 1970; Drechsel and Mayr 2008; Dürr 2008). If a crest station is not available, NWP model analysis data at the appropriate height might be used provided the model topography sufficiently resembles the actual topography. The information provided by additional concomitant variables such as gusts, the ratio between gust and 10-min average speed (gust factor), or the temporal gradients of temperature and relative humidity, respectively, did not improve the results for data from a single (valley) station. Adding relative humidity to M1 without using $\Delta\theta$ (as for M3) even worsened the classification with too many false positives, because in this constellation the model may include downslope winds with low relative humidity, which can occur in the evening when the wind has just shifted to being downslope and the spread between dry-bulb temperature and dewpoint is still large. For the combination of valley and crest station, adding the difference of the mixing ratio between mountain and valley stations did not improve the results.

Since wind speed distributions are typically non-Gaussian (cf. Wilks 2011) but the mixture model in its simplest form in Eq. (1) assumes such a distribution, we remedied a possible violation by first transforming wind speed with logarithm and square root, respectively. However, in all cases considered the results were worse.

Since weather station records at foehn locations have different length, an important practical question is the required minimum dataset length for the classification to become reliable. Figure 5 shows the effect of record length on four parameters of the mixture model [cf. Eq. (1)]. For each of the record lengths (1 month–4 years), 100 such contiguous samples were randomly drawn from the 14-yr dataset and the mixture models fitted only with these

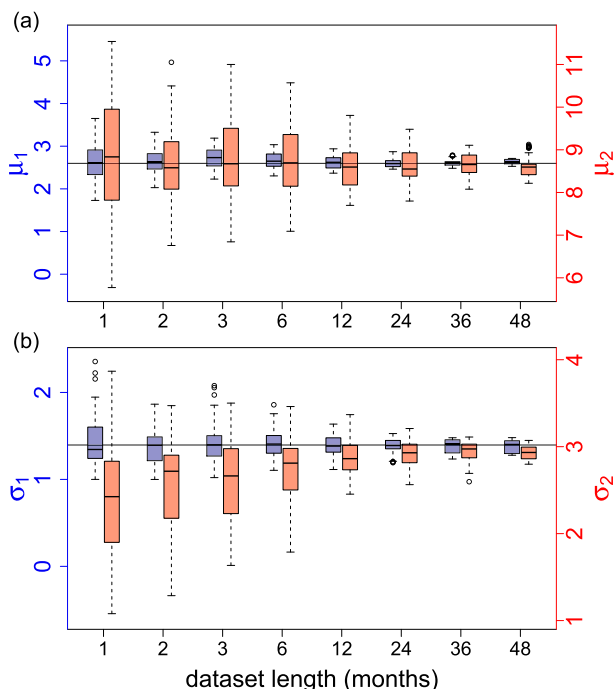


FIG. 5. Effect of data length on parameter estimation of mixture model. (a) Means μ_i and (b) standard deviations σ_i were fitted separately for 100 contiguous bootstrap samples of different dataset length (months) for mixture model M3 (M1 and M2 are almost identical). Index 1 is for radiatively driven downslope winds (blue; left axis) and index 2 is for foehn (red; right axis) from components (i) and (ii) in Eq. (1), respectively. Note that scaling for each parameter is equal for both indices (ordinates). The black horizontal lines show the parameter values fitted to the whole available dataset (152 months).

measurements. Parameters for no-foehn are already well captured for short periods since radiatively driven flows occur on many nights. Foehn, on the other hand, has a strong seasonal cycle because of, for example, local cold pools (cf. Mayr et al. 2007), with a pronounced minimum during summer in our investigated area. Therefore short records from summer will provide too small a foehn sample for reliable parameter estimation but the same duration taken in autumn might suffice. To cover the major part of foehn wind distributions, a minimum dataset of one year is required for a station with frequent foehn.

A main purpose of the paper was to introduce mixture models as an efficient way of probabilistically diagnosing foehn. We have started to explore their suitability for other foehn regions of the world. While the automatic algorithm using only wind information is successful for simple foehn locations like the Wipp Valley, other sites might require appropriate concomitants such as M2 and M3 here. The application potential of this diagnosis method

encompasses, for example, (unified) foehn climatologies, studies of foehn mechanisms, or verification of foehn forecasts.

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