Connecting Microphysical Processes in Colorado Winter Storms with Vertical Profiles of Radar Observations

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ABSTRACT

To better connect radar observations to microphysical processes, the authors analyze concurrent polarimetric radar observations at vertical incidence and roughly side incidence during the Front Range Orographic Storms (FROST) project. Data from three events show signatures of riming, aggregation, and dendritic growth. Riming and the growth of graupel are suggested by negative differential reflectivity $Z_{DR}$ and vertically pointing Doppler velocity magnitude $\lvert V_R \rvert > 2.0 \text{ m s}^{-1}$; aggregation is indicated by maxima in the downward-relative gradient of radar reflectivity at horizontal polarization $Z_H$ below the $-15^\circ \text{C}$ isotherm and positive downward-relative gradients in $\lvert V_R \rvert$ when averaged over time. A signature of positive downward-relative gradients in $Z_H$, negative downward-relative gradients in $\lvert V_R \rvert$, and maxima in $Z_{DR}$ is observed near $-15^\circ \text{C}$ during all three events. This signature may be indicative of dendritic growth; preexisting, thick platelike crystals fall faster and grow slower than dendrites, allowing for $\lvert V_R \rvert$ to shift toward the slower-falling, rapidly growing dendrites. To test this hypothesis, simplified calculations of the $Z_H$ and $\lvert V_R \rvert$ gradients are performed for a range of terminal fall speeds of dendrites and isometric crystals. The authors prescribe linear profiles of $Z_H$ for the dendrites and isometric crystals, with the resulting profiles and gradients of $\lvert V_R \rvert$ determined from a range of particle fall speeds. Both the observed $Z_H$ and $\lvert V_R \rvert$ gradients are reproduced by the calculations for a large range of fall speeds. However, more observational data are needed to fully constrain these calculations and reject or support explanations for this signature.

1. Introduction

Understanding ice growth processes is vitally important in understanding the life cycle of cloud and precipitation systems. A better understanding of these processes is thus necessary to assess the impact of clouds on the global climate (Vavrus 2004). Ice particles grow primarily through vapor deposition, aggregation, and riming. These processes are highly sensitive to temperature, supersaturation with respect to ice, and liquid water content (Marshall and Langleben 1954; Chen and Lamb 1994; Fukuta and Takahashi 1999; Bailey and Hallett 2009; Connolly et al. 2012).

In particular, the secondary ice crystal habits that develop during vapor deposition are sensitive to these parameters. A prominent example of these secondary habits is the arms and branches of dendritic ice crystals that develop at temperatures near $-15^\circ \text{C}$ and ice supersaturations above 15% (Bailey and Hallett 2009). Dendrites are a subset of the platelike crystals that form in ice supersaturated conditions and temperatures between $-12^\circ$ and $-18^\circ \text{C}$. The secondary habits of dendrites can impact how these particles grow during aggregation and riming. For example, the rate of aggregation increases because of the efficient attachment of dendrite branches (Connolly et al. 2012). The low aspect ratios and complex shapes of these particles also impact the riming process, primarily because these factors reduce the fall speed below that of more isometric particles with the same mass. The slower fall speeds of dendrites decrease the rate of particle growth by riming compared with the more spherical particles grown at temperatures near $-10^\circ$ or $-20^\circ \text{C}$ (Fukuta and Takahashi 1999; Jensen and Harrington 2015). Thus, identifying regions of growing dendrites is important in fully understanding precipitation formation (Kennedy and Rutledge 2011; Bechini et al. 2013). Polarimetric and vertically pointing radar can help to differentiate between these growth regimes by providing information about particle shapes, effective densities, and fall speeds.
Analyses of scanning polarimetric radar measurements during a number of cases show enhancements in differential reflectivity $Z_{\text{DR}}$ and increasing radar reflectivity factor at horizontal polarization $Z_H$ toward the ground at temperatures favored for platelike crystal growth (from $-12^\circ$ to $-18^\circ$C; e.g., Kennedy and Rutledge 2011; Andrić et al. 2013; Schrom et al. 2015). Studies using vertically pointing radar to understand snow growth also show unique signatures associated with dendritic growth (e.g., Moisseev et al. 2009; Surcel and Zawadzki 2010; Zawadzki 2013; Oue et al. 2016). In particular, Surcel and Zawadzki (2010) and Zawadzki (2013) show decreasing Doppler velocity magnitudes (decreasing motion toward the ground; $|V_R|$) and increasing $Z_H$ toward the ground near $-15^\circ$C. Surcel and Zawadzki (2010) show this signature to persist when averaged over 300 h of vertically pointing X-band radar data. Zawadzki (2013) uses $V_R$ spectra to examine this signature, attributing the decreasing $|V_R|$ to enhanced vertical motion at the level of nondivergence, most frequently near $-15^\circ$C at the J. S. Marshall radar observatory (near Montreal, Quebec, Canada) where the observations were collected. A shift of the entire spectra toward decreased $|V_R|$ occurs for one of the cases he presents; however, another case shows the development of a clear secondary peak in the spectra at downward motion of hydrometeors during FROST, which is a result of dendritic growth using simplified calculations. This article is organized as follows: a discussion of the data available during the FROST project is found in section 2. Section 3 contains an analysis of the radar observations taken during FROST. A discussion of the physical processes responsible for the observed radar signatures and supporting calculations are shown in section 4. The main conclusions of the study are found in section 5.

2. Data and methods

As discussed above, a synthesis of polarimetric radar observations taken at nearly side incidence and $V_R$ observations taken at vertical incidence provides a more complete picture of the ice processes ongoing within a precipitation system. The polarimetric radar variables used in this study are $Z_H$ and $Z_{\text{DR}}$: $Z_H$ depends on the size, dielectric constant, and number concentration of the particles sampled by the radar; $Z_{\text{DR}}$ is defined as follows:

$$Z_{\text{DR}} = Z_H - Z_V,$$  

where $Z_H$, $Z_V$, and $Z_{\text{DR}}$ are in logarithmic units (dBZ and dB, respectively). The quantity $Z_{\text{DR}}$ depends on the dielectric constant, shapes, and orientations of the particles contained within a sampling volume. Physically, $Z_{\text{DR}}$ is related to the reflectivity-weighted mean aspect ratio of the sampled particles (Bringi and Chandrasekar 2001). If horizontally oriented oblate particles dominate the total returned signal, $Z_{\text{DR}}$ will be positive; particles with their maximum dimension oriented vertically will be associated with negative $Z_{\text{DR}}$. The quantity $V_R$ (at vertical incidence) is a measure of the reflectivity-weighted mean vertical velocity of the sampled particles and includes contributions from the terminal fall speeds of the particles and any ambient air motion. Because of the general downward motion of hydrometeors during FROST, $|V_R|$ will be used to make interpretation of these data more intuitive in the downward-relative framework in which particle growth often occurs. In this way, increases in hydrometeor fall speed will be associated with increases in $|V_R|$; the rare instances where upward motion is detected in the $V_R$ observations will be identified in subsequent analyses.

Platelike crystals and dendrites have high intrinsic $Z_{\text{DR}}$ values of 4–7 dB because of their low aspect ratios (e.g., Wolde and Vali 2001; Kennedy and Rutledge 2011; Botta et al. 2013). Aggregates have generally lower $Z_{\text{DR}}$ values (Ryzhkov and Zrnić 1998), especially as their size increases and their effective density decreases. The more chaotic fall behavior of these particles also reduces $Z_{\text{DR}}$ below that associated with the narrow distribution of canting angles of falling plates and dendrites (Matrosov
et al. 2005; Ryzhkov et al. 2011). Graupel particles generally have near-zero $Z_{DR}$ values owing to their quasi-spherical shapes and chaotic orientations (Fukao et al. 1991; Homeyer and Kumjian 2015). However, negative $Z_{DR}$ has been calculated theoretically and observed for conical graupel (Aydin and Seliga 1984; Oue et al. 2015). Because $Z_{H}$ depends on both the number concentration and size of particles, it can take on a wide range of values depending on the microphysical situation. By itself, $Z_{H}$ provides no information about the shapes of particles within the sampling volume. When combined with $Z_{DR}$, these variables can provide a sense of the reflectivity contributions of oblate particles to the total reflectivity. The $Z_{DR}$ observations of mixtures of platelike crystals and aggregates range in value from the intrinsic $Z_{DR}$ of platelike crystals to the intrinsic $Z_{DR}$ of aggregates, depending on the reflectivity contribution of each (Schrom et al. 2015; Oue et al. 2016).

We use radar data and soundings collected in northern Colorado during FROST in our analysis. Information about these observations is shown in Table 1, with the locations of these instruments shown in Fig. 1. A transportable, polarimetric X-band radar (XPOL) was located at the Marshall Field Site (MFS) and performed vertical, plan position indicator (PPI), and range–height indicator (RHI) scans every 12 min. Approximately 73 km to the northeast, the Colorado State University–University of Chicago–Illinois State Water Survey (CSU–CHILL) radar (Junyent et al. 2015) collected X-band polarimetric observations at low elevation angles. With a scanning interval similar to XPOL, RHI scans from CHILL were performed every ~12 min in the precise direction of the MFS (along the 220.7° azimuth, with respect to CHILL). A third radar, the operational S-band Weather Surveillance Radar-1988 Doppler (WSR-88D) at the Front Range Airport [Aurora, Colorado (KFTG)], also performed PPI scans during these cases. As evident in Table 1, the CHILL and XPOL took concurrent observations during events between 2300 UTC 20 February and 0220 UTC 21 February and between 1100 and 2200 UTC 9 March. During the 1 May event CHILL observations were not taken; we use polarimetric observations from KFTG to supplement the vertically pointing observations from the XPOL for this case. Because the observations from the XPOL, soundings, and CHILL RHIs (at 220.7° azimuth and 73-km distance) are all taken above the MFS, we use vertical profiles of the observations at this location to study the winter storm events.

Given the limited vertical resolution of the KFTG PPI scans used for the 1 May event (precipitation mode with 14 elevation angles), we construct quasi-vertical profiles (QVPs; Kumjian et al. 2013; Ryzhkov et al. 2016) of $Z_{H}$ and $Z_{DR}$ using azimuthal averages of the PPI scans at an elevation angle of 9.9°. These averages are taken using only radar gates with nonmissing values. The QVPs at

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**Table 1.** The profile data used during FROST and the measurements from each used in this study.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Measurements used</th>
<th>Frequency</th>
<th>Gate width</th>
<th>Beamwidth</th>
<th>Events used</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSU–CHILL</td>
<td>$Z_{H}$, $Z_{DR}$</td>
<td>X band (9.41 GHz)</td>
<td>90 m</td>
<td>0.3°</td>
<td>20–21 Feb, 9 Mar</td>
<td>Greely, CO</td>
</tr>
<tr>
<td>NCAR-XPOL</td>
<td>$V_{R}$</td>
<td>X band (9.41 GHz)</td>
<td>75 m</td>
<td>1.3°</td>
<td>20–21 Feb, 9 Mar, 1 May</td>
<td>MFS</td>
</tr>
<tr>
<td>WSR-88D KFTG</td>
<td>$Z_{H}$, $Z_{DR}$</td>
<td>S band (2.89 GHz)</td>
<td>250 m</td>
<td>1.0°</td>
<td>1 May</td>
<td>KFTG</td>
</tr>
<tr>
<td>Soundings</td>
<td>Temperature, ice saturation</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>20–21 Feb, 9 Mar</td>
<td>MFS</td>
</tr>
</tbody>
</table>

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**Fig. 1.** Map of terrain height from the North American Mesoscale Forecast System (NAM; Rogers et al. 2009) model and the locations of the CSU–CHILL, KFTG WSR-88D, and NCAR XPOL radars.
each time are then merged to create time–height cross sections over KFTG. It is important to stress that these QVPs represent radar observations over KFTG, 60 km to the east-southeast of the MFS–XPOL. Profiles from the two radars can only be expected to reflect the same processes during relatively homogeneous widespread precipitation events. We show a comparison in section 3c of the CHILL and KFTG data from the 9 March event to justify our use of the KFTG observations for the 1 May event.

It is also important to note that the peak magnitudes of the vertical gradients in \( Z_{\text{H}} \) found near \(-15^\circ\)C were consistently higher when using KFTG QVPs than when using CHILL RHI profiles. We expect the QVPs to have larger magnitudes in gradients of the radar variables than gradients derived from RHIs because of their superior vertical resolution. The RHI-derived profiles are also negatively affected by beam broadening at the \( \sim 73\) km range at which the CHILL data are analyzed. Theoretical calculations (see the appendix) assuming a Gaussian antenna pattern and a Gaussian profile of the vertical gradient in \( Z_{\text{H}} \) show that at 73 km, the magnitude of the vertical gradient in \( Z_{\text{H}} \) measured by the CHILL will have a negative bias of 0.5–1.5 dB km\(^{-1}\). Other factors such as the procedure used to bin the CHILL data and horizontal inhomogeneities in the precipitation field may have also contributed to these negative biases relative to the QVPs.

As stated above, \( |V_R| \) contains contributions from the particle terminal fall velocity and the air motion. Therefore, to link the \( |V_R| \) observations to particle fall velocities, the magnitude of the air motion must be relatively small. Unfortunately, observations of the vertical air motion were not taken during FROST. However, analyses from the Rapid Refresh (RAP; Brown et al. 2011) show maximum vertical velocity values of 0.1–0.2 m s\(^{-1}\) throughout the events we examine in this study. These values also correspond to kinematic vertical velocity values derived from wind profiler data in northeast Colorado by Carlson and Forbes (1989). In addition, brief periods where an updraft of similar magnitude as the particle terminal fall speed is present will have a relatively small effect on the time-averaged \( |V_R| \) observations. Therefore, we expect that the errors from attributing most of the \( |V_R| \) observations to hydrometeor fall speed will be minimal.

To analyze \( |V_R| \) from the XPOL and \( Z_{\text{H}} \) and \( Z_{\text{DR}} \) from the CHILL, we need to create vertical profiles of these observations over the MFS. To produce vertical profiles of \( |V_R| \) from the XPOL, we take time averages of the \( |V_R| \) observations over each \( \sim 3\)-min sampling period at vertical incidence. We construct profiles of \( Z_{\text{H}} \) and \( Z_{\text{DR}} \) using the CHILL RHI scans at an azimuth of 220.7°. We first isolate the radar gates found within a centered 2-km window over the MFS (corresponding to horizontal distances of 72–74 km from the CHILL). These gates are then binned by 75-m height increments; these height bins thus have the same vertical resolution as the XPOL observations (75-m gate spacing; see Table 1). The median \( Z_{\text{H}} \) and \( Z_{\text{DR}} \) values of all gates within a particular height bin are taken to be the values of \( Z_{\text{H}} \) and \( Z_{\text{DR}} \) at that height bin. Combining each \( Z_{\text{H}}, Z_{\text{DR}}, \) and \( |V_R| \) profile in time produces time–height cross sections of \( Z_{\text{H}}, Z_{\text{DR}}, \) and \( |V_R| \) at the MFS. The vertical coordinate used in plotting these data is height above CHILL.

Because ice particles grow primarily during descent in stratiform precipitation (e.g., Lo and Passarelli 1982; Heymsfield et al. 2002), we may identify different growth regimes using vertical gradients of \( |V_R|, Z_{\text{H}}, \) and \( Z_{\text{DR}} \). These fields are calculated with respect to the direction toward the ground; references to gradients in the radar variables made hereinafter refer to vertical gradients in the downward-relative direction. For example, positive (negative) \( Z_{\text{H}} \) gradients herein refer to \( Z_{\text{H}} \) increasing (decreasing) toward the ground. The vertical gradients are calculated using a centered linear fit of data from seven consecutive height bins.

Because of the limited number of available soundings, we attempt to use model data to connect the radar observations to atmospheric thermodynamic conditions. For this study, we use hourly analysis fields from the RAP to construct time–height fields of temperature and relative humidity with respect to ice at the MFS and KFTG. These locations are approximated using the closest point on the 13-km horizontal grid of the RAP. The RAP-derived temperature profiles show relatively close correspondence to FROST soundings from 20–21 February and 9 March, with the mean absolute error ranging from 0.6° to 1.0°C for the entire profile (Fig. 2). The lapse rates within these soundings generally ranged from 4° to 8°C km\(^{-1}\). Therefore, the maximum error in the height of a specific isotherm will be at most 250 m. The profiles of relative humidity with respect to ice from the RAP are less accurate, with the mean absolute error ranging from 14% to 17% over the entire profile. Plots of the profiles from these cases show a general overestimation in the relative humidity with respect to ice in the lowest 3 km above radar level (ARL) and a general underestimation of the relative humidity with respect to ice in between 3 and 6 km ARL (Fig. 2). Based on this comparison, temperature information will be taken from the hourly RAP analyses and information about supersaturation will be provided by the two soundings taken during the 20–21 February event and the 9 March event and shown in Fig. 2. We will also use the operational 0000 UTC 2 May sounding from Denver, Colorado (DNR), for information about the degree
of supersaturation during the latter portion of the 1 May event.

3. Radar observations

a. 20–21 February 2013 event

A several-hour period of intense snowfall was observed in northern Colorado during the afternoon and evening of 20–21 February 2013. The snowfall accumulations as based on the National Operational Hydrologic Remote Sensing Center (NOHRSC; Carroll et al. 2001) analysis ranged from 5 to 15 cm over the Front Range region, with the maximum accumulations near and south of Denver. An overview of this case is presented in Schrom et al. (2015).

From the radar observations, two distinct periods of precipitation are found during this case: a higher-$Z_H$–lower-$Z_{DR}$ period and a lower-$Z_H$–higher-$Z_{DR}$ period. The highest $Z_H$ values (15–20 dBZ) are found between 2345 and 0130 UTC, with two distinct periods of $Z_H$. $15 \text{ dBZ}$ visible in the time–height profiles (Fig. 3a). Between 0000 and 0130 UTC, $Z_{DR}$ values $>1.0 \text{ dB}$ are found just above the $>15 \text{ dBZ} Z_H$ values with both $Z_H$ and $Z_{DR}$ decreasing in height with time (Fig. 3b). After 0130 UTC, $Z_H$ tends to decrease and $Z_{DR}$ tends to increase; the largest values of $Z_{DR}$ (1.5–3.0 dB) are collocated with $Z_H < 15 \text{ dBZ}$.

Local maxima in the $Z_H$ gradient field are found at [2345–0215 UTC, 2.5–3.0 km ARL] (locations are hereinafter designated with the convention of [time, height]) during both the higher-$Z_H$–lower-$Z_{DR}$ and lower-$Z_H$–higher-$Z_{DR}$ periods of this event (Fig. 3d). Below 2.5 km ARL, two couplets (contained within contours of 15 dBZ and as labeled in Fig. 3d) of negative and positive $Z_H$ gradients, decreasing in height with time, are found. These features are located at [2345–0130 UTC, 1.0–2.5 km ARL] and reflect the two $Z_H$ enhancements associated with the banded precipitation that passes over the MFS between 2345 and 0130 UTC (Schrom et al. 2015). As shown in Fig. 2a, the relative humidity with respect to ice from the 0017 UTC sounding indicates supersaturated conditions are present within the first of these enhancements. Observations of $Z_H > 15 \text{ dBZ}$ and $Z_{DR} > 1.5 \text{ dB}$ during this time are contained within this supersaturated region (Fig. 3b).

The highest $|V_R|$ values are associated with the two enhancements in $Z_H$. Two main regions of $|V_R| \sim 1.5 \text{ m s}^{-1}$ are found: at [2345–0000 UTC, 1.0–1.5 km ARL] and at [0045 UTC, 1.2–2.0 km ARL] (Fig. 3c). A few scattered observations of $|V_R| \sim 1.5 \text{ m s}^{-1}$ are also present at [after 0145 UTC, below 1.4 km ARL]. These larger $|V_R|$ values are collocated with the highest values of $Z_H$ observed by the CHILL.

Throughout the period, gradients in $|V_R|$ are predominantly negative (the time-averaged mean value is around $-0.2 \text{ m s}^{-1} \text{ km}^{-1}$) between 2.8 and 3.4 km ARL,
suggesting that the magnitude of the reflectivity-weighted mean vertical motion of the sampled particles is decreasing. These observations are consistent with the observations presented by Zawadzki (2013) and could indicate the presence of a consistent updraft. Other potential explanations for this signature are presented in section 4. Temperatures within this region are between 215°C and 220°C. At heights below the 215°C isotherm and above 2.0 km ARL, the gradients in $|VR|$ become predominantly positive, indicating that the reflectivity-weighted vertical motion of the particles is increasing toward the ground.

b. 9 March 2013 event

On 9 March 2013, a long-lasting period of precipitation occurred at the MFS, with concurrent data from the XPOL and CHILL collected over an 11-h subset of this event. The snowfall accumulation analysis from NOHRSC shows storm totals ranging from 2 to 20 cm over the Front Range, with the maximum accumulations reported to the south and west of Denver. A description of this event is found in Schrom et al. (2015). The time–height plots of polarimetric CHILL data show a tower of enhanced $Z_H (>15\, \text{dBZ})$ at [1200–1430 UTC, 3.0–5.3 km ARL] (Fig. 4a). Another region of $Z_H > 15\, \text{dBZ}$ below this is found intermittently at [1130–2100 UTC, 0.7–3.0 km ARL].

During the period when the tower of $Z_H > 15\, \text{dBZ}$ was observed, negative $Z_{DR}$ values are found at [1300–1430 UTC, 0.7–1.5 km ARL] (Fig. 4b), suggesting that the reflectivity-weighted aspect ratio of the particles is greater than unity; $|VR|$ values are $>2.0\, \text{m} \, \text{s}^{-1}$ at [1300–1430 UTC, 1.0–1.8 km ARL] (Fig. 4c). Values of $|VR|$ between 2.0 and 2.2 m s$^{-1}$ are also found at [1330 UTC, 4.2 km ARL]. However, positive $Z_{DR}$ values here suggest that particles with vertically oriented maximum dimensions contribute less to the overall $Z_H$ than particles with horizontally oriented maximum dimensions. Given the convective appearance of the radar observations, these $|VR|$ observations may be biased by significant vertical air motions relative to the particle fall speeds. XPOL observations of outbound $V_R$ (upward motion) indicate an updraft is likely present at the top of the tower between 1300 and 1400 UTC, supportive of enhanced supersaturation and riming (see Fig. 12c in Kumjian et al. 2014). The sounding taken at 1319 UTC confirms that supersaturated conditions are found between 4.5 and 6.0 km ARL (Fig. 2b).
After 1600 UTC, a region of predominantly weakly negative values of the $\|V_R\|$ gradient are found above the −15°C isotherm (Fig. 4f), similar to the observations shown for the 20–21 February event. The largest enhancements in $Z_{DR}$ during this period are observed between the −15°C and −10°C isotherms, with gradients in $Z_{DR}$ generally positive at heights above the −15°C isotherm and negative at heights between the −15°C and −10°C isotherms (Fig. 4e). The $Z_H$ gradient is largely positive and maximized near −15°C (Fig. 4d); generally, $|V_R|$ is <1.2 m s$^{-1}$ at −15°C and has a predominantly positive gradient below the −15°C isotherm (Fig. 4f). This signature suggests that, at heights above the −15°C isotherm, descending hydrometeors that become more oblate and have smaller fall speeds than more isometric particles dominate the changes in $Z_H$, $Z_{DR}$, and $|V_R|$. At heights below the −15°C isotherm, the changes in $Z_H$, $Z_{DR}$, and $|V_R|$ are dominated by the hydrometeors that fall faster and become less oblate.

c. 1 May 2013 event

Another lengthy period of precipitation was observed in northern Colorado between 1400 and 2300 UTC 1 May 2013. Accumulations of snowfall from this event, based on the NOHRSC analysis, were 5–30 cm across the Front Range; the greatest accumulations were observed near Boulder, Colorado. As discussed earlier, polarimetric data from the KFTG radar were used in place of CHILL data for this event, with QVPs of the 9.9° PPI scans from KFTG compared with vertically pointing XPOL measurements at the MFS. Observations of $Z_H$ and $Z_{DR}$ from the KFTG PPI scans appear relatively consistent in value between the MFS and KFTG between 1400 and 1800 UTC (e.g., Fig. 5; time averages of QVPs derived from multiple scans will further decrease the spatial variability found within a particular scan). However, by 2100 UTC the majority of the precipitation detected by KFTG was south and east of the MFS, yet still over KFTG.

To further justify our use of the KFTG $Z_H$ and $Z_{DR}$ observations to explore the microphysical processes occurring over the MFS, we compare the time–height profiles of these fields from KFTG and CHILL during the 9 March event, when both radars collected observations during the more stratiform period of precipitation from 1400 to 2100 UTC. Time–height cross sections of KFTG QVPs and CHILL RHI-derived profiles during the 9 March event show enhancements in $Z_{DR}$ near −15°C and maxima in the $Z_H$ gradient near −15°C at approximately the same times (Fig. 6). To better determine the correspondence between the mean polarimetric signatures from these radars over time, we examine time averages of the $Z_H$ and $Z_{DR}$ gradients from both the CHILL RHI-derived profiles and the
KFTG QVPs from 1400 to 2100 UTC 9 March. The profiles at each time are first mapped from height to temperature coordinates using interpolated RAP temperature profiles to compare the profiles as functions of temperature, a more microphysically relevant quantity than height ARL.

The Z_H gradient profiles from the two radars show similar shapes with values increasing as the temperature becomes closer to between −15°C and −14°C (Fig. 6c). There appears to be an offset between the peaks of the Z_H gradients from the two radars of about 1.5°C, perhaps because of uncertainties in the propagation paths of the radar beams. The magnitudes of the Z_H gradients from KFTG are greater than the magnitudes of the Z_H gradients from CHILL, likely because of the effects of beam broadening (see the appendix). The profiles of the Z_{DR} gradient from KFTG and CHILL are less similar in shape than the profiles of the Z_H gradient from the two radars. However, both profiles show negative Z_{DR} gradients at temperatures greater than −14°C and positive values at temperatures between −19°C and −14°C (Fig. 6f). Some of the discrepancies between the Z_{DR} observations may be due to differential beam blockage from stationary targets illuminated by KFTG during PPI scans. When these PPI scans are transformed into QVPs, the stationary signatures may be visible as roughly constant Z_{DR} values at a particular height (e.g., the negative Z_{DR} observations at 4.3 km ARL in Figs. 6b and 7b). Nonetheless, the similar time-averaged signatures observed in these two sets of radar observations adds confidence that the KFTG radar can provide similar fingerprints of microphysical processes as the CHILL during relatively homogeneous events.

During the 1 May event, time–height cross sections from KFTG show two main periods of enhanced Z_H: from 1430 to 1700 UTC and from 1900 to 2200 UTC (Fig. 7a). During the earlier period, the height of the 15-dBZ contour extends to near 3.2 km ARL, just below the −15°C isotherm. Based on the operational 1200 UTC sounding from DNR (not shown), the profile between 2.0 and 4.5 km ARL is supersaturated. The maximum Z_{DR} values during this time period are 1.0–1.5 dB and centered at −15°C (Fig. 7b). The gradient of Z_H also reaches a maximum of 10 dB km\(^{-1}\) near −15°C between 1400 and 1600 UTC (Fig. 7d). By 1730 UTC, the maximum Z_{DR} gradients increase to 15–20 dB km\(^{-1}\) and are found at 0.0–0.5 km below the −15°C isotherm, 1730–2100 UTC. Between 1930 and 2100 UTC, Z_{DR} values increase to 4.5–5.5 dB and the largest gradients in Z_H increase to 20–25 dB km\(^{-1}\) near −15°C. These increases in Z_{DR} and in the Z_H gradient between 1730 and 2030 UTC suggest that hydrometeors become more oblate near −15°C and increase in size more rapidly below the −15°C isotherm. The maximum |V_{R}| values during this case are 1.4–1.6 m s\(^{-1}\) and are found at [1630–1830 UTC, 1.0–1.4 km ARL]. After 1930 UTC, the maximum values of |V_{R}| are between 1.2 and 1.4 m s\(^{-1}\).

At [1700–2300 UTC, 0.0–0.5 km below the −15°C isotherm], consistent negative |V_{R}| gradients are visible (Fig. 7f), similar to the negative |V_{R}| gradients visible in the 20–21 February and 9 March events. However, the

![KFTG WSR-88D - 01 May 2013 14:42 UTC - 0.5° PPI](image)

**Fig. 5.** A PPI scan from the KFTG WSR-88D radar at 1442 UTC 1 May 2013 at an elevation angle of 0.5° for (a) Z_H and (b) Z_{DR}. The locations of the CSU–CHILL (CHILL) radar, the KFTG WSR-88D (KFTG) radar, and MFS are labeled in (a) and marked by black dots in (a) and (b).
negative $|V_R|$ gradients from those cases are found at heights above the $-15^\circ$C isotherm. Highly supersaturated (115% relative humidity with respect to ice) conditions at temperatures of $-15^\circ$C and lower are indicated by the soundings in Fig. 2 throughout the February and March events, suggesting that favorable conditions for dendritic growth were present. In the May event, however, subsaturated conditions near and above the $-15^\circ$C isotherm may have limited dendritic growth until these particles reached the regions of higher relative humidity with respect to ice closer to the ground. The 0000 UTC sounding from DNR on 2 May (not shown) has relative humidity with respect to ice values near saturation at 0 km ARL and decreasing with height, suggesting that subsaturated conditions may have been present above the $-15^\circ$C isotherm at the MFS, where the precipitation weakened earlier than at KFTG and DNR.

4. Discussion

a. Physical explanations of the radar observations

From the concurrent vertically pointing and polarimetric observations presented above, we find evidence of varying levels of confidence for the occurrence of dendritic growth, aggregation, and riming. As described earlier, $|V_R|$ observations $>2.0$ m s$^{-1}$ are collocated with $Z_{DR}$ values $<0$ dB and $Z_H$ values between 15 and 20 dBZ during a portion of the 9 March event (cf. Fig. 4). Conical graupel is known to be associated with negative $Z_{DR}$ (Aydin and Seliga 1984; Oue et al. 2015) and graupel particles of various shapes have terminal fall speeds of 1.0–3.0 m s$^{-1}$ (Locatelli and Hobbs 1974). Ground observations (Kumjian et al. 2014) indicate that graupel was found at the surface during this time period. The observations of $|V_R| > 2.0$ m s$^{-1}$ and negative $Z_{DR}$ agree with previous studies (e.g., Aydin and Seliga 1984; Oue et al. 2015) and are consistent with the presence of graupel particles with conical or more complex shapes and vertically oriented maximum dimensions during this period.

Aggregation is associated with positive $|V_R|$ gradients (e.g., Moisseev et al. 2009; Surcel and Zawadzki 2010), positive $Z_H$ gradients, and negative $Z_{DR}$ gradients (e.g., Kennedy and Rutledge 2011; Andrić et al. 2013). However, some ambiguity in the fingerprints of $|V_R|$, $Z_H$, and $Z_{DR}$ for aggregation and riming exist; aggregation and riming of oblate ice crystals can produce similar
changes in these radar variables as particle aspect ratios tend toward unity and fall speeds increase during both processes. Given this ambiguity, the temperature data from the RAP may be useful in differentiating between periods where riming or aggregation was dominant. The location of the maximum \( j_{VR} \) and \( Z_H \) gradients 0.0–0.5 km below the \(-15^\circ C\) isotherm, observed during portions of each event (cf. Fig. 4), supports the conclusion by Hobbs et al. (1974) that the local peak in the aggregation rate between \(-215^\circ C\) and \(-210^\circ C\) is caused by dendritic growth. Therefore, it is probable that the maxima in the \( Z_H \) gradient observed between \(-15^\circ C\) and \(-10^\circ C\) are associated with the aggregation of dendrites.

A third signature present during each of the aforementioned cases may reflect the presence of a consistent updraft near \(-15^\circ C\), induced by heating during depositional growth, synoptic-scale forcing, or upslope flow (Zawadzki 2013). This signature is characterized by locally enhanced \( Z_{DR} \) values (indicating oblate particles), positive \( Z_H \) gradients (indicating growth as the particles descend), and negative \( |V_R| \) gradients near \(-15^\circ C\), suggesting that the reflectivity-weighted downward motion is decreasing. The enhanced vertical motion from an updraft would decrease \( |V_R| \) within the updraft and increase \( Z_H \) because of particle growth and increased particle concentration from convergence. To highlight the persistence of this signature near \(-15^\circ C\), observations of the \( Z_H \) gradient, \( |V_R| \) gradient, and \( Z_{DR} \) gradient are mapped from height to temperature coordinates using the RAP data and averaged over time for each case. These time-averaged profiles (Fig. 8) show negative \( |V_R| \) gradients over a roughly 500 m vertical layer (average depths of 580, 430, and 350 m for the February, March, and May events, respectively), with minimum values between \(-0.15 \) and \(-0.20 \) m s\(^{-1}\) km\(^{-1}\).

These minima in the \( |V_R| \) gradient occur at \(-18^\circ C\) and \(-17^\circ C\) for the February and March events, respectively; the minimum in the profile of the \( |V_R| \) gradient from the May event is near \(-14^\circ C\). The values of the \( Z_H \) gradient at these temperatures, where the minimum \( |V_R| \) gradients are observed, are 3, 4, and 11 dB km\(^{-1}\) for the February, March, and May events, respectively. This discrepancy is likely due to the different microphysical conditions occurring between the February and March and May events. During the May event, the precipitation field weakened over the MFS (where the \( |V_R| \) observations were taken) after 1800 UTC and maintained its strength over KFTG (where the \( Z_H \) observations were taken). As shown by Cotton et al.

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**Fig. 7.** As in Fig. 3, but for the KFTG QVPs at an elevation of 9.9° taken from 1400 to 2300 UTC 1 May. The isotherms for (a), (b), (d), and (e) are derived from RAP profiles at KFTG; the isotherms for (c) and (f) are derived from RAP profiles at the MFS (the location of the XPOL). The black contour plotted in all of the panels indicates the 15-dBZ contour of \( Z_H \) from KFTG in (a).
(1986), decreasing the rate of aggregation decreases the intensity of precipitation, leaving a larger population of pristine ice crystals. Therefore, the XPOL observations from 1 May at −14°C, during weakening precipitation, may reflect more platelike crystal growth; the KFTG observations from 1 May at −14°C, in heavier precipitation, may reflect more growth through aggregation. The height offset in the time-averaged $Z_H$ gradients between the CHILL and KFTG radars, noted in section 3, may have also contributed to the $Z_H$ gradient at −14°C being more representative of aggregation than platelike crystal growth.

In order for an updraft to be responsible for the persistent negative gradients in $|V_R|$ near −15°C, the updraft would have to occur consistently near −15°C over the large number of cases where the signature has been observed [e.g., those presented in this study and those presented by Surcel and Zawadzki (2010) and Zawadzki (2013)]. During the cases presented herein, low-level upshele slope flow is a consistent feature that provides forcing for ascent. However, the height of the −15°C level and the height of the $|V_R|$ gradient signature change in tandem, seemingly independent of changes in the upshele component of the low-level wind. Another explanation is that persistent upward motion near −15°C may be a result of buoyancy associated with depositional heating in the presence of vigorous dendritic growth. However, modeling studies are needed to evaluate this speculative theory.

Alternatively, we explore the possibility that this signature in the Colorado storms is a result of enhanced dendritic growth in the presence of a faster-falling, slower-growing population of ice particles. These two populations of particles may develop as an initial size distribution of isometric crystals, generated aloft, falls into an environment conducive to dendritic growth. On the basis of theoretical work (e.g., Marshall and Langleben 1954; Chen and Lamb 1994; Sheridan et al. 2009), the smallest isometric crystals, with large curvature and thus enhanced vapor flux, will undergo the fastest growth. These particles will rapidly add mass, decrease their aspect ratios, and begin to exhibit the secondary habit characteristics associated with dendrites. In contrast, the largest isometric crystals will have slower relative growth rates and smaller decreases in aspect ratio.

The initially larger, slower-growing isometric crystals and the initially smaller, faster-growing isometric crystals are hereinafter referred to as “isometric” and “dendritic,” respectively. We therefore name each particle class by the morphology that class will exhibit after a period of growth. During this growth, $Z_H$ will increase as mass is added to each particle class. However, given their differential growth rates, the contributions to $Z_H$ from the faster-growing dendrites will increase more rapidly than the contributions to $Z_H$ from the slower-growing isometric crystals. Therefore, $V_R$ will become weighted increasingly more by the dendrites and increasingly less by the isometric crystals. This effect will decrease $|V_R|$ because the largest isometric crystals will have higher fall speeds than the dendrites; isometric particles have smaller cross-sectional areas for a given mass and typically have fall speeds of 0.1–0.3 m s$^{-1}$ (Kajikawa 1972; Mitchell 1996; Heymsfield and Westbrook 2010). In each of the cases presented above, time-averaged $|V_R|$ observations (0.7–0.9 m s$^{-1}$; not shown) above the −15°C isotherm are consistent with the fall speed of compact isometric crystals.

During depositional growth, the initially larger isometric crystals have relatively small increases in size and

![Figure 8](image-url)
therefore relatively small increases in fall speed. Initially smaller particles rapidly grow into dendrites, but increases in their fall speeds will also be small as the drag on the particles increases. Therefore, we hypothesize that the more rapid growth and slower terminal fall speeds of the dendrites may lead to the observed decreases in $|V_R|$ and increases in $Z_H$.

**b. Simplified calculations**

As a first step in testing this hypothesis, synthetic $Z_H$ and $|V_R|$ gradients are produced and compared with the time-averaged radar observations. In general, this analysis requires knowledge of how hydrometeor properties (e.g., size, density, terminal fall speed) vary within these regions of the atmosphere. However, to simplify this procedure, at any given height level each particle class is assumed to be defined by a single terminal velocity ($V_t$) and $Z_H$. Therefore, the equations for $|V_R|$ (neglecting air motion) and $Z_H$ can be written as follows:

$$ Z_h = Z_h^D + Z_h^I $$  \hbox{ and }  \hbox{ (2)}

$$ |V_R| = \frac{1}{Z_h^D} (Z_h^D V_t^D + Z_h^I V_I^I), \hbox{ (3)} $$

where $Z_h = 10^{Z_h/10}$ (mm$^6$ m$^{-3}$) and the superscripts $D$ and $I$ correspond to the dendrites and isometric ice crystals, respectively.

To assess gradients in $|V_R|$ and $Z_H$, synthetic $Z_H$ profiles for each particle type are constructed; these profiles are assumed to be linear over the $\sim$500-m vertical extent of the observed signature. The profiles for each particle type are thus defined by a constant $Z_H$ gradient, an initial $Z_H$ value at the top of the profile, and a terminal fall speed. Using these quantities, the total profiles of $Z_H$ and $|V_R|$ are calculated using Eqs. (2) and (3), respectively. Linear fits over these profiles are used to determine the total $Z_H$ and $|V_R|$ gradients.

Calculations of the total $|V_R|$ gradient as a function of the assumed terminal fall speeds for the dendrites and isometric ice crystals.

**Fig. 9.** Calculations of the $|V_R|$ gradient over a 500-m layer with $Z_H$ profiles of constant slope for dendrites and for faster-falling, more isometric particles. Initial values of $Z_H$ of 2.0 and 7.0 dBZ are used for the dendrites and isometric particles, respectively; $Z_H$ gradients of 9.0 and 1.0 dB km$^{-1}$ are used for the dendrites and isometric particles, respectively. The data are presented as a function of the assumed terminal fall speeds for the dendrites and isometric crystals.
isometric crystals are shown in Fig. 9. For a realistic range of particle fall speeds, the calculated $|V_R|$ gradients are consistent with the time-averaged observed $|V_R|$ gradients (Fig. 8c). When performing these calculations, we assign fixed $Z_H$ gradients of 9 and 1 dB km$^{-1}$ for the dendrites and isometric crystals, respectively, and fixed initial $Z_H$ values of 2.0 and 7.0 dBZ for the dendrites and isometric crystals, respectively. These values are chosen based on the observations between $-18^\circ$ and $-16^\circ$C where the signature is most commonly observed (presented above).

The lack of observational constraints makes the initial $Z_H$ and $|V_R|$ gradient values for the isometric particles and the dendrites uncertain. Given this uncertainty, we assess the sensitivity of the resulting $Z_H$ and $|V_R|$ gradient calculations to the parameters assumed for the isometric particles and dendrites. The sensitivity of the $|V_R|$ gradient calculations to $V_t$ can be seen in Fig. 9, where the $|V_R|$ gradient is proportional to $V_t^p - V_t^j$. As implied by (1), the calculated $Z_H$ gradient is unaffected by the assumed values of $V_t$.

To examine the sensitivity of the $|V_R|$ gradient to the initial $Z_H$ values for the isometric particles and dendrites, the simulated $|V_R|$ gradient is plotted as a function of these parameters in Fig. 10a. Green contours show values of the total $Z_H$ at the top of the profile, the total $Z_H$ gradient, and together bound the observed values (hatched region). These bounds correspond to the extremes in the time-averaged observed profiles of $Z_H$ for the 20–21 February and 9 March cases at temperatures between $-18^\circ$ and $-16^\circ$C and thus constrain the initial $Z_H$ for the isometric particles and the dendrites. Only the pairs of initial $Z_H$ values for the dendrites and isometric particles contained within the hatched region of Fig. 10a can produce values of $Z_H$ and the $Z_H$ gradient that fall within the range of observations. For these bounded initial $Z_H$ values, increasing the initial $Z_H$ of the dendrites increases the magnitude of the $|V_R|$ gradient, while increasing the initial $Z_H$ for the isometric particles decreases the magnitude of the $|V_R|$ gradient. For the range of initial $Z_H$ values for the dendrites and isometric particles within the hatched region, the calculated $|V_R|$ gradients are consistent with the observed time-averaged $|V_R|$ gradients.

The sensitivity of the calculated $|V_R|$ to the $Z_H$ gradients assumed for the isometric particles and for the dendrites is shown in Fig. 10b. Because the initial values of $Z_H$ are fixed at 8 dBZ for the isometric particles and 2 dBZ for the dendrites in Fig. 10b, only the observed $Z_H$ gradients constrain the assumed $Z_H$ gradients directly. As in Fig. 10a, the assumed $Z_H$ gradients must produce a total $Z_H$ gradient that falls within the hatched region bounded by the 2.5 and 4.0 dB km$^{-1}$ contours to be consistent with the observed profiles near $-15^\circ$C. Within this region, increasing the $Z_H$ gradient for the dendrites increases the magnitude of the $|V_R|$ gradient; increasing the $Z_H$ gradient for the isometric particles decreases the magnitude of the $|V_R|$ gradient. Negative biases induced by the beam broadening of the CHILL will lead to underestimates of the true $Z_H$ gradient (see the appendix).

For fixed isometric and dendritic $Z_H$ gradients, correcting for the beam-broadening bias will increase the magnitude of the $|V_R|$ gradient (Fig. 10a).

The $|V_R|$ gradient ranges between $-0.07$ and $-0.26$ m s$^{-1}$ km$^{-1}$ within the constrained region of Fig. 10b, suggesting that the assumed $Z_H$ gradients for the isometric particles and the dendrites have a relatively large impact on the $|V_R|$ gradient. The assumed $Z_H$ gradients are directly related to vapor depositional growth and therefore could be explored and likely better constrained using microphysical model simulations. In comparison, when varying the initial $Z_H$ values, the $|V_R|$ gradient ranges from $-0.13$ to $-0.19$ m s$^{-1}$ km$^{-1}$ within the bounded region in Fig. 10a, suggesting that the $|V_R|$ gradient is less sensitive to the initial values of $Z_H$ assumed for the isometric particles and dendrites. Values of the $|V_R|$ gradient corresponding to the observations from $-0.15$ to $-0.20$ m s$^{-1}$ km$^{-1}$ are calculated for a relatively large range of realistic terminal fall speeds for both the dendrites and isometric crystals (Fig. 9). Values that produce the observed $|V_R|$ gradients range from 0.10 to 0.30 m s$^{-1}$ for $V_t^p$ and from 0.45 to 0.80 m s$^{-1}$ for $V_t^j$. The resulting total $Z_H$ gradient of 3.7 dB km$^{-1}$, fixed in the calculations shown in Fig. 9, also falls within the range of time-averaged observations above the $-15^\circ$C isotherm (cf. Fig. 8a). The relatively large parameter space that can reproduce the observed $|V_R|$ and $Z_H$ gradients may help explain the pervasiveness of this signature in the observations. Further constraints such as $Z_{DR}$ and $V_R$ spectra could also be applied to the set of parameters and potentially increase confidence in this explanation for the signature. Nonetheless, these simple calculations are consistent with the differential growth hypothesis.

5. Conclusions

Concurrent vertically pointing and polarimetric observations for three winter storm cases in Colorado were presented. These data, along with model analyses, uniquely identified periods of heavy riming (where irregularly shaped graupel was observed), suggested with high confidence regions of aggregation, and highlighted potential dendritic growth. Observations of $Z_{DR} < 0$ dB and $|V_R| > 2.0$ m s$^{-1}$ were indicative of graupel and thus associated with riming; coincident surface observations of graupel confirmed the presence of riming aloft.
Aggregation was suggested by positive gradients in $|V_R|$ and $Z_H$, negative gradients in $Z_{DR}$, and the green contours represent constant values of the total $Z_H$ and the total $Z_{DR}$ as labeled. The green hatched region represents the set of initial values for the isometric particle $Z_H$ and the dendritic particle $Z_{DR}$ bounded by these contours. In (b), initial values of $Z_H$ are 2.0 and 8.0 dBZ for the dendrites and isometric particles, respectively, and the green contours represent constant values of the total $Z_H$ gradient as labeled. The hatched region represents the set of $Z_H$ gradient values for the isometric and dendritic particles that are bounded by the contours of the combined $Z_H$ gradient. Both sets of calculations use terminal fall speeds of 0.20 and 0.65 m s$^{-1}$ for the dendrites and isometric particles, respectively.

The errors in assuming $|V_R|$ corresponds to the particle fall speed would be large in those cases. Doppler velocity spectra may be used to estimate the vertical air velocity, assuming particles with negligible fall speeds are present (Shupe et al. 2008). If this estimate of vertical air velocity is small relative to typical ice particle fall speeds, the $|V_R|$ observations will likely be due primarily to microphysical processes.

To test the dendritic growth explanation for this signature, we performed simple calculations assuming differential $Z_H$ gradients for dendrites and isometric crystals through the layer where the signature is observed. Using a range of realistic assumptions, these calculations produced $Z_H$ and $|V_R|$ gradients within the range of observations. An alternative explanation suggested previously by Zawadzki (2013) of persistent updrafts near $-15^\circ$C, perhaps due to heating during dendritic growth, could have also enhanced the negative $|V_R|$ gradient and made conditions more favorable for dendritic growth.

Future work with microphysical models of vapor deposition (e.g., Harrington et al. 2013) could better constrain the gradients of $Z_H$ prescribed for the isometric crystals and dendrites in the calculations. Coincident vertically pointing and scanning polarimetric radar observations also provide a number of additional observations.
(e.g., Doppler velocity spectra, differential reflectivity, and specific differential phase) that may be useful in constraining these calculations and help to reveal the physical processes responsible for the observed signature.

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APPENDIX

Smoothing of a Gaussian dZ_H/dz profile by beam broadening

Given the ~73-km distance between CHILL and XPOL, the effect of beam broadening on the dZ_H/dz profiles necessitates further analysis. At low elevation angles, the smoothed dZ_H/dz profile (dZ_H/dz) that results from the broadening of the radar beam can be expressed as a convolution of the antenna weighting function W(z_0) with the true dZ_H/dz profile (dZ_H/dz),

\[ \frac{dZ_H'}{dz}(z) = W(z_0) \ast \frac{dZ_H}{dz}(z), \]  

(A1)

where the asterisk denotes the convolution, \( z \) is the height of the beam center above the radar, and \( z_0 \) is the vertical distance from the beam center. Function \( W(z_0) \) is related to the radiation pattern of the antenna \( f(\theta) \) (expressed in spherical coordinates), where \( \theta \) is the elevation angle relative to the boresight direction. We only consider the radiation pattern as a function of this elevation angle since we are not concerned with beam broadening in the azimuthal direction. \( f(\theta) \) is typically approximated using a Gaussian function as (Bringi and Chandrasekar 2001)

\[ f(\theta) = \exp\left[-4 \ln(2) \frac{\theta^2}{\theta_1^2}\right], \]  

(A2)

where \( \theta_1 \) is the half-power beamwidth of the antenna. In terms of distance from the center of the beam,

\[ f(z_0) \approx \exp\left[-4 \ln(2) \frac{z_0^2}{z_1^2}\right], \]  

(A3)

where \( z_1 \) is the vertical beamwidth of the antenna in Cartesian coordinates. Hereinafter, references to beamwidth refer to \( z_1 \). Function \( W(z_0) \) is then related to \( f(z_0) \) by (Bringi and Chandrasekar 2001)

\[ W(z_0) = \frac{f(z_0)^2}{\int_{-\infty}^{\infty} f(z_0)^2 dz_0}. \]  

(A4)

On the basis of the time-averaged profiles of dZ_H/dz (not shown), dZ_H/dz is modeled as a Gaussian function, where

\[ \frac{dZ_H'}{dz} = -a \exp\left[-\frac{(z - b)^2}{2c^2}\right]. \]  

(A5)

and \( a, b, \) and \( c \) are the amplitude (dB km\(^{-1}\)), height of the peak (m ARL), and width of the peak (m), respectively. Thus, the convolution of the Z_H gradient profile and the antenna weighting function gives

\[ \frac{dZ_H'}{dz} = -a \sqrt{\frac{2c^2}{2c^2 + \frac{z_1^2}{8\ln(2)}}} \exp\left[-\frac{(z - b)^2}{2c^2 + \frac{z_1^2}{8\ln(2)}}\right]. \]  

(A6)

where \( dZ_H'/dz \) is the Z_H gradient profile smoothed by beam broadening.

In comparing (A6) with (A5), it is evident that beam broadening (increasing \( z_1 \)) will increase the width of \( dZ_H'/dz \) and decrease its amplitude. When the

![FIG. A1. Smoothing bias at the peak of the vertical gradient in Z_H.](image-url)
beamwidth becomes small (e.g., at close range to CHILL). \((A6)\) converges to \((A5)\). By subtracting \((A6)\) from \((A5)\) at the height of the peak in the \(Z_H\) gradient (e.g., at \(z = b\)), the smoothing bias due to beam broadening \(B^s\) can be found with

\[
B^s = -a \left[ 1 - \sqrt{\frac{2c^2}{2c^2 + \frac{z_1^2}{8 \ln(2)}}} \right]. \tag{A7}
\]

For a given amplitude of \(dZ_H/dz\), \(B^s\) is a function of the beamwidth and the width of \(dZ_H/dz\). Increases in beamwidth and decreases in the width of \(dZ_H/dz\) cause the smoothing bias to become more negative. From the time-averaged observations, \(a\) and \(c\) are chosen to range from 4 to 8 dB km\(^{-1}\) and from 150 to 250 m, respectively, resulting in a smoothing bias that ranges from \(-1.5\) to \(-0.5\) dB km\(^{-1}\) (Fig. A1). Given that this range of amplitudes is based on \(Z_H\) profiles taken 73 km from the CHILL, and therefore already biased by beam smoothing, these values are likely underestimates of the true \(dZ_H/dz\) amplitude. Thus, the true smoothing bias associated with these observations is likely of greater magnitude than the values plotted in Fig. A1.

REFERENCES


