Normalized Hail Particle Size Distributions from the T-28 Storm-Penetrating Aircraft

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ABSTRACT

Hail and graupel are linked to lightning production and are important components of cloud evolution. Hail can also cause significant damage when it precipitates to the surface. The accurate prediction of the amount and location of hail and graupel and the effects on the other hydrometeor species depends upon the size distribution assumed. Here, we use ~310 km of in situ observations from flights of the South Dakota School of Mines and Technology T-28 storm-penetrating aircraft to constrain the representation of the particle size distribution (PSD) of hail. The maximum ~1-km hail water content encountered was 9 g m$^{-3}$. Optical probe PSD measurements are normalized using two-moment normalization relations to obtain an underlying exponential shape. By linking the two normalizing moments through a power law, a parameterization of the hail PSD is provided based on the hail water content only. Preliminary numerical weather simulations indicate that the new parameterization produces increased radar reflectivity relative to commonly used PSD representations.

1. Introduction

Hail is observed on every continent but Antarctica (Cecil and Blankenship 2012). Significant hail damage to crops and structures occurs often in regions along the flanks of mountain ranges in Europe, North America, South America, southern and eastern Africa, the European portion of Russia, and in China (Court and Griffiths 1986). Large hail is often produced by thunderstorms forming during the warm season in interior continental plains regions, such as the High Plains of the United States (Changnon 1977), the steppes of Russia (Cecil and Blankenship 2012), and central China (Ni et al. 2016). And storm electrification is intimately tied to the growth of graupel and hail in these storms [see, e.g., MacGorman and Rust (1998, chapter 3)].

Our objective is to provide guidance on how to parameterize graupel/hail particle size distributions (PSDs) for use in cloud models. The representation of graupel (heavily rimed particles < 5-mm diameter) and hail (heavily rimed particles > 5-mm diameter) in models has been shown to be a source of large uncertainty in terms of cloud coverage, precipitation, and cloud evolution. The 5-mm size threshold for graupel to hail is taken from the American Meteorological Society’s Glossary of Meteorology definition, but it recognized that model representations that separate graupel and hail will do so based on differing
process rates or process pathways. Gilmore et al. (2004) demonstrated using idealized simulations that the precipitation amounts and condensed water species mixing ratios in deep convection were sensitive to the representation of the hail size distribution. Van den Heever and Cotton (2004) described how supercell development could be strongly modified by changing the mean size of hail particles. Similarly, Cohen and McCaul (2006) noted that modifying the hail mean size affects the evaporative cooling in downdrafts that then goes on to influence the subsequent evolution of convective storms. But we note that some regional simulations have also shown less sensitivity (e.g., Van Weverberg et al. 2012). Clearly, there is great uncertainty related to the representation of graupel and hail that can have an impact on the prediction and simulation of extreme weather phenomena such as large convective systems. Therefore, there is a need to constrain the representation of the particle size distribution of these species in numerical simulations of clouds and storms.

Graupel and hail PSDs have been previously derived from hailpads at the surface, from aircraft using foil impactors, and from optical array probes (Ulbrich and Atlas 1982; Cheng et al. 1985; Federer and Waldvogel 1975; Spahn and Smith 1976; Morgan 1982; Smith and Jansen 1982; Peterson et al. 1991; Musil et al. 1991; Heymsfield and Musil 1982). Airborne observations have recorded hail water contents up to 3 g m$^{-3}$ and number concentrations up to 20 m$^{-3}$ for sizes larger than 5 mm (Spahn and Smith 1976; Musil et al. 1991; Heymsfield and Musil 1982), while for hail observations at the surface lower hail water contents (<0.8 g m$^{-3}$; Cheng et al. 1985) and number concentrations (<4 m$^{-3}$ for sizes > 4 mm) have been reported.

Aircraft-based observations have indicated that hail particle sizes are distributed as a negative exponential function with increasing size when sampling is restricted to particles larger than ~5 mm (Spahn and Smith 1976). Size distribution shapes other than a simple exponential have been proposed such as a double exponential to represent different size ranges (Musil et al. 1976; Smith and Jansen 1982), power laws (Auer and Marwitz 1972), gamma distributions (Wong et al. 1988), or truncated exponential distributions (Morgan and Summer 1986). Inclusion of sizes smaller than 5 mm can include particles such as raindrops and ice aggregates that can contaminate the hail PSDs. Measurements of hail PSDs at the ground can be affected by the loss of smaller hail and graupel due to melting and sublimation, or size sorting effects reducing the frequency of occurrence of smaller particles resulting in gamma-distribution-shaped PSDs (e.g., Milbrandt and Yau 2005; Kumjian and Ryzhkov 2012; Loftus et al. 2014). In particular, Jameson and Srivastava (1978) used Doppler and radar reflectivity information to determine hail particle size distributions. They showed that below cloud base the size distributions display markedly modal distributions with a mean size of ~1.5 cm while higher up (in the cloud) the hail size distributions become exponential, in agreement with in situ observations.

Graupel and hail particle densities are often represented as effective densities for a spherical particle with a diameter equal to some characteristic dimension of the actual irregular particle. Previous work, based on observed graupel and hail particles, suggests that for sizes up to 20 mm effective spherical densities (mass/volume of sphere with a diameter equal to the maximum span of the particle) can span a range from 100 to 910 kg m$^{-3}$ (Magono 1953, 18–40; Braham 1963; Bashkirova and Pershina 1964; Zikamunda and Vali 1972; Locatelli and Hobbs 1974; Heymsfield 1978; Knight and Heymsfield 1983; List 1985). As the particles become larger, the specific density of hail derived from the immersion method of estimating density approaches that of solid ice (Prodi 1970; Vittori and Di Caporiacco 1959; Macklin et al. 1960). However, as hail grows larger, it tends to become less spherical and so the equivalent spherical density will be lower. Recently, Heymsfield et al. (2018) combined multiple datasets using 3D laser scans of individual hailstones collected at the ground to estimate hail volume to show that the effective density of hail particles decreases with size for hail particles (5 mm–5 cm).

For numerical cloud models the representation of graupel and hail density is often done by assuming a constant density. For example, densities of 400 and 917 kg m$^{-3}$ for graupel and hail, respectively, are assumed by Ferrier (1994). Or a power-law relationship can be adopted that continuously varies the effective hydrometeor density with size [Heymsfield et al. 2018, hereafter H18: mass = 89.2(diameter)$^{2.69}$ in SI units]. Other models have attempted to represent the evolution of density from low values to solid ice density by predicting continuous changes to the density throughout a cloud’s lifetime as particles become more heavily rimed (e.g., Mansell et al. 2010; Morrison and Milbrandt 2015).

Airborne hail spectrometer data have been reported previously but usually on a case study basis (e.g., Spahn and Smith 1976; Smith et al. 1976; Smith and Jansen 1982). For this study, we have synthesized hail spectrometer data from multiple flights of the South Dakota School of Mines and Technology (SDSMT) T-28 storm-penetrating aircraft (Detwiler et al. 2012) to produce a normalized PSD that can be used in models that represent hail at heights close to and above the 0°C temperature level.
The parameterization can also be potentially used for graupel, but the 5-mm lower-size threshold of the observations would constitute an assumed extrapolation of these results into the size range more appropriate for graupel. We briefly test our results within a modeling framework in this study but leave the challenge of a more detailed comparison of hail in observed and simulated storms to a later paper. These normalized PSDs do not necessarily apply to observations made at the surface due to melting and evaporation experienced by hail falling below cloud base.

2. Hail spectrometer description

The SDSMT Hail Spectrometer was designed and built for use on the T-28 aircraft and is described in detail by Johnson and Smith (1980). The probe is a 1D optical array probe, and although modified to a 2D probe in the 1980s, the particle data were still recorded in the archive data used here as 1D vertical size information collected along roughly horizontal aircraft tracks. The probe was mounted as two pylons under the left wing of the aircraft. A sheet of laser light emitted from one pylon illuminates a photodiode detector array behind a window in the other pylon. The detector array has 128 photodiodes with 0.9-mm separation. The pylon spacing is 90 cm, leading to a sample volume of \( \sim 10 \text{ m}^3 \text{ s}^{-1} \), or \( 100 \text{ m}^3 \text{ km}^{-1} \) for a typical 100 m s\(^{-1}\) aircraft speed. The maximum number of vertically arrayed photodiodes occluded as a particle passes through the light sheet is taken as a measure of hail size. Although the photodiode array had a total height of 11.5 cm, size distributions are recorded only in the 5-mm–5-cm range, with increasing size bin widths as the size increases. During missions, guidance from a meteorologist on the ground with access to data from a research-grade weather radar were provided to the pilot so that areas with hail larger than 5 cm could be avoided. Hail this large could have caused serious damage to the armored aircraft.

3. Data treatment

The data analyzed here were obtained with the hail spectrometer on a number of flights during different projects. The counts per size bin (particles that occluded the edge of the detector array were excluded) in 1-s records (Honomichl 2011; Honomichl et al. 2013) were combined with air temperature to filter out regions warmer than the 0°C level. Pilot reports were used to identify the time periods where hail was encountered. Depending on pilot workload during the flight and the main objectives of the project in which the aircraft was participating, hail encounters may not have been always reported by the pilot. But if hail was reported by the pilot, then it was present. A time window of ±1 min was used to recover 10-s PSDs (\( \sim 1 \text{ km} \) horizontal resolution for a typical 100 m s\(^{-1}\) airspeed) from that reported time, which given the probes’ sample volume, would be able to detect a concentration as low as 0.01 m\(^{-3}\). Overall, this meant that we used \( \sim 310 \) PSDs of 10 s, or \( \sim 310 \text{ km} \) of along-track cloud sampling, from 18 flights over Colorado, Oklahoma, and Kansas from 1995 to 2003 (see Table 1). These data were from altitudes where the air temperature was between 0°C and −12°C and the aircraft was flying straight and level (some profiles were not included due to potential fogging of the optical surfaces in the probe on descent to warmer lower altitudes). Surface radar information was relayed from the ground to the pilot in order to avoid flying in regions with reflectivity > 55 dBZ. Therefore, there is some sampling bias that will mean that the largest hailstones in these storms may have been avoided.

Other ways of determining when the hail was present were attempted. These included (i) listening to the aircraft audio record, which included a track recorded from a microphone attached to the front windscreen, but there was too much background noise to distinguish the impacts of hail; (ii) inspecting imagery from a Particle Measuring Systems 2D-C optical array probe, but shape information is only robust for particles smaller than 500 \( \mu \text{m} \) and these size particles cannot confidently be linked to the population starting at 5 mm measured by the hail spectrometer; and (iii) the occurrence of large particles observed with the hail spectrometer (diameter > 4 cm) was also considered, but this does not always correlate with when hail was reported. (These large particles might have been large snow aggregates in some cases, for instance.) Therefore, taking PSDs centered around the pilot’s hail reports seems the most reliable way to capture hail PSDs. But it is accepted that these will potentially be contaminated by nonhail particles and may miss some that were not reported. If the properties of particles larger than 5 mm (the minimum size of the particles detected) are different between the hail regions and nonhail regions, then we should be able to observe this by changing the length of the averaging window centered around the pilot report of hail. We tested the impact of varying the length of the time window centered on the pilot report of hail to determine if the choice of ±60 s was justified. This was done by examining the mean values of measured moments of the PSDs as a function of the window length. Because of the 5-mm minimum size threshold for the observations, it is expected that hail particles will have higher concentrations than other particle types in this size range. Figure 1 shows the result for the geometric mean of the
concentration (other moments show the same behavior). This plot indicates that the mean concentration remains approximately constant for small time periods centered around the pilot report, but rapidly departs toward the mean of the whole dataset that includes nonhail regions as $D_t$ exceeds 100 s, becoming constant again for $D_t > 1000$ s as the nonhail regions dominate the statistics. Therefore, a choice of $\pm 60$ s for the window length appears acceptable. Later, we will show that inspection of histograms of the PSD moments indicates that the hail population is distinct from the distributions of the whole population. For each PSD the (truncated, from 5 mm to 5 cm) moments are calculated and used to define the fit parameters.

Additional filtering of the PSDs included removing PSDs that appeared to be contaminated by electronic noise. These PSDs were identified by filtering out anomalously flat distributions of particles counted. Visual inspection of the PSDs indicated that the 2.5–3-cm size bin sometimes reported a much higher number of counts than the neighboring two size bins. This was believed to be possibly due to electronic noise affecting a group of detectors on the probe. To alleviate this problem, the particle count in this bin was replaced by the mean of the adjacent size bins. No particle-by-particle information or interarrival time data were available to assess for the effects of particle shattering, but we note that lower-resolution probes are less susceptible to the effects of shattering that dominate particle sizes of a few hundred microns and smaller (e.g., Field et al. 2006).

4. Normalizing the particle size distributions

Process rates involving hydrometeor species in bulk microphysics schemes used in cloud models need to make some assumption about the shape of the size distribution. This is commonly done by assuming a functional form and determining the parameters that define it.

### Table 1. List of T-28 campaigns and flights used in this analysis. Flight numbers increase serially from flight to flight, beginning in 1972.

<table>
<thead>
<tr>
<th>Project name and date</th>
<th>Flights used</th>
<th>Location airport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Polarization Experiment/Thunderstorm Electrification and Lightning Experiment (JPOLE/TELEX), 15 Mar–15 Jun 2003</td>
<td>798, 803</td>
<td>Norman, OK</td>
</tr>
<tr>
<td>CHILL T-28 Experiment (CHILL-TEX), 3–18 Jun 2002</td>
<td>781</td>
<td>Greeley, CO</td>
</tr>
<tr>
<td>Severe Thunderstorm Electrification and Precipitation Studies (STEPS), May–July 2000</td>
<td>754, 756, 757, 759, 761</td>
<td>Goodland, KS</td>
</tr>
<tr>
<td>Turbulence Characterization and Detection (TCAD), June 1999</td>
<td>728, 729, 735</td>
<td>Fort Collins, CO</td>
</tr>
<tr>
<td>VORTEX, April–June 1995</td>
<td>658, 667, 668, 670</td>
<td>Fort Collins, CO/Norman, OK</td>
</tr>
</tbody>
</table>
We can make an assessment of the underlying shape of the PSD by normalizing the observations. To normalize the PSD, no assumption needs to be made about the final shape of the distribution (e.g., Testud et al. 2001; Lee et al. 2004; Field et al. 2007). But because the measured distribution is truncated, we will assume a functional form to allow extension of the PSD to smaller and larger sizes. As we will see, an exponential distribution will be adequate to describe the data and we define it as

\[ N(D) = N_g \exp(-\lambda D), \]  

(1)

where \( N(D)dD \) is the particle number concentration (m\(^{-3}\), assuming SI units) between sizes \( D \) (m) and \( D + dD \). The “intercept” and “slope” parameters that define the exponential distribution are \( N_g \) (m\(^{-4}\)) and \( \lambda \) (m\(^{-1}\)). We ran a trial using a generalized gamma function but found that it did not improve the fit much and would still require assumptions about the shape parameter to carry out the exercise of adjusting for the PSD truncation described below.

Numerical weather prediction models that represent hail and/or graupel prognose the water content of this species. To be able to predict the PSD as a function of the total hail water content, the mass–size relation needs to be introduced:

\[ m_g = aD^\beta, \]

where \( m_g \) is the particle mass (kg). If we assume a spherical geometry, then we are assuming a constant bulk density for the hail and \( \beta \) would be 3. However, we permit the exponent to be variable to allow for changing effective spherical density with size, where the effective density is the density that a sphere of the same maximum size of a nonspherical particle would possess to have the same mass as the particle.

For the normalization we define the \( n \)th complete moment of the PSD as

\[ M_n = \int_0^\infty D^n N(D) dD = \frac{N_g \Gamma(n + 1)}{\lambda^{n+1}}, \]  

(2)

where \( \Gamma \) is the gamma function. We note that we use the observed size distribution necessarily means that we are dealing with truncated distributions that in this case start at 5 mm (\( D_l \)) and end at 5 cm (\( D_u \)). Therefore, we determine the \( n \)th truncated moment of the observed distribution as

\[ m_n = \int_{D_l}^{D_u} D^n N(D) dD = \frac{N_g}{\lambda^{n+1}} \left[ \gamma(n + 1, \lambda D_u) - \gamma(n + 1, \lambda D_l) \right], \]  

(3)

where \( \gamma \) is the incomplete gamma function.

If the characteristic size of the distribution approaches these thresholds sizes (5 mm or 5 cm), the measured moments will be biased relative to a distribution that extends from 0 to infinity. We also note that due to the relatively small sample size available that moment estimates are likely to be biased (e.g., Smith and Kliche 2005). Using ratios of moments can mitigate this effect to some extent.

Two moments (integrating from 0 to infinity) can be combined to define a characteristic size for the PSD. Here, we choose the \( \beta \)th and the \( \beta + 1 \)st moments. This quotient yields the mass-weighted mean size \( D_m \).

\[ D_m = \frac{\int \alpha D^{\beta+1} N_g \exp(-\lambda D) dD}{\int \alpha D^\beta N_g \exp(-\lambda D) dD} = \frac{M_{\beta+1}}{M_\beta} = \frac{\beta + 1}{\lambda}, \]

(4)

and rearranging gives the slope parameter,

\[ \lambda = \frac{(\beta + 1)M_\beta}{M_{\beta+1}}. \]

(5)

We can now use the assumption about the size distribution shape and the estimate of the slope parameter to compute complete moments from the measured truncated moments. By using the initial estimate of \( \lambda \) derived from the measurements, we can perform a rearrangement of (2) and (3).

\[ M_n = m_g \frac{\Gamma(n + 1)}{\gamma(n + 1, \lambda D_u) - \gamma(n + 1, \lambda D_l)}, \]

(6)

to provide improved estimates of the complete moment. The new estimates of the complete moment are then used to update the estimate of \( \lambda \) and the process is iterated until \( \lambda \) values become unchanging (within 1%). An approach like this was previously used by Vivekanandan et al. (2004) for droplet distributions and by Tian et al. (2010) for ice crystal size distributions.

Once we have the complete moments and the updated exponential parameters, we can proceed by assuming integrals from zero to infinity. The intercept parameter for the exponential can be linked to hail water mass \( W \) (kg m\(^{-3}\)) through the \( \beta \)th moment:

\[ W = \alpha M_\beta = \frac{\Gamma(\beta + 1)}{\lambda^{\beta+1}} a N_g. \]

(7)

Rearranging for \( N_g \) and substituting for \( \lambda \) gives

\[ N_g = \frac{(\beta + 1)^{\beta+1} M_\beta^{\beta+2}}{\Gamma(\beta + 1)^{-1} M_{\beta+1}^{\beta+1}}. \]

(8)
Finally, substituting $N_g$ and $\lambda$ into (1) leads to

$$N(D) \frac{M_{\beta+1}^{\beta+1}}{M_{\beta}^{\beta+2}} = (\beta + 1)^{\beta+1} \frac{M_{\beta}^{\beta}}{\Gamma(\beta + 1)} \exp \left[ - (\beta + 1) \frac{M_{\beta}}{M_{\beta+1}} D \right].$$

(9)

This is similar to the normalization proposed by Sekhon and Srivastava (1971) but differs in that this expression is independent of density assumptions about hail if we assume a constant bulk density. For spheres, the density information resides in $\alpha$, which has canceled out. If a variable bulk density ($\beta \neq 3$) is assumed, then density will start to enter the normalization through the value of $\beta$. If we assume $\beta = 3$, then plotting $(M_3/M_4)N(D)$ against $(M_3/M_4)D$ should collapse the data onto an exponential distribution with an intercept of $256/6$ and slope of $-4$ if the data are well represented by an exponential distribution. If this collapse agrees with the predicted behavior, then this supports our choice of assuming an exponential distribution as the functional form for the hail PSD.

For an exponential distribution, to predict the PSD, two moments are required that a cloud model would ideally predict to completely define the distribution. Typically a “double-moment microphysics scheme” would predict number concentration and mass concentration as required. However, many models used for numerical weather prediction currently use single-moment representations and only predict mass concentration. If one moment can be parameterized as a function of the other, then it will be possible to predict the PSD given one moment alone (e.g., see Testud et al. 2001):

$$M_{\beta+1} = a M_{\beta}^\beta,$$

(10)

which allows the PSD parameters ($N_g, \lambda$) to be defined by the hail water content alone and its link to $M_{\beta}$ as follows through combining (5) and (7):

$$\lambda = \frac{\Gamma(\beta + 2)}{\Gamma(\beta + 1)} \frac{W^{1-b}}{a^{1-b}}$$

and

$$N_g = \frac{\Gamma(\beta + 2) [\Gamma(\beta + 1)^{\beta+1}]^{1/(1-b)}}{\Gamma(\beta + 1)^{\beta+2}} W^{1+\beta/(1-b)}.$$  (12)

5. Results

Size distributions from the hail periods (∼310 periods of 10 s, equivalent to ∼310 km of sampling) are shown in Fig. 2. The sizes cover the range from 5 mm to 5 cm, which is a range of sizes that large snowflakes as well as hail can attain, potentially leading to overlap between the populations. Overplotted are mean PSDs for 0.01, 0.1, 1, and $10 \text{ g m}^{-3}$ hail water contents [using H18 mass = 89.2(diameter)$^{3.69}$]. This indicates a tendency for the PSD to become broader as the intercept parameter increases. Normalized histograms of moments of the PSD show a distinct difference between the PSDs dominated by the hail population and when all 10-s PSDs from the set of flights are considered (Fig. 3). All of the moments for the hail population exhibit higher modal values than for the background population.
indicating higher water content and number concentrations for particles with size > 5 mm. Figure 3e uses the H18 relation to estimate the hail water content that reaches a maximum of 9 g m$^{-3}$ for one 10-s period (approximately 1-km distance) and exhibits a mode in the observations ~0.1 g m$^{-3}$. Characteristic size is the mass-weighted mean size assuming a mass–size exponent of 2.69. This histogram (Fig. 3f) indicates that the maximum mass-weighted mean sizes encountered reach ~3 cm, while the mean is ~1 cm.

Values from the literature for $N_g$ and $\lambda$ have been presented in Fig. 4a to provide some comparison to the
observations. Using the 310 PSD moments, the values for $N_g$ and $\lambda$ for each PSD have been calculated and plotted in Fig. 4b. For this study $N_g$ and $\lambda$ have ranges of $2 \times 10^1$–$3 \times 10^4$ m$^{-4}$ and 100–900 m$^{-1}$, respectively. The range of values for this study is in agreement with previous work and toward the lower $\lambda$ end (i.e., broader distribution) of the range of reported values. Also shown in Fig. 4a are some examples of the intercept parameter used in cloud microphysical representations of graupel and hail. These intercept values used in the models tend to be above the observed range reported here. For the same water content this would mean that the model particle mean sizes would be smaller and their fall speeds slower, which would increase the residence time of the hail within the cloud.

The hail PSDs have been normalized using the third and fourth moments and results are plotted in Fig. 5a. The collapse of the data reduces the spread from 2.5 orders of magnitude (Fig. 2) to about 1 order of magnitude. The normalized distribution is approximated quite well by an exponential distribution (the expected exponential curve for a third- and fourth-moment normalization is overplotted: intercept $= 256/6$ and slope $= -4$), supporting our choice of an exponential distribution to represent the PSD.

Finally, cloud microphysical representations that use a single moment, such as the hail water content to represent hail, need to parameterize one of the moments in terms of the moment prognosed by the model. Figure 5b shows power-law relationships between the $b$ and $b+1$ moments where here $b = 3, 2.69$. It can be seen that the power laws vary slightly in terms of the exponent. The best fit lines to relate the moments shown in Fig. 5b are

$$M_4 = 0.10M_1^{1.15}$$

and

$$M_{3.69} = 0.10M_1^{1.69.}$$

Table 2 uses the power-law relation for the moments to generate the parameters required for estimating the PSD based on hail water content only from (11) and (12).

The results indicate that as the water content increases, $N_g$ increases and $\lambda$ decreases, as was seen in Fig. 2. This decrease in $\lambda$ with increasing water content is similar to behavior reported by Knight et al. (1982) in

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FIG. 4. (a) Previous slope and intercept parameters for exponential fits to the PSDs. The boxes [solid, Cheng et al. (1985), also shown is their $N_g$–$\lambda$ relationship; dotted, Federer and Waldvogel (1975); dotted–dashed, Spahn and Smith (1976)] show ranges from the literature where the slope and intercept were given. The horizontal lines toward the bottom of the panel show the range of slope values from the literature where only the slope was known (usually derived from hailpads; Morgan and Summers 1986; Ulbrich and Atlas 1982; Fraile and Sánchez 1999; Morgan 1982). The symbols to the left of the figure indicate intercept values used for microphysics schemes in cloud models. The open squares denote the range used in Thompson et al. (2008). The $+$ is taken from Hong and Lim (2006), and the $x$ is from Lin et al. (1983). (b) The open circles are the slope and intercept parameters for the hail PSDs in this study. The gray solid curve, marked Sphere, represents the $\lambda$ and $N_g$ values assuming a constant bulk density (it is insensitive to density, but different densities will sit at a different point along the line for the same water content). The black solid curve uses the H18 mass-size relationship. The dotted lines show contours of constant hail water content based on the 500 kg m$^{-3}$ sphere density.
the U.S. National Hail Research Experiment (conducted in 1972–76) for increasing precipitation rate based on data from the hail spectrometer and a foil impactor on the SDSMT armored T-28. This means that the intercept parameter (or concentration) increases at the same time as the distribution gets broader (or as the mass-weighted mean particle size gets larger).

Figure 4b includes two results for the single-moment parameterization, using (13) with values given in Table 2 overplotted, as curves. The gray curve uses a constant bulk density to relate size to mass, while the black curve is based on the mass–size relationship from H18. In principle a double-moment representation of hail would be able to better cover this phase space. But because the hail PSD representation has been reduced to a single moment, it is not able to cover all of the phase space that the observed size distributions explore and, instead, follows a trajectory that bisects the data.

Microphysics process rates or diagnostics ultimately use different moments of the size distribution. For a parameterization of the PSD based on the mass moment that is close to 3 the least well-predicted moments of interest are expected to be the number concentration (zeroth) and the radar reflectivity (sixth). Figure 6 shows the predicted and measured (adjusted to represent a PSD extending from zero to infinity in particle size, as described above) zeroth and sixth moments, as well as the exponential distribution parameters ($\lambda$, $N_g$). The geometric means and standard deviations suggest that, over the range of the data used, the mean predicted values are a factor of 1.4 and 0.6 of the measured values for $M_0$ and $M_6$, respectively. Geometric standard deviations indicate that the variability is a factor of 3 around the mean value. Similarly, the parameterized values of $\lambda$ and $N_g$ based on water content (Table 2) can be compared to those derived from the PSDs, and the mean bias and standard deviation can be assessed using the ratio of parameterized to observed. It was found that for $\lambda$-parameterized/$\lambda$-observed ratios the mean and standard deviation were 1.2 and 0.5, respectively. In addition, for $\log_{10}(N_g$ parameterized)/$\log_{10}(N_g$ observed ratios) the mean and standard deviation were 0.2 and 0.6, respectively. The parameterized $N_g$ is more biased than the parameterized $\lambda$ because it is more closely related to the number concentration than the mass-defined reference moment used in the analysis. A parameterization using the concentration could be constructed to reduce the bias in $N_g$, but because the moments of the process rates that are important (sedimentation, collection) are closer to the moment linked to the water content (~3), it would be less useful for modeling.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\lambda$</th>
<th>$N_g$</th>
<th>$N_{08}$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 500$ kg m$^{-3}$</td>
<td>262</td>
<td>3</td>
<td>0.1</td>
<td>1.15</td>
<td>98 $W^{-0.15}$</td>
<td>57 770 $W^{0.39}$</td>
<td>7.9 $\times 10^9$</td>
<td>-2.58</td>
</tr>
<tr>
<td>$\rho = 910$ kg m$^{-3}$</td>
<td>473</td>
<td>3</td>
<td>0.1</td>
<td>1.15</td>
<td>107 $W^{-0.15}$</td>
<td>45 820 $W^{0.39}$</td>
<td>7.9 $\times 10^9$</td>
<td>-2.58</td>
</tr>
<tr>
<td>H18</td>
<td>89.2</td>
<td>2.69</td>
<td>0.082</td>
<td>1.14</td>
<td>85 $W^{-0.19}$</td>
<td>36 570 $W^{0.30}$</td>
<td>4.7 $\times 10^7$</td>
<td>-1.61</td>
</tr>
</tbody>
</table>

FIG. 5. (a) Normalized size distribution using the third and fourth moments for the hail PSDs. The solid line indicates the theoretically expected curve for an exponential distribution normalized with the third and fourth moments. The variability bars indicate one standard deviation in log space (correlation coefficient $r = -0.87$). (b) Power-law relations between $M_3$ and $M_4$. As in (b), but for $M_{2.69}$ and $M_{3.69}$. The relationships between $M_3$ and $M_4$ and between $M_{2.69}$ and $M_{3.69}$ are shown (correlation coefficients of 0.99 and 0.98, respectively).
FIG. 6. (a) Predicted complete zeroth moment vs measured adjusted zeroth moment (i.e., concentration). Correlation coefficient $r = 0.79$. (b) As in (a), but for sixth moment. Correlation coefficient $r = 0.94$. (c) Predicted and measured $\lambda$, and correlation coefficient $r = 0.72$. (d) Predicted and measured $N_g$, and correlation coefficient $r = 0.57$. The 1:1 lines are overplotted for all panels. Right panels show histograms of the logarithm of the ratio of the predicted to...
6. Model testing

We have used the Met Office Unified Model to test the impact of changing the rimed particle PSD relationship. The model uses a single mass-only representation of graupel based on a gamma distribution: \( N(D) = N_g D^\mu \exp(-\lambda D) \), where \( N_g \) is given by (13).

In the operational model the values are \( N_{0g} = 5 \times 10^{25}, \delta = -4.0, \mu = 2.5 \) (control) and an effective density of 500 kg m\(^{-3}\) is assumed. For the test we take the values for the same density in Table 2, using \( N_{0g} = 7.9 \times 10^9, \delta = -2.58, \mu = 0 \) for this study (where \( \mu = 0 \) comes from the assumption of an exponential distribution) and the values for a more widely used assumption based on Lin et al. (1983), \( N_{0g} = 4 \times 10^4, \delta = 0.0, \mu = 0 \), for comparison. Strictly, the PSD observations and parameterization are for hail particles between 5 mm and 5 cm in size. However, we have applied the PSD to all rimed particles represented in the model. We note that radar reflectivity is derived directly from the hail size distribution (\( Z_e \sim D^6 \)) parameters assuming a constant density of 500 kg m\(^{-3}\) assuring consistency between the microphysical treatment of the hail and the radar response.

The case study is from 20 May 2013, when an EF5 tornado caused significant damage in and around the city of Moore, Oklahoma. The model configuration is as described in Stratton et al. (2018), but with a finer horizontal grid resolution of 1.5 km and 70 vertical levels with stretched vertical spacing (~100 m at 1 km). The domain of the simulation is as shown in Fig. 7. The model is initialized at 0000 UTC 20 May 2013 and run for 24 h. Model fields are inspected and compared for \( T + 20 \) (to coincide with the reported timing of the tornado on the ground). Here, we will comment on the qualitative differences in the simulated reflectivity patterns due to changing the PSD alone and we leave the challenge of verification with data for a later paper. The ability to reproduce observed radar reflectivities is a challenging problem. It relies on the accurate representation of not only the hail/graupel particle size distribution but also 1) the microphysical process rates that provide sources and sinks for graupel/hail, which are highly uncertain; 2) the radar forward model to convert the model PSD into reflectivity and the radar wavelength assumed; and 3) accurate reflectivities from the other condensed water species that will contribute to the radar response and impact the sources and sinks of graupel/hail.

Points 1–3 will differ for different numerical weather models. Our approach here is to demonstrate the relative response of the model using the new PSD relative to using a classic example from the literature to provide motivation for others to assess the impact of this PSD in their model. This work provides an in situ–based observational constraint around which more uncertain aspects such as microphysical hail sources (e.g., riming and droplet freezing) and sink rates (e.g., melting and shedding) can be tuned.

From Fig. 4 it is clear that the diagnosed concentrations will be lower in the parameterization proposed in this study than is ordinarily used. This will mean that for the same water mass there will be lower concentrations of particles but the mean size and hence the mean fall speed will be larger. Therefore, for larger particles we would expect that, all things being equal, we will see reduced hail water paths and potentially increased radar reflectivity signals (if not offset by reduced water mass). Figure 7 shows the result of the test compared to the UM control PSD representation and the widely used Lin et al. (1983) PSD representation. The control and Lin PSDs produce similar results in terms of composite reflectivity, hail water path, and a lack of hail seen at the surface. For the new PSD presented in this work, it can be seen that the radar composite reflectivity (maximum reflectivity in the vertical column) increases, while the hail water path decreases. The new PSD is the only representation that indicates hail at the surface with maximum hail sizes of up to 25 mm.

7. Conclusions

Using a comprehensive hail dataset collected in situ at temperatures 0°C and below with an airborne instrument that has large sample volumes relative to data collected at the ground, normalization of the PSD using moments of the distribution indicates that the hail PSD can be represented as an exponential between diameters of 5 mm and 5 cm. Hail water contents of up 9 g m\(^{-3}\) (in 10 s or 1 km of flight sample) were inferred. Exponential distribution intercept parameters derived from these results suggest that commonly used exponential intercept values for models are larger than those observed in cloud. By linking two moments of the size

\( \delta \)

measured parameters depicted in the left panels. The geometric mean is shown as a vertical dashed line with one geometric standard deviation on either side of the mean shown as dotted lines.

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FIG. 7. (top) Location map, where the blue rectangle is the region of interest. (bottom) Model sensitivity tests for 2000 UTC 20 May 2013 for PSD settings used for the (left) control, (center) this study, and (right) Lin et al. (1983) simulations. Results for the (first row) composite radar reflectivity, (second row) maximum hail size at surface, and (third row) hail water path.
distribution together with a power law, the parameters of the exponential distribution are predictable from hail water content alone. However, the variability exhibited by the intercept parameters suggests that the ability to predict two moments of the hail distribution may be advantageous for modeling the evolution of hail. The results of our study have considerable utility for modeling the development of graupel and hail within convection. A preliminary test of the new PSD parameterization indicates that radar reflectivities are increased, and more hail is able to survive to fall to the surface at warmer temperatures, relative to simulations with a previous more commonly used PSD representation.

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