Relationship of Multiwavelength Radar Measurements to Ice Microphysics from the IMPACTS Field Program

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Abstact: Coincident radar data with Doppler radar measurements at X, Ku, Ka, and W bands on the NASA ER-2 aircraft overflying the NASA P-3 aircraft acquiring in situ microphysical measurements are used to characterize the relationship between radar measurements and ice microphysical properties. The data were obtained from the Investigation of Microphysics and Precipitation for Atlantic Coast-Threatening Snowstorms (IMPACTS). Direct measurements of the condensed water content and coincident Doppler radar measurements were acquired, facilitating improved estimates of ice particle mass, a variable that is an underlying factor for calculating and therefore retrieving the radar reflectivity Ze and snowfall rate S. The relationship between the measured ice water content (IWC) and that calculated from the particle size distributions (PSDs) using relationships developed in earlier studies, and between the calculated and measured radar reflectivity at the four radar wavelengths, are quantified. Relationships are derived between the measured IWC and properties of the PSD, Dm, Zm at the four radar wavelengths, and the dual-wavelength ratio. Because IWC and Zm are measured directly, the coefficients in the mass–dimensional relationship that best match both the IWC and Zm are derived. The relationships developed here, and the mass–dimensional relationship that uses both the measured IWC and Zm to find a best match for both variables, can be used in studies that characterize the properties of wintertime snow clouds.

Significance Statement: The goal of this study is to provide reliable microphysical measurements and algorithms to facilitate improvements in cloud model microphysical parameterizations and in retrieval of snow precipitation properties from spaceborne active remote sensors and to characterize ice and snow precipitation development within clouds. This work draws upon a unique set of in situ measurements of the ice and total water content coupled with overflights of aircraft radar measurements at four radar wavelengths. Better estimates of the contributions of the ice phase to the total global precipitation using spaceborne radar data pave the way for assessing and advancing global climate modeling, thereby strengthening predictions of global climate change.

Keywords: Cloud microphysics; Cloud retrieval; Ice particles

1. Introduction

With the development of algorithms to characterize the microphysical properties of ice clouds, radar is becoming a reliable tool for remotely measuring the properties of snow precipitation. Relating a radar-measurable cloud property, such as the equivalent radar reflectivity factor Z (e.g., Waterman 1993), to the snow precipitation rate S or ice water content (IWC) can be used to estimate the development of snow precipitation. The use of multiwavelength radars further improves the accuracy of the estimates of S and IWC (Greco et al. 2018; Huang et al. 2019; Skofronick-Jackson 2019). For example, with the ability to map out snow precipitation globally using W-, Ku-, and Ka-band radar measurements from spaceborne radars [CloudSat; the Global Precipitation Measurement (GPM) Mission], it is possible to characterize the global distribution of snow precipitation, both at the surface and aloft (Heymsfield et al. 2020). Several studies suggest using the dual-wavelength ratio (DWR) to improve the accuracy of radar-based estimates of snowfall rate and to retrieve microphysical properties of ice hydrometeors (e.g., Matrosov 1998; Liao et al. 2005; Huang et al. 2019). The DWR, also called the dual-frequency ratio, is defined as the logarithmic difference between equivalent radar reflectivity factors Z at two frequencies. Some studies extended this methodology by using triple-frequency measurements for deriving two different DWR values (e.g., Kneifel et al. 2015; Chase et al. 2018; Leinonen et al. 2018).

The use of the DWR relies on retrieval algorithms that derive the particle size distribution (PSD) properties and their moments. Recent studies (Chase et al. 2021, 2022) based on a
machine learning approach and observations reached the conclusions that the GPM retrievals of the PSD intercept parameter, median mass diameter, ice water content, and snowfall rate and the CloudSat snowfall rate were systematically different than what they were expected to be.

Deriving microphysical parameters from radar reflectivity observations requires knowledge of the PSDs and $Z_r$–$S$ or $Z_r$–IWC relationships. These relationships are usually in a power-law form, $Z_r = AS^b$ or $Z_r = \alpha IWC^b$, where $A$ and $\alpha$ are coefficients and $P$ or $\beta$ is the exponent; most relationships to date are independent of the air temperature and surface pressure. Both IWC and $Z_r$ require knowledge of the PSD and ice particle mass as a function of particle diameter.

There are a number of ways to derive $Z_r$–$S$ and $Z_r$–IWC or related relationships. In one method, radar reflectivities measured by a ground-based radar are matched to coincident snowfall rates measured with precipitation gauges (Puhakka 1975), a Hotplate (Wolfe and Snider 2012), or indirectly with a 2D video disdrometer (Huang et al. 2010; von Lerber et al. 2017).

In a second method, polarimetric radar, yielding reflectivities at orthogonal orientations, together with coincident ground-based snowfall rate measurements, offers improvements to the estimates (Hassan et al. 2017). For a third method, $Z_r$–$S$ or $Z_r$–IWC relationships are derived from PSD measured at the ground. Sekhon and Srivastava (1970) drew upon the ground-based measurements of Gunn and Marshall (1958) to derive $Z_r$–$S$ and $Z_r$–IWC relationships. Matrosov (2007) modeled snowflake scattering properties and the PSDs from Braham (1990) to develop $Z_r$–$S$ relationships at multiple radar wavelengths. Using the discrete dipole approach, Liu (2008) developed a relationship applicable to the CloudSat W-band radar measurements. More recently, Grecu et al. (2018) and Chase et al. (2021) used observed PSD to simulate multiple-frequency radar observations and derived machine learning methodologies to estimate IWC from the simulated observations.

In yet a fourth method, $Z_r$ at one or more radar wavelengths measured from an overflying aircraft is related to IWC and $S$ calculated from the PSD and assumptions about the mass–dimensional relationship $m(D)$ derived from in situ aircraft measurements. Alternatively, direct measurement of the IWC (or derived indirectly through calculations of $S$) are compared with $Z_r$ calculated from the PSD. A fine example of the former approach is the study by Duffy et al. (2021), that used multiwavelength radar data collected during three field programs to relate the melted equivalent median mass diameter $D_{me}$ to $Z_{K_a}$, $Z_{K_a}$, and the DWR in different temperature ranges. They found that the $D_{me}$–$Z_{K_a}$ relationships were well correlated for each of the field programs but poorly correlated between the different field programs, the $D_{me}$–$Z_{K_a}$ relationships were poorly correlated for each field program, and the $D_{me}$–DWR relationship had a good degree of correlation among the three field programs. Ding et al. (2020) used a combination of PSDs collected during the Olympic Mountain Experiment to calculate the IWC and coincident radar to represent the $a$ and $b$ coefficients as a function of the median mass diameter $D_{me}$. They found a good correlation between coefficients $a$ and $b$ and $D_{me}$, with both parameters decreasing with $D_{me}$. A major finding was that the use of a constant $m(D)$ relationship was not sufficient to explain particle mass when aggregates $> 3$ mm dominate. Rather, they recommend multiple $m(D)$ relations to apply for different $D_{me}$ ranges.

In a fifth method, zenith-pointing radars measuring reflectivity and Doppler velocity at two wavelengths are used to estimate the $m(D)$ relationship and properties of the particle shape, terminal velocity, and other microphysical parameters (Mason et al. 2018). The DWR provide a means of constraining the estimates of particle size.

In a sixth method, $Z_r$ measurements at one or more radar wavelengths on board the in situ aircraft or from an overflying aircraft are related to direct measurements and calculations of the IWC from the PSD derived from an in situ aircraft. As compared with the fourth method, this method constrains the mass–dimensional relationship $m(D)$, facilitating more accurate calculation of $S$. Heymsfield et al. (2005, 2016) used coincident measurements from two field programs, each with direct measurements of $Z_r$, at two radar wavelengths, and direct measurements of the IWC to develop $Z_r$–IWC relationships. Fontaine et al. (2014) derived the density of ice hydrometeors in tropical clouds from analysis of particle probe data and coincident (measured from the same aircraft) W-band Doppler radar. They developed a theoretical study of particle shapes in 3D to show that the coefficient $b$ could be related to the particle surface area as measured by the particle probes (see also Heymsfield et al. 2002). They then constrained the value of coefficient $a$ by matching their theoretical calculations and the PSDs to the measured radar reflectivities.

Potential sources of errors for the methods used to relate $S$ to $Z_r$ are as follows. For methods 1 and 2, there are marked differences in sample volumes between the ground-based instruments and radars (Huang et al. 2015). Averaging times needed to develop the relationships, spanning minutes to tens of minutes, collocation errors, and assumptions related to deriving the masses of individual particles (i.e., 2D video disdrometer) produce errors (Huang et al. 2010; Wood et al. 2013). For method 3, PSDs depend upon the type of cloud that produces them. Also, assumptions about particle masses and terminal velocities lead to errors (Liu 2008). Method 4 requires assumptions to derive particle mass and terminal velocity. Non-Rayleigh scattering becomes significant when large ice particles are measured with Ku- through W-band radars. Therefore, at the shorter wavelengths, uncertainties about the non-Rayleigh scattering of the larger particles potentially lead to errors. Method 5 involves assumptions about particle mass, particle cross-sectional area, backscatter cross sections, and terminal velocities. Especially when coupled with observations at the surface (Mason et al. 2018), it offers promise for reliable retrievals of ice cloud properties. Fewer studies have employed the sixth method. Given the differences in speed of measurements from two aircraft, there are relatively few instances of collocation in convective clouds and the yield of collocation data points can be relatively small.

Overall, the sixth method probably is the most accurate of the six under most circumstances, and therefore it is the method adopted in this study. Using the results from method 6, we can evaluate the relative accuracy of method 4. We draw upon the data from the Investigation of Microphysics and Precipitation for Atlantic Coast-Threatening Snowstorms (IMPACTS)
field program in 2019 based out of Wallops Island, Virginia (McMurdie et al. 2022; section 2). The joint dataset from the two aircraft is compared with calculations from the PSD (section 3). Conclusions are drawn from the results of the comparisons (section 4).

2. Data

IMPACTS is a NASA-sponsored field campaign to study U.S. East Coast wintertime snowstorms, with a focus on cyclonic systems and snowbands (McMurdie et al. 2022). The dataset examined here took place January–February 2020. Two NASA aircraft flew coordinated flight patterns and sampled a range of storms from the Midwest to the East Coast. The ER-2 aircraft flew above the clouds and carried a suite of remote sensing instruments including cloud and precipitation radars, lidar, and passive microwave radiometers. The P-3 aircraft flew within the clouds and sampled environmental and microphysical quantities. The ER-2 data were measured with multiple frequency Doppler radars: X band (9.6 GHz), Ku band (13 GHz), Ka band (35 GHz), and W band (94 GHz) (Li et al. 2015; Heymsfield et al. 2021).

Measurements from the in situ NASA P-3 aircraft were collocated with the overflying NASA ER-2 aircraft radars operated at four wavelengths. Microphysical properties measured with instruments on the P-3 include particle probes measuring the PSDs in the size range from 2 μm to 2 cm or larger and particle shapes at sizes above about 25 μm (Table 1). Particles 2–50 μm were measured with a Droplet Measurement Technologies LLC cloud droplet probe (CDP) and a Stratton Park Engineering Co. (SPEC) Fast-CDP. In middle size ranges, a SPEC two-dimensional stereo (2D-S) probe (10–2000 μm) was used. For the larger particles, two orthogonally oriented SPEC High Volume Precipitation Spectrometer, version 3, (HVPS-3) probes (0.3–19.2 mm) were used. Composite PSDs were generated from a combination of the small and large probes. A SPEC cloud particle imager (CPI) was used for qualitative estimates of particle habit. The resolution of the pixel images is about 2.3 μm for the CPI, 10 μm for the 2D-S, and 150 μm for the HVPS-3. Total condensed content was measured by the Water Isotope System for Precipitation and Entrainment Research (WISPER; D. Henze et al. 2020, unpublished manuscript; IMPACTS). The accuracy of WISPER is approximately ±15% [see Twyohy et al. (1997), where they discuss the measuring system used by WISPER].

Data from four flights on 25 January and 1, 5, and 7 February 2020 were examined in this study. These flights were chosen because there were good collocations between the ER-2 and P-3 aircraft. A “good” collocation, which is relatively subjective, was chosen such that the two aircraft were within a horizontal distance of 5 km within a 10-min time frame. Collocations used 1-s P-3 and ER-2 data. A longer averaging time and closer separation distance would improve the accuracy but would lead to considerably fewer data points. On 25 January, elevated convection and banded precipitation associated with a warm occlusion moving across New England were sampled. On 1 February, the cloud field was associated with a developing surface low off the North Carolina coast. On 5 February, snowbands and convection were sampled. On 7 February, a rapidly deepening surface cyclone was located over eastern Pennsylvania and snowbands were penetrated.

In the online supplemental material, cross sections of X- and W-band data when the two aircraft were nearly collocated on 5 and 7 February are plotted. Sloping snowband precipitation and the melting layer are noted in Figs. S1 and S3 in the online supplemental material.

In what follows, we include data at temperatures 0°C and below and include regions of mixed phase, unless otherwise noted. Mixed-phase regions where the LWC > 0.03 g m⁻³ composed about 28% of the data points, LWC > 0.1 g m⁻³ composed about 10%, and LWC > 0.2 g m⁻³ composed 5%.

3. Calculations

The methods we use to derive properties of the particle size distributions and to calculate moments derived from them are discussed in this section.

The PSDs have been represented in terms of gamma functions of the form

\[ N = N_0 \Gamma(D) e^{-\lambda} D^a \mu^b \]

(1)

where \( N_0 \) is the intercept, \( \mu \) is the dispersion, and \( \lambda \) is the slope of the PSD. The fit parameters are derived using a moment-matching method that favors the first, second, and sixth moments of the PSD (Heymsfield et al. 2002).

The ice water content and radar reflectivity as derived from the gamma PSDs, when compared with the measurements, will facilitate a better understanding of errors involved in our calculations of each of these parameters. Integrating the fitted form of the PSDs from 0 to \( \infty \) yields

\[ \text{IWC } (\text{g} \text{ m}^{-3}) \approx 10^6 a N_0 \frac{\Gamma(b + 1 + \mu)}{\lambda^{b+1+\mu}} \]

(2)

where \( \Gamma \) is the gamma function, and the intercept parameter \( N_0 \) and slope \( \lambda \) are in cgs units. The terms \( a \) and \( b \) are the coefficient and exponent in the mass–dimensional relationship

<table>
<thead>
<tr>
<th>CPI particle imagery pixel size</th>
<th>2D-S</th>
<th>HVPS-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size range (μm)</td>
<td>No. of bins</td>
<td>Bin size (μm)</td>
</tr>
<tr>
<td>2.3 μm</td>
<td>10–2000</td>
<td>29</td>
</tr>
</tbody>
</table>

TABLE 1. The size range measured by the optical array probes: 2D-S probe and HVPS-3 probe, and imagery from the CPI probe.
\[ m(D) = aD^b. \] (3)

Mason et al. (2018) provide a very thorough discussion of the \( a \) and \( b \) coefficients from a wide variety of studies of unrimed aggregates and rimed particles (their Fig. 1). Ding et al. (2020) present and evaluate several \((a, b)\) combinations reported in the literature. For reasons discussed in the appendix, we use the \( a \) and \( b \) coefficients for \( m(D) \) from Heymsfield et al. (2010, hereinafter HS) of \( a = 0.0061 \) (cgs units) and \( b = 2.05 \) and for \( m(D) \) from Brown and Francis (1995, hereinafter BF) of \( a = 0.00294 \) (cgs) and \( b = 2.05 \) and 1.9.

The radar reflectivity, assuming ice scatterers, can be calculated from (Heymsfield et al. 2005)

\[ Z_e \approx \left( \frac{6}{\pi \rho_0} \right)^2 \times 10^{12} a^2 \sigma \left( 1.09 \frac{K_i}{K_w} \right)^2 N_0 \frac{\Gamma(2b + 1 + \mu)}{\lambda^{(2b+1+\mu)}}. \] (4)

where \( \rho_0 \) is the density of liquid water and \( K_i \) and \( K_w \) are the complex index of refraction of ice and liquid water, respectively. In Eq. (4), the backscatter cross section \( s \) is included to account for non-Rayleigh scattering. In what follows, two models are used. Matrosov et al. (2005) developed a \( \mathbf{T} \)-matrix scattering model that adequately describes the radar polarization backscattering properties of most nonspherical (oblate) atmospheric hydrometeors, including ice cloud particles, pristine snowflakes, and raindrops. Our forward model uses the Matrosov et al. (2005) model with an assumed particle aspect ratio of 0.6. The self-similar Rayleigh–Gans approximation (SSRGA) is also used for the computation of the backscatter cross section of ice aggregates (Hogan et al. 2017, hereinafter H17). The model is shown to perform well relative to other models (H17), while being significantly less computer intensive than more complex approaches based on the discrete dipole approximation (DDA) method of Draine and Flatau (1994).

Because both IWC from WISPER and \( Z_e \) from the ER-2 radars are measured, the ratio \( IWC/Z_e \) from the measurements can be used to evaluate the \( a \) and \( b \) coefficients that provide a good representation of both the IWC and \( Z_e \). This ratio is given by [Heymsfield et al. (2005), who did not include the term \( r = \sigma/\sigma_r \), the ratio of the mean backscatter cross section to the mean backscatter cross section for Rayleigh scattering]

\[ \frac{IWC}{Z_e} = \left( \frac{\pi a^b \rho_0^2}{36 \times 10^6} \right) \left( 1.09 (K_i/K_w) \right)^2 \times \frac{\Gamma(2b + 1 + \mu)}{\Gamma(2b + 1 + \mu)} \times \frac{1}{r}. \] (5)

In Eq. (5), the \( a \) and \( b \) coefficients can be varied to find the best set of coefficients that match the measurements obtained at the longer wavelengths (Ku and X band) when \( r \approx 1 \). For the shorter wavelengths, knowledge of the \( a \) and \( b \) coefficients derived at Ku and X bands together with the PSD could be used to estimate the approximate value of \( r(D) \) that would match the measurements.

Another parameter useful for radar applications is the median mass diameter \( D_m \). For a gamma-type PSD, it can be derived from the PSD and \( m(D) \) relationship and represented by the analytic expression (Mitchell 1991)

\[ D_m = \frac{b + \mu + 0.67}{\lambda}. \] (6)

4. Results

In the first part of this section, the properties of the PSDs from all nine IMPACTS P-3 flights are characterized, and, where possible, compared with PSD properties reported in earlier studies. These PSD properties, at the times of the collocations of the overflying and in situ aircraft, are used as a basis for developing better calculations of the ice microphysics from multiwavelength radar. The second part of this section directly relates the P-3 observations of the PSD and IWC to those from the ER-2 four-wavelength radar data for the four collocation flights.

a. Properties of the PSDs

Figure 1a illustrates how the PSD slope from the IMPACTS observations is related to the median mass diameter \( D_m \). The range of \( \lambda \) is about 3–100 cm\(^{-1} \) and that for \( D_m \) is about 0.08–0.4 cm, with most of the values about 0.08–0.2 cm. As can be inferred from Eq. (6), \( D_m \) and \( \lambda \) are inversely related; steep slopes imply small \( D_m \), and vice versa. Using the HS coefficient \( b = 2.1 \) in our processing, Eq. (6) can be used to approximate the relationship (see Fig. 1a, green line; the \( \lambda-\mu \) relationship in Fig. 1b is used). It agrees very well with the curve fitted to the PSD data.

Having a well-defined relationship between the PSD slope and dispersion provides another means of reducing the number of variables in the \( N(D) \) relationship [Eq. (1)]. It also simplifies our relationships based on the PSD. The \( \lambda-\mu \) relationship found here bisects earlier relationships derived for stratiform clouds–cirrus and stratiform clouds (Fig. 1b).

With a \( \lambda-\mu \) relationship directly derived from the PSD measurements, Eq. (6) can be used to get an approximate estimate of the accuracy of the derived values of \( D_m \) (Fig. 1a). If the \( b \) coefficient is 1.7 rather than the HS-assumed value of 2.1, the ratio of the lower to higher value of \( b \) from 25 to 100 m\(^{-1} \) is 0.86–0.9 and decreases from 25 to 1 m\(^{-1} \).

Both direct measurements of the PSD for particle sizes extending to above 2 cm, which include sizes expected for most snowfall events, and measurements of the IWC, over the range of about 0.01–2.5 g m\(^{-2} \) are available, facilitating the derivation of relationships between them that are expected to encompass most snow events. A direct relationship between IWC and \( D_m \) is found (Fig. 2a), and, not surprisingly [from Eq. (6)], IWC is inversely related to the PSD slope (Fig. 2b).

The relationship between \( N_0 \) and IWC is inverse (Fig. 2c), which can be accounted for because IWC and \( \lambda \) are inversely related.

The relationships derived in Figs. 1 and 2 are very well defined and can be used for some interesting calculations. For example, \( N_0 \) can be derived from both the WISPER
measurements (Fig. 2c) and the analytical relationships. From Eq. (2),

\[ N_0 = \frac{\text{IWC} \times \lambda^{(b+1+\mu)}}{a \times 10^b \Gamma(b+1+\mu)} \]  

(7)

From Figs. 1b and 2c, respectively,

\[ \mu = 1.41 \times \lambda^{0.328} - 3 \]  

(8)

\[ \lambda = 15.31 \times \text{IWC}^{-0.611} \]  

(9)

The \( N_0 \) [Eq. (7)] can be derived from knowledge of the IWC (which can be prognosed from a model), \( \mu \), and \( \lambda \). The IWC is specified in Eq. (7), and \( \mu \) can be derived from Eq. (8) and \( \lambda \) from Eq. (9). A curve representing this \( N_0 \)-IWC relationship is plotted in Fig. 2c. This curve follows the data quite closely and represents the data better than the power-law fit (plotted in Fig. 2c). The gamma function in Eq. (7) allows the curve to deviate from a power law to better match the data.

The accuracy of the BF and HS \( m(D) \) relationships can be estimated for the IMPACTS data at temperatures below 0°C by comparison with the direct measurements (Fig. 3). For the HS relationship, the median ratio of IWC\(_{\text{calc}}\)/IWC\(_{\text{meas}}\) is 1.06 (Fig. 3a), whereas for BF the median ratio of IWC\(_{\text{calc}}\)/IWC\(_{\text{meas}}\) is 0.68 (Fig. 3b). The mean ratios are nearly constant across the range of the measured IWCs, although the BF relationship is flatter, which might mean that a value of coefficient \( b \) lower than 2.1 could be expected.

If a particle were a perfect sphere, it would have an area imaged by the particle probes that is equivalent to that of a circle. The ice particle area in comparison with the area of a

**FIG. 1.** Properties of the particle size distributions as a function of particle slope: (a) median mass diameter relationship and (b) PSD dispersion. Fitted relationships from observations (red) are shown in both panels, along with earlier relationships in (b).

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If a particle were a perfect sphere, it would have an area imaged by the particle probes that is equivalent to that of a circle. The ice particle area in comparison with the area of a
circle is termed the area ratio $A_r$, which gives an indication of the roundness of a particle. For each 1-s datum from the particle probes, a power-law curve has been fitted to the mean particle area ratio as a function of the particle diameter. The derived exponents in the relationship are shown as a function of the measured IWC in the right panels of Fig. 3 (plotted values are the same in each panel). Ice crystal aggregates imaged by aircraft particle-imaging probes often appear to be fractal in nature (Schmitt and Heymsfield 2010). As such, their dimensional properties, mass, and projected area can be related using fractal geometry. Based on fractal theory, this exponent is related to the exponent in the $m(D)$ relationship. From Schmitt and Heymsfield (2010), the exponent in the $m(D)$ relationship $b$, can be estimated from the exponent in the $A_r(D)$ relationship $p$, through

$$b = (p + 2) \times 1.275. \quad (10)$$

Using the mean value of $-0.19$ for the exponent in the $A_r(D)$ relationship would imply a mean value of $b = 2.31$. Doing the derivation of $p$ for all the actual PSDs (not shown) leads to a median value of $b = 2.3$. The agreement between the estimated and derived values of $b$ is excellent. By applying the value of 2.3 to all of the PSDs and using the measured IWC to find the coefficient $a$ leads to a median value of $a = 0.018$ (cgs).

We can now compare the $m(D)$ relationship to $D$ derived from the fractal dimension with the HS and BF relationships. In Fig. 4a, the HS $m(D)$ and the fractal $m(D)$ are normalized by the BF $m(D)$. Note diameters in the range of about 0.03-0.08 cm, which comprises the sizes that dominate the median mass diameters (Fig. 3a). The highest masses relative to BF are given by the

![Fig. 2. Properties of the particle size distributions as a function of the IWC measured by the WISPER instrument: (a) calculated median mass diameter, (b) PSD slope, and (c) intercept parameter. Relationships fitted to each variable are shown (aqua) in (c), along with earlier relationships in (b).](image-url)
HS relationship, which is why the HS $m(D)$ relationship is closer to the measured IWCs (Fig. 3). The fractal relationship yields even lower IWCs than the BF relationship (not shown), as suggested by Fig. 4a. At sizes closer to 0.1 cm, which dominate the reflectivity-weighted mean diameter (discussed later), the BF relationship provides lower masses than the other two.

Drawing on the results shown in Fig. 4a, we explore values of $b$ of 1.7, 1.8, 1.9 (BF), 2.0, and 2.1 (the HS value). This method was used by Leinonen et al. (2018) to explore changes in DWR due to changes in $b$. The $a$ coefficient is derived in two ways (Fig. 4b). Using the PSDs, a mean value of $a$ is derived for each value of $b$ by matching the calculated and measured IWC (solid lines). The second method matches the calculated X-band reflectivity, assuming Rayleigh scatterers, to the measured reflectivity (dashed lines). The masses derived by these methods are normalized by the BF mass. The first very interesting thing to note is that for each value of $b$ the $a$ coefficient is quite different when calculated from the IWC or $Z_c$. The narrowest span of the $a$ coefficient is found for $b = 1.7$, and the widest difference is noted with $b = 2.1$. This effect is seen later when comparing measured and calculated reflectivities versus a comparison of the IWCs (Fig. 3).

b. Radar/aircraft collocation observations

Using the 1-s aircraft data, there were approximately 6000 times when 1) the two aircraft were within 5 km laterally, 2) the temperatures were ≤0°C and the IWC from WISPER was >0.1 g m$^{-3}$, and 3) the radar reflectivities were < 30 dBZ (to remove spurious data points at higher reflectivities). The cumulative distributions of the magnitudes of the IWC and radar reflectivity at the four wavelengths are plotted in Fig. 5. The most numerous collocations occurred where IWCs were below 1 g m$^{-3}$ (Fig. 5a) and the $Z_c$ at X band were at about 13–25 dBZ. These are typical IWCs and reflectivities for snow (Heymsfield et al. 2016; Matrosov et al. 2009). Not surprisingly, the X- and

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**FIG. 3.** Comparison of IWC calculated from the PSD with the measured IWC: (a) HS and (b) BF $m(D)$ relationships.
Ku-band distributions of reflectivity are similar, with progressively lower reflectivities from Ku to Ka bands and Ka to W bands.

We were surprised by measurements of IWC as high as 7.5 g m\(^{-3}\) and 600 points above 2 g m\(^{-3}\). Are these high IWCs artifacts? To address this point, we examined the relationship of IWC to \(Z_e\) at X band for these times. For the IMPACTS data, there should be a direct relationship between the two. For IWC > 1 g m\(^{-3}\) the median X-band reflectivity is 21.3 dBZ, whereas for IWC > 1.5 g m\(^{-3}\) the mean reflectivity is 20.6 dBZ (see Figs. 5a,b). Because of this apparent inconsistency, the IWC is limited to 1.5 g m\(^{-3}\) and the \(Z_e\) to 30 dBZ, the expected ranges for stratiform snow clouds and snowbands.

The measured radar reflectivity versus IWC at each of the radar wavelengths shows approximately a linear relationship on a log(IWC) versus dBZ plot or by a power-law \(Z_e\)-IWC relationship (Fig. 6). The nonlinearity noted at the higher reflectivities for Ka and W bands is due to non-Rayleigh scattering effects.

Also plotted in Fig. 6 are previously derived relationships, mostly based on measured PSDs and assumed \(m(D)\) relationships. At X band, where non-Rayleigh effects are minimal for \(Z_e < 30\) dBZ, the Hogan et al. (2006) temperature-dependent relationship, here assuming a temperature of \(-10^\circ\)C, falls considerably below the data from IMPACTS (Fig. 6a). For Ka band, the relationships based on GPM Cold Season Precipitation Experiment (GCPEx) and CRYSTAL-FACE data (Matrosov and Heymsfield 2017), fall considerably below that for the IMPACTS data (Fig. 6c). Direct measurements of the IWC and \(Z_e\) from CRYSTAL-FACE (Sayres et al. 2008) and...
the high–ice water content experiments at Darwin (Protat et al. 2016) compare favorably to our observations (Fig. 6d). Indeed, the latter relationship fits our data extremely well, even though our observations are from quiescent versus convective clouds. Retrievals of IWC using a combination of radar and lidar data for three years of data from Darwin, Australia, are quite consistent with our W-band data.

Comparisons of the calculated to measured reflectivities for the four wavelengths are presented in Figs. 7 and 8 using the HS and BF $m(D)$ relationships, respectively. The median difference between the calculated and measured reflectivities, which use the $T$-matrix and SSRGA methods, is shown in each panel. Consider first the comparisons at X and Ku bands, where non-Rayleigh scattering effects can be ignored for the PSDs measured here (Figs. 7a,b and 8a,b). The HS-calculated $Z_e$ are consistently higher than those measured, whereas the BF values are mostly lower than the measured $Z_e$. Use of the SSRGA method is found to have the advantage that the difference between the calculated and measured reflectivities does not change much with increasing $Z_e$. Also, using the BF $m(D)$ relationship, the calculated and measured $Z_e$ compare very favorably. Because non-Rayleigh scattering is expected to be relatively minimal when the reflectivities are below about 10 for Ka and 5 for W bands, one would not expect much more difference than was found for X and Ku bands, and generally there is not (Figs. 7c,d and 8c,d). The picture changes, though, at the higher reflectivities for Ka and W bands. The comparisons suggest that the backscatter cross sections we use from the $T$-matrix approach inadequately represent the cross sections, especially when relatively large particles are present. However, The SSRGA method provides consistent results across the full range of $Z_e$. Using the BF $m(D)$ relationship provides excellent agreement with the measurements (Figs. 8c,d).
Inspection of Eq. (4) suggests that the coefficient \( a \) in the HS \( m(D) \) relationship could be adjusted to get a better match with the measured \( Z_e \). The BF relationship provides a good estimate of \( Z_e \) for all wavelengths when the SSRGA method is used. However, reducing the coefficient \( a \) in the HS \( m(D) \) would decrease the IWC, which would worsen the "error" in the calculated versus measured IWCs.

A better understanding of how favorably both the calculated IWC and \( Z_e \) relate to the measurements can be gained through the use of Eq. (5), the ratio IWC/\( Z_e \). Plotting this ratio as a function of \( \lambda \) or, alternatively, \( D_m \) eliminates a degree of freedom and is informative as it provides an indication of the sizes most in error (small particles: low \( D_m \), high \( \lambda \); large particles: high \( D_m \), low \( \lambda \); Fig. 9). At X and Ku bands, the calculated ratios using the HS \( m(D) \) relationship and the SSRGA method are somewhat below the measured values across the full range of \( \lambda \), whereas in general the BF values are slightly above and closer to the measured values at reflectivities above about 10 dB. Note that the

![Graph showing reflectivity vs IWC from WISPER](image-url)

**Fig. 6.** Measured IWC vs measured radar reflectivity at (a) X, (b) Ku, (c) Ka, and (d) W bands. The median and standard deviations of the data are shown. Earlier relationships are plotted for comparison.
SSRGA method provides a much better match than the T-matrix approach, and for that reason only the SSRGA approach is plotted. Using the rhs of Eq. (5), we can take the ratio of what it would be if the $b$ coefficient was 1.7 rather than 2.1 and use a value for coefficient $a$ that is about 20% larger. The resulting ratios are now reasonably close to the measured ratios across the full range of $\lambda$ (Fig. 9; calculated, rescaled). For the BF method, only shown are the values calculated using the BF relationship and not the rescaling, as the BF values are fairly close to the rescaled coefficients. The BF coefficients generally do not compare favorably to the measured values for $Z_e < 10$ dB. We conclude that adjustments to both the $a$ and $b$ coefficients are needed for $Z_e < 10$ dB to provide better estimates of both the IWC and $Z_e$ calculated from the PSDs.

The relationship between $Z_e$ and Doppler velocities $V_Z$ is shown in Fig. 10 with and without the measured aircraft vertical
velocity subtracted from the Doppler velocity. The Doppler velocity is normalized to a pressure of 1000 hPa using the equation

\[ V_Z(1000) = V_Z(1000)/P^{0.4} \]  

(11)

that is based on Heymsfield et al. (2013). For X band, there is nearly linear relationship between \( V_Z \) and \( Ze \), with a linear curve fitted to the data (Fig. 10a). The net effect of the air vertical motion \( V_a \) on this relationship is relatively negligible; for all flights considered here, the median \( V_a \) is 0.1 m s\(^{-1}\). Also shown in Fig. 10a and subsequent plots in the figure are median values of the true \( V_Z \), found by subtracting \( V_a \) from the measured \( V_Z \), and a curve fitted to the data.

An interesting approach advanced by Orr and Kropfli (1999) was to average the Doppler velocities from a ground-based radar over an hour or two to remove the effects of small-scale vertical air motions. The \( V_Z-Ze \) relationship developed can then be used to derive the small-scale vertical motions. In essence, that is what we have done here, resulting in the relationship shown in Fig. 10a. A \( V_Z-Ze \) relationship could be developed

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**Fig. 8.** As in Fig. 7, but for the BF relationship.
for each flight day that could potentially be used to derive the vertical air motions from the ER-2 radar (\(Z_e\) and \(V_Z\)) data.

Figures 10b and 10d show the \(V_Z-Z_e\) measurements for the other ER-2 radar wavelength, along with second-order polynomials fitted to the data. What is surprising is that the relationship for Ku band (Fig. 10b) is somewhat different than for X band. This could possibly be a bias in the Ku Doppler velocities due to aircraft motions not being fully removed, which could possibly lead to a bias in \(V_Z\).

The non-Rayleigh effects become progressively more noticeable from Ka to W bands (Figs. 10c,d), resulting in a flat or slightly decreasing \(V_Z\) relationship with increasing \(Z_e\).

We wondered whether the radar reflectivity provided useful information about the PSD mean mass-weighted diameter \(D_m\) or the reflectivity-weighted mean diameter \(D_Z\). For \(Z_e < 15\) dBZ, there is a fairly strong correspondence between \(Z_e\) and these characteristic diameters, \(D_m\) and \(D_Z\), for all four wavelengths (Figs. 11, 12). For \(Z_e > 15\) dBZ, non-Rayleigh effects become

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**FIG. 9.** Ratio of ice water content to radar reflectivity at the four radar wavelengths, as derived using different methods.
significant at Ka and W bands, resulting in very different relationships for the different wavelengths.

What errors are likely to occur in the HS estimates of $D_m$ as a result of a possible overestimate in the coefficient $b$? These can be assessed through use of Eq. (6). If $b$ is 1.7 rather than 2.1, the error would be about 20%. The error in $Z_e$ would be about 35%. If these errors were the case, the points representing $D_m$ and $D_Z$ would be closer and shifted downward.

When radars with two wavelengths are available, the DWR presents an opportunity to improve the accuracy of retrievals of the snow PSD properties (Hou et al. 2014; Matrosov et al. 2019). Given the four radar wavelength measurements from

**Fig. 10.** Measured Doppler velocity as a function of radar reflectivity for (a) X, (b) Ku, (c) Ka, and (d) W bands (red symbols), along with values found by subtracting the measured air vertical velocity (aqua symbols). The Doppler velocities are normalized to a pressure level of 1000 hPa (see the text). Curves are fitted to the data.
IMPACTS and the collocation with in situ measurements, the particle probe data can help us evaluate whether $D_m$ can be reliably derived from DWR measurements. As a means of possibly identifying anomalous DWR values and/or $D_m$ values that do not have a significant number of data points, the distribution of number of events per DWR bin when appropriate collocations were made as a function of DWR for the four radar wavelengths is plotted in Fig. 13 (black curve). Also plotted in the figure are median values of $D_m$ (mean value of $D_m$ in the specific DWR bin).

The reason for looking at X–Ku DWR is to provide some guidance on how well the two radars are collocated in our data and how good the relative calibrations of the two radars are (Fig. 13a). For X–Ku bands, the DWR should be very small because the particles from the dataset are likely to be mostly in the Rayleigh regime. The data suggest that for DWR above

![PSD Median Mass Weighted Diameter vs Measured Reflectivity](image_url)
about 2 dB (see * symbol), where the number of counts decreases significantly from the peak and $D_m$ values begin to vary quite a bit, there are mismatches between the two wavelengths. For Ku–W bands, the variations begin somewhere between 4 and 6 dB, where again the counts drop significantly (Fig. 13b). Below those DWR, $D_m$ progressively increases with DWR. For Ka–W bands, $D_m$ progressively increases with DWR until about 8 dB, where they progressively decrease (Fig. 13c). This decrease is probably due to non-Rayleigh scattering at W band. Overall, there is information content in the DWR–$D_m$ relationship, even for the GPM radar wavelengths.

Before addressing relationships for snowfall rate $S$, it is desirable to compare the measured to calculated Doppler velocities $V_Z$ because this provides an indication of the accuracy of the HS-calculated terminal velocities $V_t$. In Fig. 14a, the air vertical velocities (Fig. 14b) are subtracted from the measured X-band Doppler velocities (non-Rayleigh scattering is minimal for the reflectivities measured) and plotted as a function of the measured

![Diagram](image-url)
radar reflectivity. The HS-calculated $V_Z$ after subtracting $V_a$ yields $V_Z$ that are about 21% too large for $Z_e < 15$ dB and about 10% too low where $Z_e > 15$ dB. This decrease might indicate that the HS $m(D)$ relationship—a fundamental component of $V_T$—is underestimating the mass of the larger particles, which could indicate that the particles are becoming progressively more rimed with increasing size. The median ratio of the HS-calculated mass to reflectivity-weighted terminal velocities $V_m/V_Z$ is 1.04 (Fig. 14a), providing guidance on how to relate errors in $V_Z$ to those in $V_T$. Generally, there is a progressive decrease in the ratio of the calculated to measured $V_Z$ with increasing $Z_e$. The HS $V_T$ are probably within ±15% of the actual $V_T$.

Few studies have related in situ calculations of the snowfall rates when the IWC was directly measured to collocated over-flying aircraft Doppler radar measurements, especially for multiple radar wavelengths. The IMPACTS dataset provides such information. In Fig. 15, the median distribution of $S$ versus dBZ is plotted for the four radar wavelengths, along with curves fitted to the data over the range of $Z_e$ where the particles are likely to be in the Rayleigh regime. Also shown are relationships from other studies over the respective wavelength. First consider the Olympic Mountain Experiment (OLYMPEX) dataset, where, although IWC was not directly measured, the curves were developed based on collocated in situ and radar data from the ER-2 aircraft (Heymsfield et al. 2018). The $S$ were derived as in the current study. What is remarkable is that the $S$–$Z_e$ curves fall almost on top of the curves from the current study; this agreement suggests that the curves developed in the current study are fairly general for stratiform ice clouds at temperatures warmer than about −10°C, where most of our data are contained. Relationships developed from collocated in situ and overflying aircraft X- and W-band radar data from dissipating convective clouds at temperatures below −20°C (Heymsfield et al. 2016) also compare favorably with the IMPACTS data (Figs. 15a,d); IWC was directly measured in that study, providing a reference for the accuracy of the calculations of particle mass.
The recent study by Falconi et al. (2018) compared direct measurements of $Z_e$ at X, Ka, and W bands with the calculations of $S$ from PSDs measured from a ground-based spectrometer for three stratiform ice clouds. For X and Ka bands, the results are actually quite different from those observed here (Figs. 15a,c). This difference could possibly be the result of measurements of the PSD with a ground-based instrument with a fairly limited sample volume or of changes in the PSD as particles fall from the lowest range gate of the radar to the surface.

Also shown in Fig. 15 are retrievals of $S$ from $Z_e$ from spaceborne radars. Figure 15b, from retrievals from the Tropical Rainfall Measuring Mission (TRMM) and GPM Ku-band radars, used the mass flux (MF) method to estimate $S$ (H18). In this technique, the rainfall rate $R$ at the base of the melting layer (ML) was related to the $Z_e$ measured at the top of the ML, assuming conservation of MF through the ML; this method is thought to be a more accurate way of deriving a $S-Z_e$ relationship (H18).

The in situ products are also compared with GPM Ku and Ka band and Dual-Frequency Precipitation Radar (DPR)-based products in Fig. 16. These are derived from GPM data collected globally for the period 1–30 June 2018, randomly selected to be representative of the GPM data. They consist of the version V7 of the official GPM DPR product (GDPR; Iguchi et al. 2021; Seto et al. 2021). The GPM combined lookup tables are derived using the method described in Olson et al. (2016).

Figures 16c and 16d compare the IMPACTS with the GPM estimates of $R$ for both Ku and Ka bands, respectively. Our calculated precipitation rate values are considerably higher than the GPM values by as much as a factor of 10 in overlapping reflectivity ranges. Figures 16e and 16f compare GPM $D_{m}$ and $R$
with those from the in situ data. Most of the DWR values from the IMPACTS data were in the range $0 < \text{DWR} < 5 \text{ dB}$, and those are plotted in the figures. In that overlapping range, the IMPACTS values are considerably higher than those from GPM.

c. Analysis of the best coefficients for use in the mass–dimensional relationship
Can a single $m(D)$ relationship be used for the calculations of both IWC and $Z_w$, which, together with the PSD and

FIG. 15. Calculated snowfall rate, normalized to a pressure level of 1000 hPa, as a function of radar reflectivity for (a) X, (b) Ku, (c) Ka, and (d) W bands. Median values and the standard deviation are shown (red symbols), along with curve fits to the data. Also plotted are relationships developed earlier.
assumed backscatter cross sections, provide estimates that are consistent with the measurements? Heymsfield et al. (2005) have shown that a single \( m(D) \) relationship, with an exponent \( b \) of about 2.0, cannot simultaneously provide accurate measurements of both IWC and radar reflectivity. It yielded the correct IWC but too large a \( Z_e \), indicating that too much mass is given to the larger particles. They found that an exponent \( b \) of about 1.6, giving smaller mass to the larger particles and a larger mass to the smaller ones, assured quite accurate estimates of IWC and \( Z_e \).

We wondered whether the IMPACTS data would produce similar results. In this section, we evaluate the \((a, b)\) coefficient pairs that, together with each 1-s PSD, give the best estimates of the measured IWC and the \((a, b)\) coefficients that give the
best estimates of the measured X-band $Z_e$. Calculated IWC and $Z_e$ are not used in this analysis. The X-band radar data are used, as non-Rayleigh scattering at X band is mostly negligible for the reflectivities measured here. The assumed $a$ value is varied from 0.0001 to 0.08 (cgs) and the $b$ value is varied from 1.0 to 2.5, each with 100 equally spaced intervals. The ($a$, $b$) combination that provided the best match with the measured IWC was derived for each PSD. Likewise, the ($a$, $b$) combinations that provided the best match with the measured X-band radar reflectivities were also derived.

Contour plots summarizing the results, along with the median ($a$, $b$) combination that best fit the data for IWC and $Z_e$, are derived (Fig. 17; left and right panels). The contoured purple region comprising the best coefficients for the IWC calculation slopes to the right, indicating that the particles relatively higher in the region have more mass, and the larger the particle the relatively more mass they have than lower in the region. The contours for $Z_e$ are more focused than for IWC, and the purple region for $Z_e$ is much smaller than for IWC. On average, the ($a$, $b$) coefficients yielding the best estimates of the measured IWC were $a = 0.0037$ (cgs) and $b = 1.76$. The most accurate ($a$, $b$) pair found for X-band reflectivity was $a = 0.0024$ (cgs) and $b = 1.71$

Most of the values of coefficient $b$ fall in the range of 1.4–2.1 for IWC and 1.6–1.9 for $Z_e$; values of $a$ range from about 0.001 to 0.007 (cgs) and 0.002 to 0.004 (cgs), respectively. The most representative coefficient $b$ is similar for both IWC and $Z_e$, whereas coefficient $a$ differs by 54%.

Also plotted in Fig. 17 are the ($a$, $b$) combinations derived from the PSD together with the fractal coefficient $b$ derived from the particle images through fractal geometry (Schmitt and Heymsfield 2010). The ($a$, $b$) coefficients are very different from those derived from the IWC and $Z_e$ measurements. Fractal geometry does not give a good match when compared with the best estimates of the ($a$, $b$) coefficients in these cases.

It is reasonable to assume that the use of the combination (IWC, $Z_e$) at X band is the most accurate way to calculate the ($a$, $b$) pair when one $m(D)$ is used. In our calculations, ($a$, $b$) pairs are specified. The selected range of $a$ is chosen to be 0.001–0.01 (cgs) and $b$ from 1.6 to 2.6 (cgs) based on the results from Fig. 17 using the aforementioned ($a$, $b$) values. The mean error for each ($a$, $b$) pair is derived, and the pair that has the minimum mean error is identified. This procedure is related to the one used by Finlon et al. (2019), who had collocated ground-based radar measurements at S band and in situ observations of the IWC.

FIG. 17. Best coefficients in the $m(D)$ relationship found by matching the (a) measured IWC to that derived from the PSD, and (b) that derived from the PSD to the radar reflectivity at X band. In (a), dots show the $b$ coefficient derived from the exponent in the particle area ratio–diameter distribution (fractal dimension). From the $b$ coefficient and the measured IWC, the associated coefficient $a$ is derived. The best estimates of the $a$ and $b$ coefficients from each measurement of IWC [in (a)] and $Z_e$ [in (b)] are shown with red dots.
from the Nevzorov probe from an overflying aircraft to find the best \((a,b)\) pair in the mass–dimensional relationship.

The result of this analysis is shown in Fig. 18a, with the individual data points and median values for the composite data plotted with large symbols. The relationship between \(a\) and \(b\) is direct and nonlinear on average (Fig. 18a), consistent with the results shown in Fig. 17. In Fig. 18a, solid and dashed curves are lines of constant mass showing the \((a,b)\) coefficients.
that would yield the same mass derived from the BF (solid lines) and HS (dashed lines) \( m(D) \) relationships for particles of 0.1, 0.2, and 0.4 cm diameter. Most values of \( D_m \) are consistent with these sizes (Fig. 19a); 47\% fall in the range 0.05-0.15 cm, 13\% in the size range 0.15-0.25 cm, and 8\% from 0.25 to 0.4 cm, with rapidly decreasing percentages with increasing size. The explanation for plotting these lines is as follows. If either the BF or HS relationships provided the best \((a, b)\) coefficients—that is, as

![Fig. 19. (a) Cumulative distribution of the median mass diameter (most points are between the two horizontal lines). Also shown are the relationships (b) between the \( a \) coefficient and \( D_m \) and (c) between the \( a \) and \( b \) coefficients. In (b), median values of \( b \) are shown for temperatures less than (blue dots) and greater than (red dots) \(-9^\circ C\). Also noted are fits to the mean values of \( a \) and \( b \).](image-url)
derived from the combination of (IWC, \( Z_a \)) measurements—the derived \((a, b)\) coefficients would fall on top of or close to either the + or * in the figure. What is revealing is that 1) the median values (connected by a red line) do not fall on either of the + or * values, 2) the median values are considerably closer to the BF than HS curve, and 3) the trend noted for the median values suggest that the \((a, b)\) coefficients increase with increasing \( D_m \) and therefore increasing IWC (Fig. 2a) and \( Z_a \) (Fig. 11). The latter point is consistent with Figs. 18c and 18d, where it is noted that at temperatures from \(-5^\circ\) to \(-10^\circ\)C, most of the points are to the right of the curve for 0.2 cm, from \(0^\circ\) to \(-5^\circ\)C where larger particles are expected, most points are to the left of that curve, and where the reflectivities are \(>20\) dBZ, the \((a, b)\) coefficients are larger than for lower reflectivities.

Curve fits to the temperature-sorted \((a, b)\) coefficients are shown in Figs. 18a and 18b. The variability noted in Fig. 18a can be traced in small part to regions containing liquid water (>0.03 g m\(^{-3}\), Fig. 18d), which is expected to perturb the \(m(D)\) relationship.

If the median mass diameter of the PSD is derived from either radar dual wavelength data or model calculations, the PSD slope [Fig. 1a and Eq. (6)] and thereby other properties of the PSD could be derived. Most of the values of \( D_m \) fall in the range between the horizontal lines in Fig. 19, or from about 0.06 to 0.18 cm (Fig. 19a). Using that range of \( D_m \), a correspondence between \(a\) and \( D_m \) could be derived using a relatively simple relationship (Fig. 19b). Once \( D_m \) and \(a\) are derived, then \(b\) can be obtained (Fig. 19c).

The \(b-a\) relationship is consistent with previous discussions that particle mass is relatively higher in the larger particles as the median mass diameter increases. This is consistent with the findings from earlier studies.

5. Conclusions

Direct measurements of the ice water content and coincident X-, Ku-, Ka-, and W-band radar data are used to derive directly the relationships at the four radar wavelengths between IWC and \( Z_a \); IWC and the Doppler velocity, accounting for vertical air motions; and IWC and properties of the particle size distributions. The method used here also allowed us to evaluate two mass-dimensional relationships, one of which is widely used and the other developed based on direct measurements of the IWC, and to find the best combination of mass-dimensional relationships that facilitate forward modeling the particle size distribution to yield properties of the PSD with improved accuracy over earlier relationships.

The accuracy of the BF and HS mass-dimensional relationships used for deriving both the IWC and \( Z_a \) yielded the interesting result that the median IWC from the HS relationship was almost equal to that measured. Conversely, for the minimally attenuated X-band radar data, the HS relationship using Mie backscatter coefficients appreciably overpredicted \( Z_a \), whereas the SSRGA backscatter cross sections yielded somewhat higher \( Z_a \) but provided consistency in \( Z_a \) for all the radar wavelengths. The BF relationship produced significantly underestimated IWCs and yielded about the same accuracy over earlier relationships.

The median mass diameter increases with the dual wavelength ratio for Ku-Ka and Ka-W bands until apparent non-Rayleigh effects become significant at about 3 and 8 dB, respectively. It is found that, relative to the \( D_m \) values derived here, the GPM DWR retrievals are low. With algorithm improvements, the GPM radar data can be used to provide reasonable estimates of \( D_m \).

The relationship between the radar reflectivity and Doppler velocities at the in situ aircraft location, subtracting the air vertical velocities, are examined for the four wavelengths. The non-Rayleigh ice particle backscatter results in a decrease in \( V_z \) above about 15 and 10 dB for Ka and W bands, respectively. The ratio of the Doppler velocity calculated from the PSD and HS \(m(D)\) relationship to the measured velocity, with the vertical air velocity subtracted, indicates that the HS-derived \( V_z \) are overestimated by as much as 25%–30% at the lower X-band reflectivities, with a nearly linear trend toward and then below 1.0 as the reflectivity increases. These results are consistent with the finding that the exponent in the \(m(D)\) relationship used in the HS calculations is too high; rather than 2.1, it is closer to 1.7. The relationship between the snow precipitation rate and single- and dual-wavelength radar measurements are improved over earlier relationships that did not have direct measurements of both or either of the IWC and \( Z_a \) measurements.

A future study will use the new mass-dimensional relationship to refine the calculations using the HS relationship and apply that relationship to studying the properties of both snowbands and generating cells, as well as examining how well the new relationship fits the data from the 2022 and 2023 IMPACTS field programs.

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APPENDIX

Discussion of Mass–Dimensional Relationships

The BF relationship has been used and cited in more than 304 refereed articles in the formal literature as of June 2022. The publication topics employing this relationship include radar/lidar/passive remote sensing (including spaceborne) retrievals, deriving IWC or related parameters from aircraft particle probes, microphysical modeling and parameterization development, and estimating ice cloud radiative properties and retrievals, deriving IWC or related parameters from aircraft radar/lidar/passive remote sensing (including spaceborne) than 304 refereed articles in the formal literature as of June ground by Locatelli and Hobbs (1974, hereinafter LH), using Braham, R. R., Jr., 1990: Snow particle size spectra in lake effect Brown, P. R. A., and P. N. Francis, 1995: Improved measure- Chase, R. J., and Coauthors, 2018: Evaluation of triple-frequency relationship for aggregates of unrimed radiating assemblages of den-

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DISCUSSION

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