A Calculation of Percentage Area Covered by Convective Clouds from Moisture Convergence

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ABSTRACT

In large-scale tropical flows an estimate of the percentage area covered by active convective cloud elements may be made according to certain theoretical formulations involving parameterization of convective-scale motions. Such an example is Kuo's model cloud which is based on replacing the ambient tropical atmosphere with a moist adiabatic cloud element over a portion of a synoptic-scale grid network. Quantities such as friction layer mass convergence, moisture convergence, and other relevant parameters like net moisture convergence over vertical columns extending over the depth of the troposphere are determined to evaluate the percentage area occupied by convective clouds.

An application to a case study of an easterly wave below a cold low is made. Some radar pictures are presented for comparison purposes.

1. Kuo's theory of parameterization

According to Kuo (1965) the fraction of a unit synoptic-scale area covered by active (newly formed) convective elements is given by the relation

$$a=I/Q, \tag{1}$$

where I is the net moisture convergence in a vertical column extending from the top of the friction layer to the top of the atmosphere. We may define I by the relation

$$I=\frac{1}{g}\int_{p_B}^{p_T}\nabla\cdot q\mathbf{V}dp-\frac{w_B q_B}{g}, \tag{2}$$

where q is the specific humidity, p_B the pressure on top of the friction layer (assumed equal to 800 mb), and p_T the pressure at the top of the atmosphere (assumed equal to 100 mb). The subscript B stands for quantities on top of the friction layer. w is vertical velocity, g the acceleration of gravity, and $\mathbf{V}$ the horizontal velocity on a pressure surface.

The quantity Q is the net amount of moisture needed to produce a model cloud over the entire unit synoptic-scale grid. The unit grid in our studies is an area approximately 2° latitude square. Kuo defines Q as the sum of two integrals, Q_1 and Q_2, which are, respectively, the amounts of moisture convergence required to i) saturate and ii) warm the column according to simple moist adiabatic ascent considerations. Thus,

$$\left.\begin{aligned} Q_1&=\frac{1}{g\Delta t}\int_{p_B}^{p_T}(q_s-q)dp \\ Q_2&=\frac{1}{g\Delta t}\int_{p_B}^{p_T}\frac{C_p}{L}(T_s-T)dp \end{aligned}\right\}, \tag{3}$$

where C_p is the specific heat at constant pressure, L the latent heat from the liquid to the vapor phase, and q_s and T_s are, respectively, the saturation specific humidity and temperature of a moist adiabatically ascending parcel; they are determined from a knowledge of T_B and q_{sB}. In the vertical above the friction layer (in the cloud region), we assume a conservation of equivalent potential temperature θ_e defined by the relation

$$\theta_e=\theta\exp\left(\frac{Lq_s}{C_pT_s}\right). \tag{4}$$

Along a moist adiabat the stability relation

$$-C_p\frac{T}{\theta_e}\frac{\partial\theta_e}{\partial p}=-\frac{\partial}{\partial p}(gz+C_pT_s+Lq_s) \tag{5}$$

is valid, and hence the following equations were solved in an iterative manner for the three variables T_s,

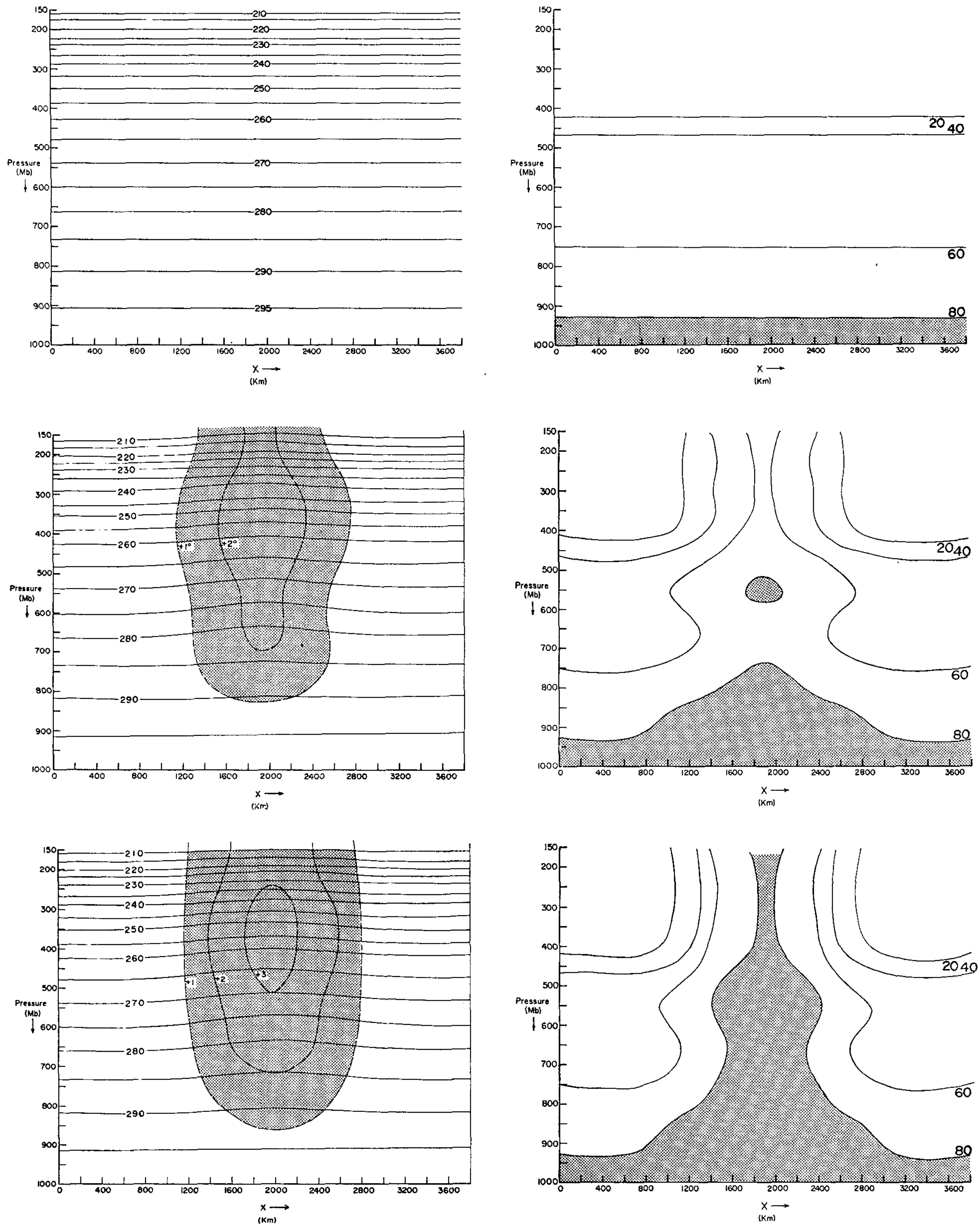


FIG. 1. Distribution of temperature (left) and relative humidity (right) in a vertical plane (x,p) for $t=0$, 6 and 12 hr. Vertical grid distance is 50 mb, horizontal, 200 km. The evolution of large-scale temperature and moisture distribution from sub-grid scale process is illustrated.

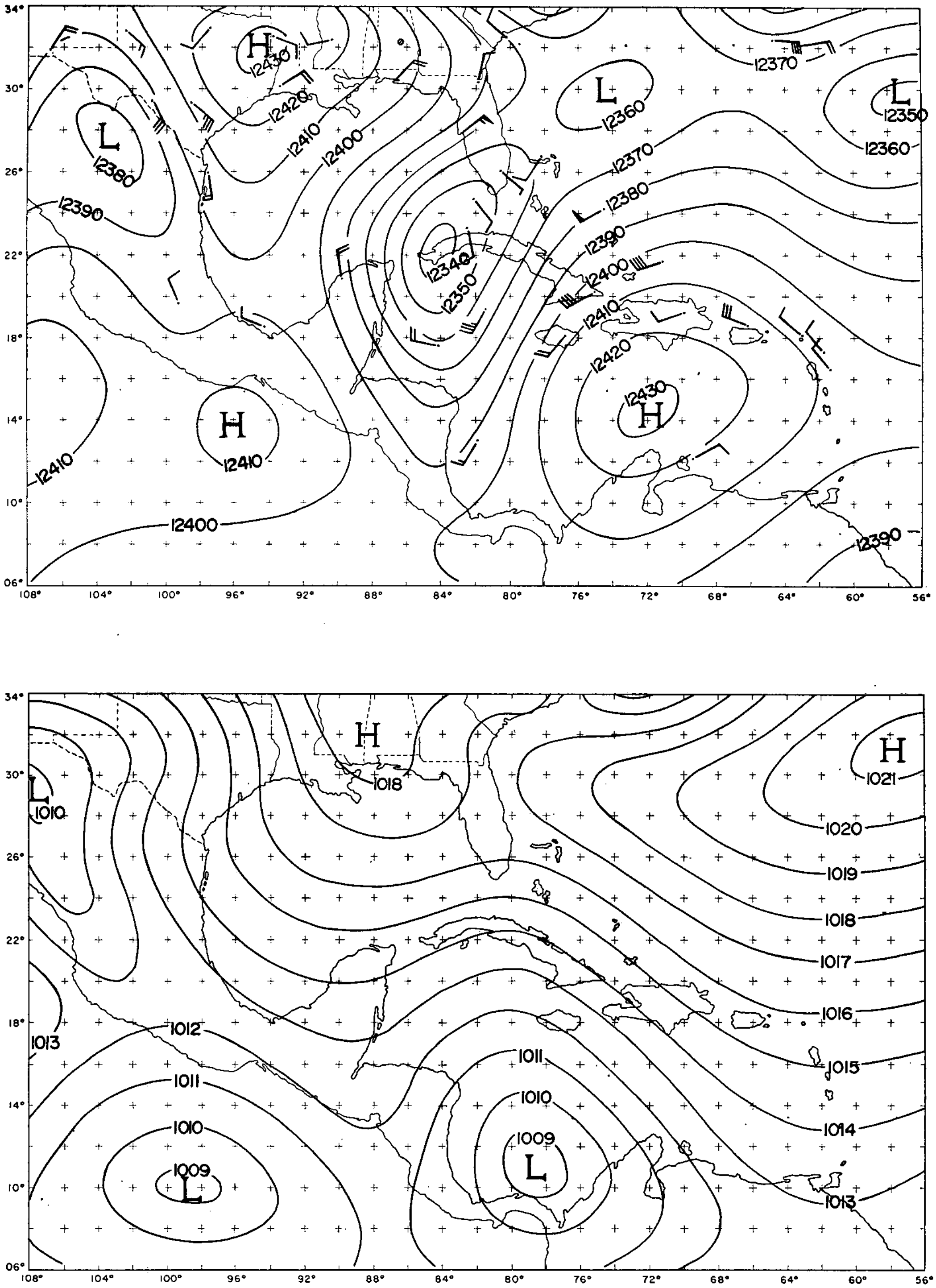


FIG. 2. 200-mb geopotential heights and surface isobars for 0000 GMT 14 August 1961. Contours are analyzed for every 10 m, and isobars for every millibar.

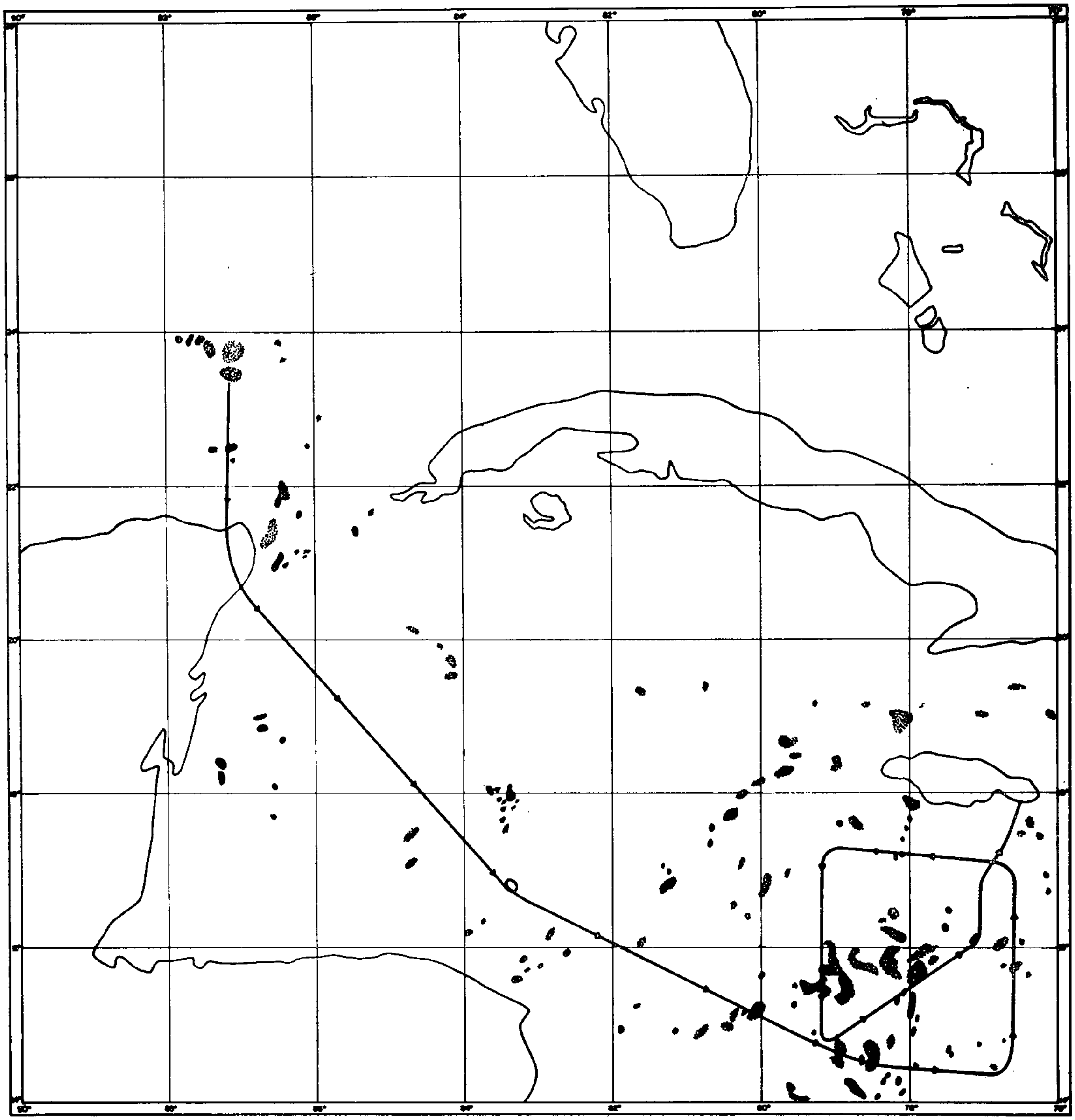


Fig. 3. Radar composite chart for 13–14 August 1961. The flight (track shown) originated from Miami and terminated at Kingston. Hard and light echoes are marked by different shading. This chart was obtained through the courtesy of Dr. William Gray of Colorado State University.

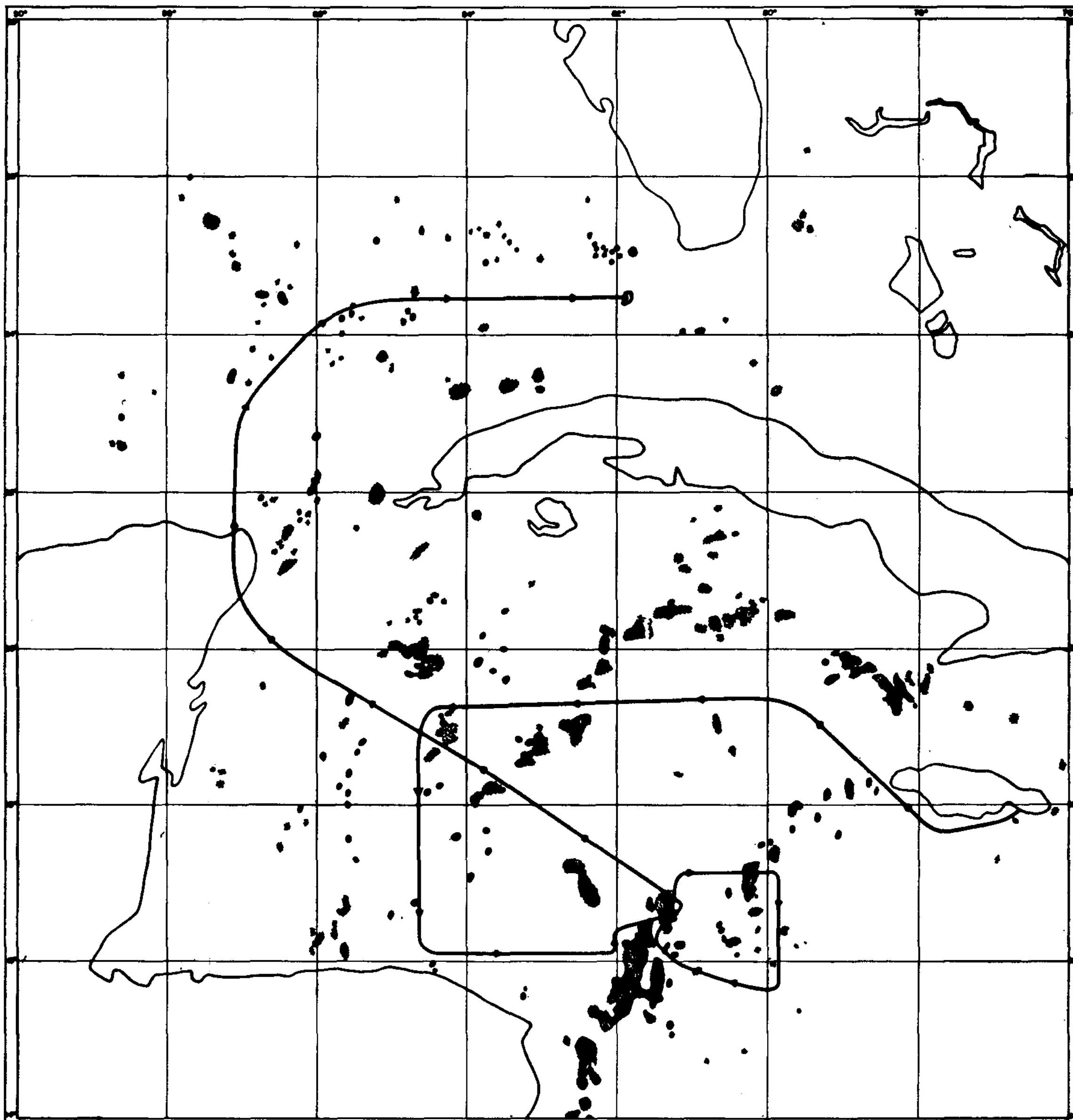


FIG. 4. Radar composite chart for 13–14 August 1961. The flight (track shown) originated from Kingston and terminated at Miami. Hard and light echoes are shown by different shading. The chart was obtained through the courtesy of Dr. William Gray of Colorado State University.

e_s and q_s:

$$gz+C_pT_s+Lq_s=C_1, \qquad (6)$$

$$e_s=6.11\, e^{25.22(1-273/T_s)}\left(\frac{273}{T_s}\right)^{5.31}, \qquad (7)$$

$$q_s=0.621\, e_s/(p-0.379e_s), \qquad (8)$$

where C_1 is a constant determined from the condensation level. Eq. (7) is a well-known relation for expressing saturation vapor pressure as a function of temperature and may be obtained from the Clausius Clapeyron equation. A similar calculation was performed by Kuo (1965). Δt is an arbitrary parameter, a measure of the time required to form a cloud element. We may arbitrarily assign any value, of the order of few hours, to represent this time scale (anywhere from 1–6 hr). The results do, to a certain degree, depend on the choice of this parameter, but as we shall show, this is not very critical if we are aware of this arbitrariness of Δt.

2. An application to an idealized situation

We present an idealized example in a two-dimensional (x,p) plane of the evolution of the synoptic-scale temperature and moisture distribution for a given fixed distribution of $a(x)$, the fractional area covered by convective clouds. Fig. 1, top left and top right, respectively, denotes an initial distribution of temperature T and relative humidity in a vertical plane with the horizontal vertical grid distances of 200 km and 50 mb, respectively. This initial distribution of temperature is that of a mean tropical atmosphere and the moisture distribution is somewhat arbitrary. It shows relative humidity $>80\%$ below 900 mb, similar to conditions in the tropics. Along the horizontal axis we assume $a(x)$ to be zero for the first five and the last five grid points and a linear increase from the 6th to the 10th point, with a increasing from 0.1 to 1.0% at the 11th point. At $x=12$, a is also $=1.0$, and decreases symmetrically back to zero from the 12th to the 16th point. Assuming that the large-scale forcing somehow maintains these values of a fixed for 24 hr, we have integrated the thermodynamic energy equation and a moisture conveyance equation for 24 hr to determine heating and moisture advection above the 900-mb surface, below which moisture and temperature values are assumed constant. A choice of $\Delta t=2$ hr is used in these integrations. In a matter of approximately 12 hr a near steady state was reached in the values of relative humidity, while on a synoptic scale q and T approached q_s and T_s and their differences became rather small.

In Kuo's scheme steady state is reached via a becoming zero when large moisture amounts are at first brought into a region by moisture convergence; later, convergence changes into divergence of flux of moisture because relatively drier air is brought into this region. The 6- and 12-hr evolutions of temperature and relative humidity are illustrated in Fig. 1, middle and bottom. The warm core attains an intensity of +3C and the 80% relative humidity isoline is carried up through the depth of the troposphere (a phenomenon on a synoptic scale very familiar to synoptic analysts in the tropics). It may be noted that the synoptic-scale changes in Fig. 1 were obtained by defining the sub-grid scale phenomenon according to a simple cloud model. Fig. 1 should be kept in view in the following calculations of a in actual meteorological situations. As we shall see in the case study we present, the largest values of a are only 0.6%; hence, the efficiency of the heating and of moisture advection are small. In a tropical storm (hurricane Carla, 1961) we found values of a close to 6%. According to Kuo, a should be interpreted as the percentage area occupied by newly formed convective clouds. The theory has several shortcomings:

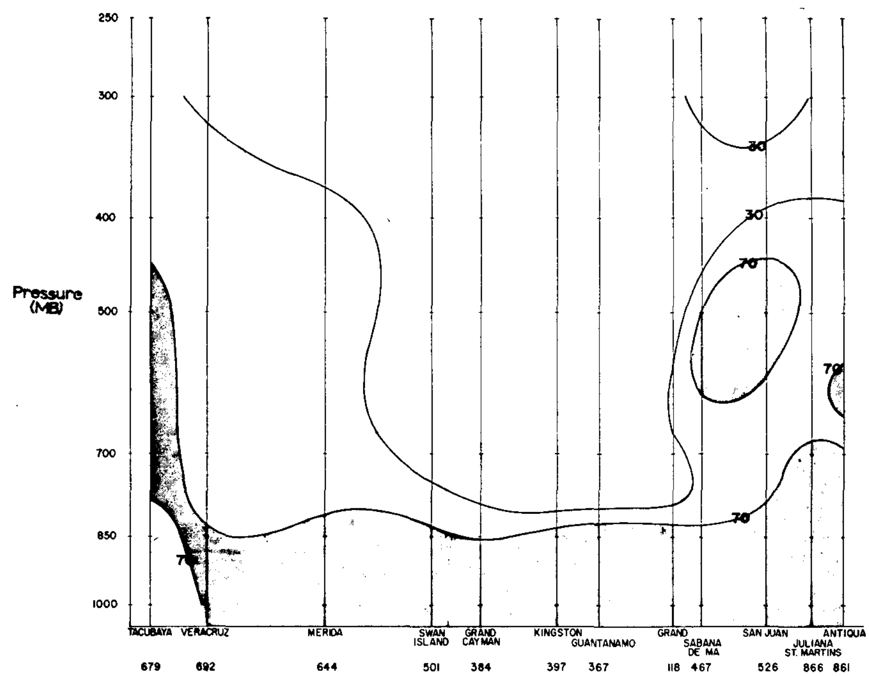
1) Moisture convergence always leads to formation of clouds in conditionally unstable states without any formal cloud physics.
2) No liquid water is permitted in the moisture conveyance equations.
3) The cloud model does not account for any entrainment.
4) Heat is released instantaneously as a function of the parameter a.
5) No dynamical effect of thunderstorm downdrafts are included.
6) The parameter Δt is somewhat arbitrary.
7) There is a tendency for saturation on a synoptic scale in this formulation which at first sight seems to be an important drawback; however, for reasons stated earlier, it does not appear to be serious.
8) No cumulus-scale vertical advection of momentum is included.

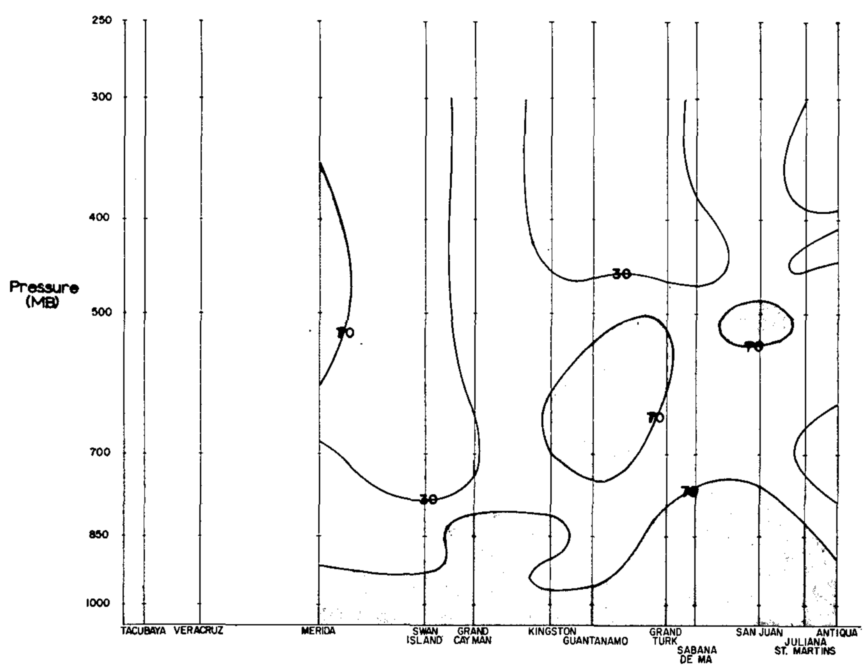
Some of these shortcomings are more obvious if one is viewing them from the point of view of cloud physics. However, when one is viewing them from the point of view of the dynamics of large-scale systems, they need not represent true shortcomings of the model since statistical influences of deep convection are being sought.

The entire theory of parameterization is thus very crude. The points in its favor are that Kuo was able to simulate the formation of a tropical storm by releasing heat of the cloud-scale motions which is easy to see from Fig. 1 presented here. We are looking for a scheme that would have wider application for studies of large-scale tropical circulations. Calculation of the parameter a may give a clue to dynamical processes of development on a large scale, at least to a degree. With this tacit assumption we are presenting results for the case studies.

3. Synoptic description of easterly wave and cold low

Fig. 2 portrays the surface and 200-mb maps for 1200 GMT 13 August 1961. The region of interest is over and south of Cuba. The synoptic situation consists of a wave perturbation of large amplitude in the basic easterly flow in the low latitudes and a closed low (a cold core low) in the upper levels. Ahead of the upper trough in the region of strong southerly flow, the wind shear vector in the troposphere is from the south-southwest and is a region of strong convective banded motions, as revealed in Figs. 3 and 4. No tropical storm formed in this situation. Figs. 3 and 4 were loaned to us by Dr. William Gray of Colorado State University. An aircraft flight made by the Flight Research Facility of the National Hurricane Research Project provided





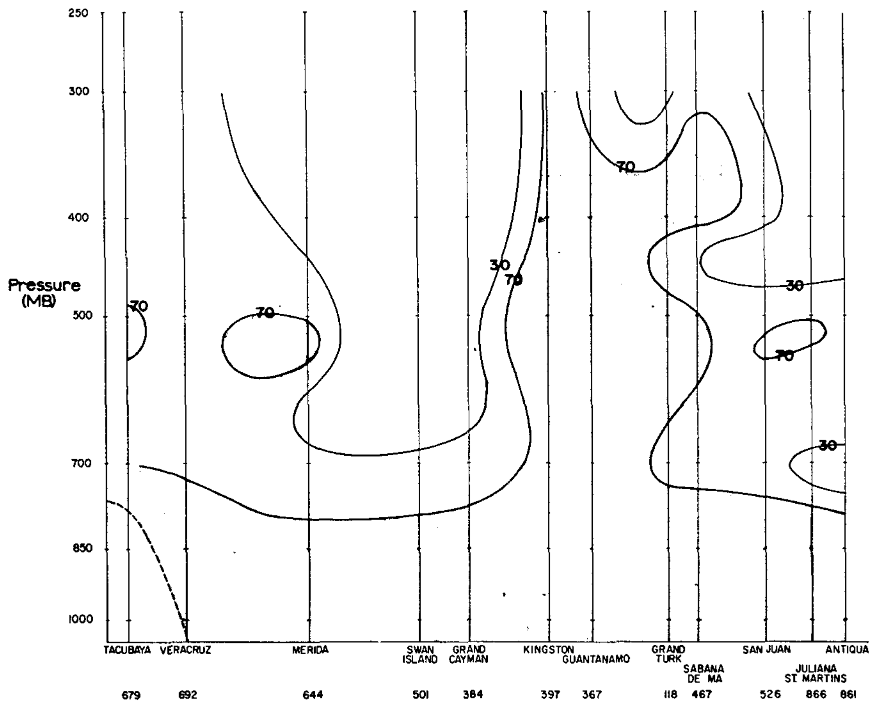


FIG. 5. Vertical cross section of relative humidity at 20N from Tacubay (Mexico) to Antigua: top, 0000 GMT 13 August 1961; middle, 1200 GMT 13 August 1961; bottom, 0000 GMT 14 August 1961. Moist regions are indicated by relative humidity $>70\%$, dry regions by values $<30\%$.

the data. The flights were made during 13 and 14 August 1961 starting from Miami to Kingston, Jamaica, and back. As mentioned, convective activity is most marked south-southwest of Cuba and in the Caribbean Sea. Several radar echoes appear very intense surrounded by weaker returns. It is possible to obtain a crude measure of the percentage area covered by hard and soft core echoes in the 2° by 2° latitude grid. Largest values for soft echoes were close to 10% and for hard echoes around 2%. A west-to-east cross section along 20N containing isolines of relative humidity is shown in Fig. 5 for three map times, 0000 and 1200 GMT 13 August, and 0000 GMT 14 August. The important feature of this cross section is the large moisture content with relative humidity $>70\%$ through the depth of the troposphere at Kingston, Guantanamo and Grand Turk Island.

Large-scale vertical motions for this case study are reported by Krishnamurti and Baumhefner (1966) and by Baumhefner (1967). The largest values of vertical velocity w were only around 10^{-3} mb sec^{-1} and could not account for the vertical advection of moisture. A more efficient small-scale rapid transfer in convection-scale motions must account for this observed large-scale distribution of moisture.

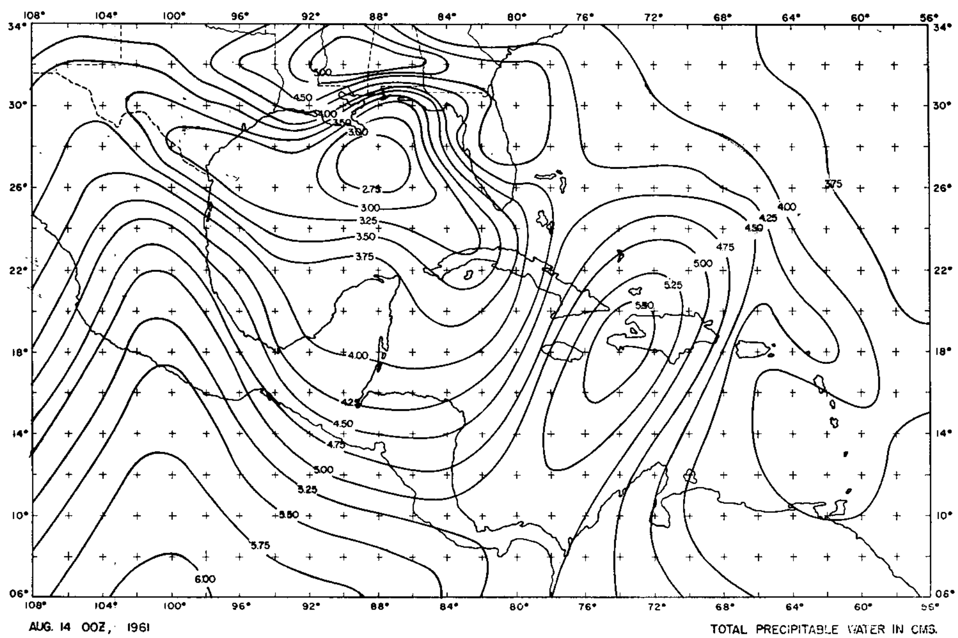
The funnelling effect of moisture on a small convective scale must undoubtedly have resulted in this large-scale distribution of moisture. The close similarity of Figs. 1 and 5 is perhaps more than a coincidence.

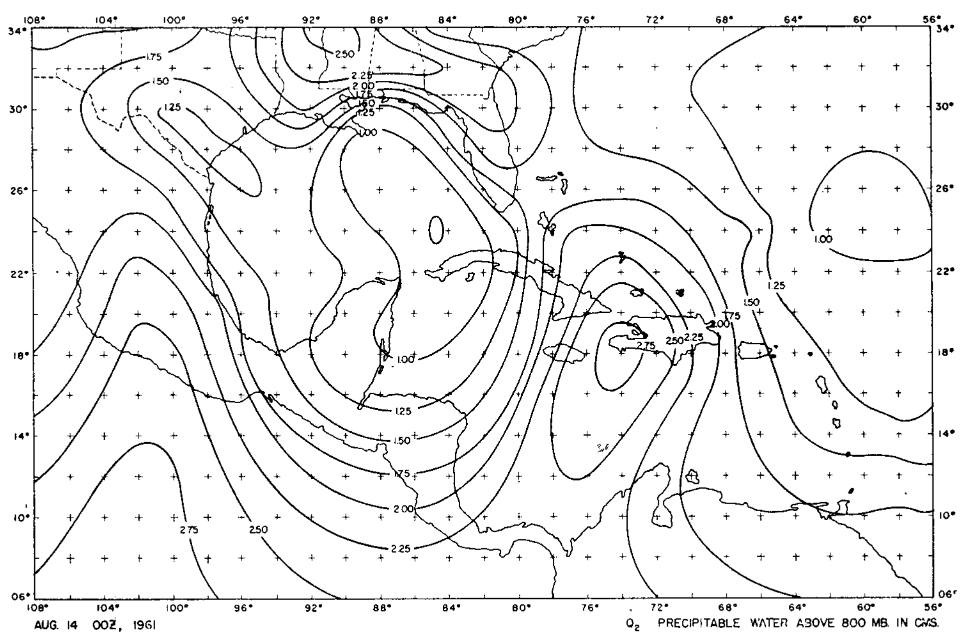
4. Some calculated fields

The calculations we present are made from synoptic-scale maps which are resolved on a 2° by 2° mesh size. Kuo in his numerical simulation study of a tropical storm has a mesh size of the order of 10 km. The calculations are meaningful only where the data density is large, the data voids in this study lying in the northeastern portion over the Atlantic and the southwestern portion over central America and the Pacific Ocean.

Precipitable water below and above the friction layer. While it is well known that large quantities of moisture in the tropics reside below the trade inversion, large amounts of moisture (after major convective activity) are also frequently found above the friction layer. The normal state above the friction layer is generally dry air with relative humidity $<50\%$. In Fig. 6 we present a somewhat arbitrarily prescribed partitioning of the total precipitable water below and above the friction layer (1000–800 mb). Fig. 6, top, shows total precipitable water, middle, total precipitable water above the friction layer, and bottom, the moisture in the friction layer. These maps were obtained from an analysis of the specific humidity at the 1000-, 850-, 700-, 500- and the 300-mb surfaces. Of pertinent interest here is the large moisture amounts over the region of active convection (Figs. 3 and 4), where on a synoptic scale, precipitable water amounts are as large as 2.75 cm (water depth) both below and above the friction layer. In the dry region ahead of the easterly wave the air is considerably drier above the friction layer, suggesting strong subsidence above the friction layer. Most of the active convection on 14 August 1961 (the period for which the precipitable water charts were constructed) was found in the region where the gradient of the moisture field is largest (16N, 80W), while the largest magnitudes of total precipitable water are found in those regions where the convection was active in the preceding 24 hr.

Fig. 7 shows the distribution of friction layer mass divergence, divergence of flux of moisture, integrated convergence of flux of moisture, and an evaluation of the percentage area occupied by convective clouds. Overall





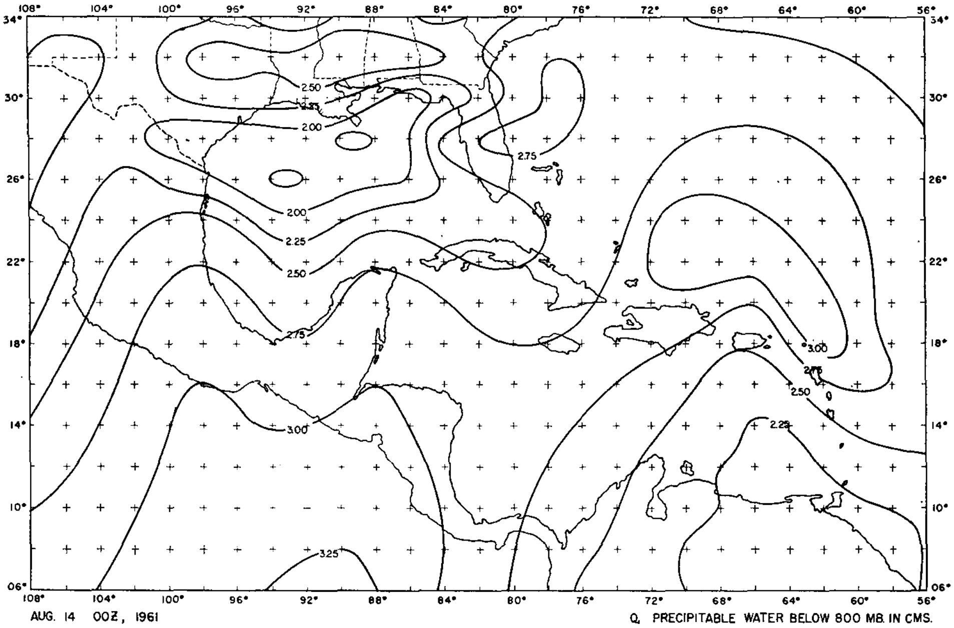


FIG. 6. Total precipitable water (cm) for 0000 GMT 14 August 1961: top, total; middle, above 800 mb; bottom, below 800 mb.

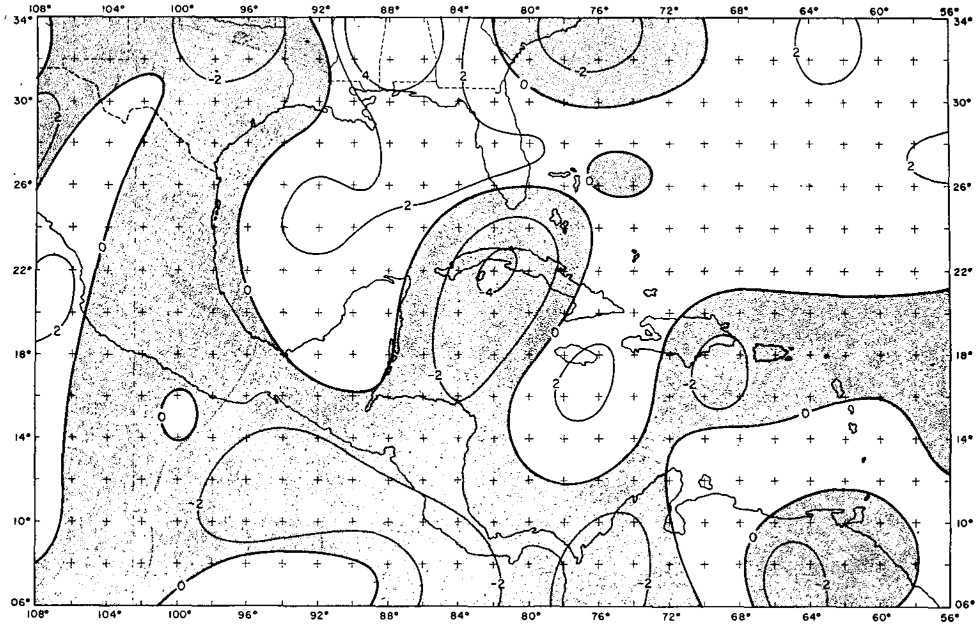
in the friction layer there is a good correlation between mass and moisture convergence, but there are regions of discrepancy. North of the Gulf of Mexico, where horizontal advection of moisture is large, the fields of $\nabla\cdot\mathbf{V}$ and $\nabla\cdot q\mathbf{V}$ are not too well correlated. In the region of interest south of Cuba there are fields of strong mass and moisture convergence and over the Gulf of Mexico, the dry belt of Fig. 6, we find strong mass and moisture divergence in the friction layer. Further, Fig. 7c shows the distribution of the net moisture convergence I in columns. Only $I>0$ has physical significance here, so the region $I<0$ is not analyzed. The typical largest magnitudes of I are of the order 200×10^{-8} mb m^{-1} sec. The distributions of moisture convergence $-\nabla\cdot q\mathbf{V}$ in the friction layer and that of I are very similar, as is to be expected. Finally, Fig. 7d shows the distribution of a, the percentage area occupied by convective clouds. It was somewhat unexpected that the largest values of the percentage area covered by convective clouds came out around 0.6%, or near 1%. This number is directly proportional to the parameter Δt which we had assumed equal to 2 hr. Since Δt cannot be larger than 5–6 hr in any case, we may state that the active convection is only present over 1–2% of the area of the synoptic-scale grid network. The correspondence to what is observed as hard echoes in the radar composite charts, Figs. 3 and 4, is thus quite encouraging. In Fig. 7 we have excluded calculations in regions where $I<0$, because the parameterization of small-scale convection is made only when the two criteria

$$-\frac{RT}{p\theta_e}\frac{\partial\theta_e}{\partial p}<0, \quad I>0$$

are satisfied.

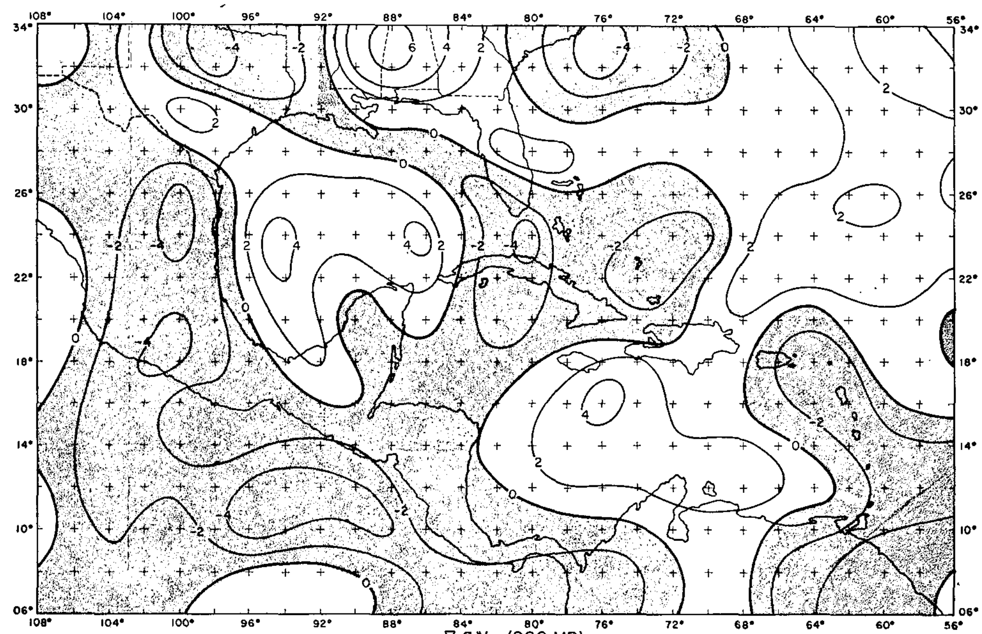
5. Concluding remarks

This is a preliminary study of the problem of interaction of the large- and small-scale motions in the tropics. The large scale is described from data on a synoptic scale through analysis of weather systems. The small-scale effects are incorporated as a function of the large-scale through use of models such as those of Kuo. Frictional convergence of mass and moisture in the lowest layers appears to have a strong correlation to the observed cloud distribution (radar). To perform more studies of this kind, maps of radar echoes in and around tropical disturbances are highly desirable. This is often available when special reconnaissance missions are flown into such systems. This is an expensive proposition, but experiments such as the Line Island and the Barbados projects may provide some possibilities for research on this problem in greater detail. Estimates of vertical motion, mass and moisture convergence, and cloud parameters like I and a can be made through use of sophisticated numerical models such as those used in this study. Great care should be exercised in interpretation of the results if a simple kinematic approach is used for this purpose, because large-scale vertical mo-



∇· V (900 MB)

(a)



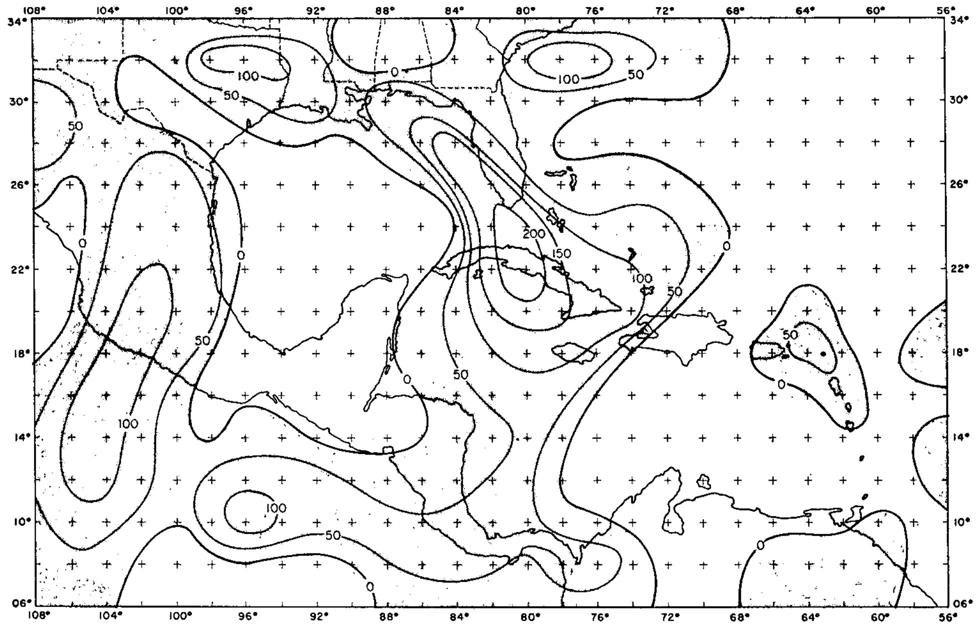
∇·qV (900 MB)

(b)

FIG. 7. Friction layer mass divergence (10^{-6} sec^{-1}), a.; friction layer moisture divergence (10^{-8} sec^{-1}), b.; net moisture convergence in columns extending from top of friction layer to 100 mb (10^{-8} mb m^{-1} sec), c.; percentage area covered by convective clouds (estimated according to Kuo's theory), d.

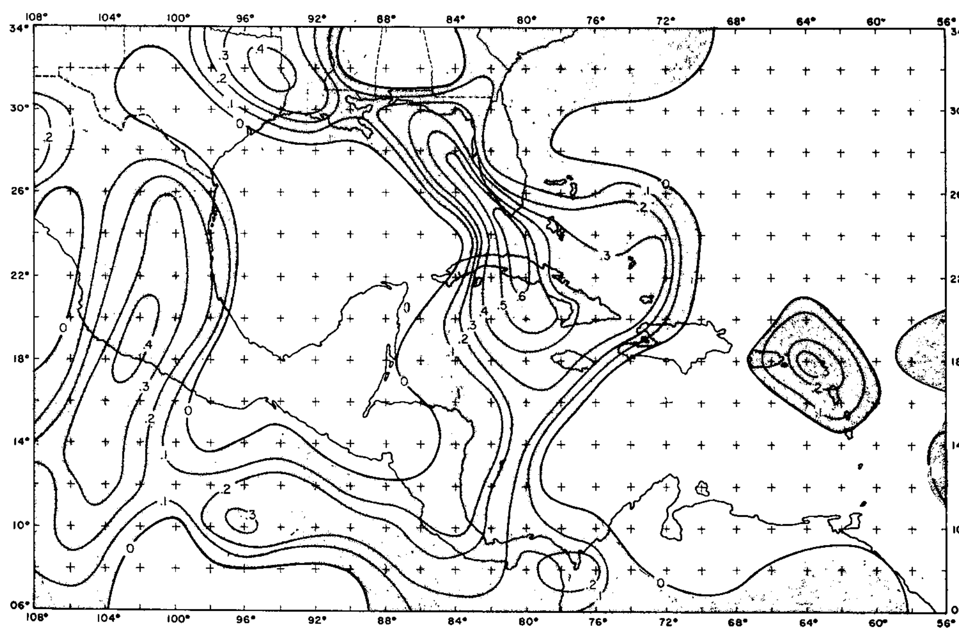
tions in the tropics are generally very small, and small errors in the estimation of divergence can give quite erroneous fields of these quantities.

This problem is more interesting when the effect of this parameterization process is included in a numerical prediction problem. Questions such as the stability of the scheme, over-saturation of the convergence regions, and development through moist conditional instability can only be answered by incorporating the procedure in a forecast model. The author is currently working on this problem and will be reporting the results of these studies in other papers to follow.



CONVERGENCE OF FLUX OF MOISTURE IN COLUMNS

(c)



PERCENT AREA COVERED BY CLOUDS

(d)

FIG. 7 (*continued*)

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