Reply

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Careful consideration was given to the comments of Thompson relative to the thin-film mounted rocketsonde thermistor. It would certainly be improper to knowingly use the stated corrections to the atmospheric temperature measured by the STS instrument (Ballard, 1966; Ballard, 1967a) if these temperature corrections were as large and uncertain as Thompson has stated.

To understand the basis for Thompson's comments, a detailed study was made of the paper by Thompson and Keily (1967). Thompson's other reference (Thompson, 1966), primarily an extensive laboratory study of the heat transfer characteristics of thermistors with various lead lengths, had been studied previously in connection with a review of seven papers (Ballard, 1967b) dealing with the measurement of temperature in the 60–30 km region of the atmosphere.

It is believed, after consideration of Thompson's comments, plus the study of the work upon which he based them, that there is little justification for Thompson's last conclusion and that Ballard's results do give a reliable indication of the corrections to be applied to the temperatures measured by the thin-film mounted thermistor of the STS instrument.

The justification for the above statement, plus an answer to each of Thompson's comments, are presented in that which follows.

The heat transfer equation is given here [Eqs. (1) and (1a)] for a thermistor which is attached to an instrument package by some type of mounting configuration, the instrument in turn being supported at an altitude z by a parachute which is moving with a speed v relative to the atmosphere. The leads are considered "short enough" that the rate of heat transfer from the cylindrical lead wires by convection as well as solar irradiation of the lead wires is negligible. The terminology "short enough" is discussed.

This equation, plus its steady state solution given in Eq. (2), serves to indicate the fundamental ideas used in the design of the STS instrument. The steady state solution to the equation which includes the effect of convection from thermistor lead wires as well as their solar irradiation is next given in Eq. (3), and Eqs. (2) and (3) are compared. Eq. (3) is then utilized to calculate the corrections to the observed thermistor temperature at altitudes of 65, 60 and 50 km for a 10-mil spherical (2.54×10^{-2} cm diameter) thermistor which is attached to a thin-film mount by leads which are 3 mm in length (the STS instrument). The values stated by Thompson and Keily are used for the pertinent parameters that occur in Eq. (3). These results then serve as a basis for an answer to each of Thompson's comments and for making a comparison between the correction values calculated originally by Ballard and the correction values obtained when the effects noted by Thompson are included.

Following the steps outlined, the steady-state heat transfer equation for the thermistor is

$$\sigma\epsilon_l A_t T_e^4+(2kA_c/L)[(T_s-T_e)-(T_t-T_e)]$$
$$+h_t A_t[T_e+rv^2/(2c_p)-T_t]$$
$$+\tfrac{1}{4}J\epsilon_s A_t-\sigma\epsilon_l A_t T_t^4=0, \quad (1)$$

where ϵ_l is the thermistor long wave emissivity, A_t the thermistor surface area, A_c the lead cross-sectional area, ϵ_s the thermistor short-wave emissivity, T_s the thermistor support temperature, T_e the environment temperature, and T_t the thermistor temperature.

Let $\Delta T_{se}=T_s-T_e$, $\Delta T_{te}=T_t-T_e$ with $\Delta T_{te}\ll T_e$. Then Eq. (1) becomes

$$4\sigma\epsilon_l T_e^3\Delta T_{te}+(2kA_c/L)\Delta T_{te}+h_t A_t\Delta T_{te}$$
$$=h_t A_t rv^2/(2c_p)+(2kA_c/L)\Delta T_{se}+\tfrac{1}{4}J\epsilon_s A_t. \quad (1a)$$

An algebraic solution of Eq. (1a) for the difference in temperature ΔT_{te}, between the thermistor and the environment, results in

$$\Delta T_{te}=\frac{h_t A_t rv^2/(2c_p)+(2kA_c/L)\Delta T_{se}+\frac{1}{4}J\epsilon_s A_t}{4\sigma\epsilon_l A_t T_e^3+(2kA_c/L)+h_t A_t}. \quad (2)$$

Eq. (2) shows that the total correction to the thermistor temperature is the sum of the aerodynamic heating correction, the correction brought about by heat conduction along the thermistor leads (which in turn is dependent upon the temperature difference between the thermistor support and the environment), and the correction due to incident solar radiation. Perhaps it is worthy of note that the correction for aerodynamic heating is not $(rv^2/2c_p)$ but $(h_t A_t/S_{tm})rv^2/2c_p$ where S_{tm} is the denominator of Eq. (2) [Ballard's Eq. (3.4)] which gives the total heat transfer rate for the processes of conduction, convection and radiation. The quantity $rv^2/2c_p$ is the aerodynamic heating correction in the absence of any heat transfer process other than convection.

The corresponding correction to the thermistor temperature to include convective heat transfer from the thermistor leads and their solar irradiation is given as

$$\Delta T_{te}=\frac{h_t A_t rv^2/(2c_p)+(2kA_c/L)(pL)(\operatorname{csch}pL)\Delta T_{se}+\frac{1}{4}J\epsilon_s A_t+(4\epsilon_s JRL/pL)[\coth pL-\operatorname{csch}pL]}{4\sigma\epsilon_l A_t T_e^3+(2kA_c/L)(pL)\coth pL+h_t A_t}. \quad (3)$$

The omission of $h_tA_t(rv^2/2c_p)$ from the numerator and $4\sigma\epsilon_lA_tT_e^3$ from the denominator of (3) gives Thompson and Keily's Eq. (6).

The quantity pL determines the conditions under which Eq. (3) reduces to Eq. (2) and the leads are then "short enough." In the notation of Thompson and Keily, let $p=(2h/kR)^{\frac{1}{2}}$, where h, k and R are, respectively, the convective heat transfer coefficient, the thermal conductivity, and the radius associated with the thermistor lead wires. If $pL \ll 1$, then $\coth pL$ and $\operatorname{csch} pL$ approach $1/(pL)$. From the values stated by Thompson and Keily for a 10-mil thermistor ($R=1.3\times10^{-3}$ cm) with leads 3 mm in length, the quantity $pL \simeq 0.7$, corresponding values of $1/(pL) \simeq 1.4$, $\coth 0.7 \simeq 1.7$ and $\operatorname{csch} 0.7 \simeq 1.4$. For leads 1.5 mm in length, $1/(pL)=1/0.45$, $\coth 0.45=1/0.42$, while $\operatorname{csch} 0.45=1/0.47$. Thus, for thermistor leads 1.5 mm in length or shorter and with the stated characteristics, Eq. (3) reduces to Eq. (2).

To obtain numerical values for ΔT_{te}, the difference between the thermistor and environment temperatures, numerical values must be assigned in Eq. (3) to ΔT_{se}, the difference between the support and environment temperatures. It was believed at the time Ballard's material was submitted for publication that the most uncertain quantity was the excess temperature of the film mount relative to the environment temperature as caused by aerodynamic heating of the film. This problem was studied, and the results presented (Rubio and Ballard[1]). The dimensions of the film mount place it in the continuous flow regime while in the 65–30 km region of the atmosphere with a corresponding recovery factor, $\sqrt{\mu c_p/k}=\sqrt{\Pr}=0.9$. Based upon this value, the $rv^2/2c_p$ *temperatures* of the film at altitudes of 65, 60 and 50 km were calculated to be 11.9, 6.6 and 1.6C, respectively. From the principle indicated in Eq. (2), the quantity $\Delta T_{se}=(h_fA_f/S_f)(r_fv^2/2c_p)$ is less then $r_fv^2/(2c_p)$ by virtue of $4\sigma\epsilon_{lf}A_fT_e^3$ occurring in the expression for the dissipation factor of the film [Ballard's Eq. (3.1)]. Thus, the values of ΔT_{se} for altitudes of 65, 60, and 50 km, corresponding to the respective temperatures of 11.9, 6.6 and 1.6C were calculated to be 7.5, 4.3 and 1.1C, respectively. Based upon the comments of Thompson concerning the effective long-wave emissivity of mylar (subsequently measured to be 0.7 rather than 1.0), the numbers of 7.5, 4.3 and 1.1C should be increased by a factor of approximately 1.1 to give values of 8.3, 4.7 and 1.2C for ΔT_{se} at altitudes of 65, 60 and 50 km, respectively. These values for ΔT_{se} are used in Eq. (3) for the determination of ΔT_{te} at these altitudes.

From Eq. (3) at 65 km, with $rv^2/(2c_p)=15.0$C, $pL=0.7$, $L=0.3$ cm, $\operatorname{csch}0.7=1.4$, $\coth0.7=1.7$, $\Delta T_{se}=8.3$C, $4\sigma\epsilon_lA_tT_e^3=1$ μW $(°\text{C})^{-1}$, $2kA_c/L=10$ μW $(°\text{C})^{-1}$, $h_tA_t=3\mu$W $(°\text{C})^{-1}$ and $J=0$ (corresponding to a shaded thermistor), $\Delta T_{te}=7.9$C. This is the correction at 65 km to the observed thermistor temperature for the effects of aerodynamic heating of the thermistor and thermistor mount as well as heat conduction between the thermistor and mount.

From Eq. (3) at 60 km, with $rv^2/2c_p=9.0$C, $pL=1.0$, $L=0.3$ cm, $\operatorname{csch}1.0=0.9$, $\coth1.0=1.3$, $\Delta T_{se}=4.7$C, $4\sigma\epsilon_lA_tT_e^3=1$ μW $(°\text{C})^{-1}$, $2kA_c/L=10$ μW $(°\text{C})^{-1}$, $h_tA_t=6.3$ μW $(°\text{C})^{-1}$ and $J=0$ (corresponding to a shaded thermistor), $\Delta T_{te}=4.8$C. This is the correction at 60 km to the observed thermistor temperature for the effects of aerodynamic heating of the thermistor and thermistor mount as well as conduction between the thermistor and mount.

From Eq. (3) at 50 km, with $rv^2/2c_p=1.7$C, $pL=1.5$, $L=0.3$ cm, $\operatorname{csch}1.5=0.5$, $\coth1.5=1.1$, $\Delta T_{se}=1.2$C, $4\sigma\epsilon_lA_tT_e^3=1$ μW $(°\text{C})^{-1}$, $2kA_c/L=10$ μW $(°\text{C})^{-1}$, $h_tA_t=15.5$ μW $(°\text{C})^{-1}$ and $J=0$ (corresponding to a shaded thermistor), $\Delta T_{te}=1.1$C. This is the correction at 50 km to the observed thermistor temperature for the effects of aerodynamic heating of the thermistor and thermistor mount.

Thompson's comments are now considered relative to the emissivity of the film, convection to the air from the thermistor lead wires, and aerodynamic heating of the thermistor and the thermistor mount. The calculation based upon Eq. (3), which accounts for all of these effects, gives a value of $\Delta T_{te}=7.9$C at 65 km as opposed to a value of $\Delta T_{te}=12$–15°C at 65 km, as claimed by Thompson. The difference here lies in the failure by Thompson to recognize that the calculated $rv^2/2c_p$ temperature is not the aerodynamic heating correction when processes of heat transfer other than convection are active. This is shown in Eqs. (2) and (3). Consideration of the local effects at the point of attachment of the bead thermistor lead wires to the film and radiation from lead wires 0.3 cm in length and 2.5×10^{-3} cm diameter indicates a modification to the above calculated corrections of a few tenths of a degree, a refinement which has not been incorporated, since in the reduction of the temperature data, the data is smoothed such that a recorded temperature point may differ by as much as, but no more than, 2C from an actually observed point.

Relative to Thompson's various comments concerning the solar radiation correction, calculations by a number of investigations (Ballard, 1967b) indicate that the corrections to the observed temperature in the 65–30 km region of the atmosphere as caused by long-wave radiation also amount to a few tenths of a degree. In light of the temperature reduction techniques discussed above, this refinement in the temperature corrections is considered negligible. In addition, it was realized at the time of construction of the STS instrument that the application of a reflective coating of aluminum could be detrimental rather than helpful in reducing the solar radiation correction by reason of the heat energy absorbed in

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[1] Rubio, Robert, and H. N. Ballard, 1966: Time response and aerodynamic heating of atmospheric temperature sensing elements. Paper presented at Amer. Meteor. Soc. Conference on Dynamic Structure of the Free Atmosphere, El Paso. Tex.

the aluminum film, as Thompson suggests. At the same time the aluminum coating was helpful in disseminating over the film surface the heat energy transmitted from the thermistor leads to the film. Many laboratory tests were conducted in this regard, and it was found that an aluminum thickness of 400Å would partially reflect the incident radiation while producing a negligible temperature rise of the film since the mass of the aluminum coating was approximately one-hundredth that of the mylar film. The discussion of this matter in Ballard's paper indicates that the magnitude of the temperature corrections should lie between 4C for uncoated mylar to 1C for the coated mylar. Thompson in his comments indicates that in his opinion the temperature rise of the coated film should be 20–40 times the above value, thus amounting to observed temperature oscillations ranging from 20–40C. This is probably an extreme estimate on the part of Thompson. Based upon an examination of numerous temperature records, temperature oscillations with amplitudes of approximately 3C are observed at 60 km, decreasing in amplitude to approximately 1C at 50 km. These oscillations do not occur on temperature records obtained at night so they are at least solar induced. Recent experiments with solar shields indicate that they are produced by solar radiation directly incident on the thermistor and film mount. If Eq. (3) is used to determine the effects of direct solar radiation upon the thermistor, thermistor leads and thin-film mount, the calculated temperature increase is 4.5C at 65 km, 2.5C at 60 km and 1.2C at 50 km, agreeing quite well with the amplitude of the observed thermistor temperature oscillations. The above calculations are based upon an absorption of solar energy by the film at the rate of 28,800 μW, corresponding to the conditions discussed by Ballard where $\frac{1}{4}J\epsilon_s A_t = 16$ μW, $4\epsilon_s JRL = 32$ μW with $L = 0.3$ cm, and with all other values as previously stated for the specified altitudes. Thus, there appears to be nothing really dubious, as Thompson maintains, in taking the temperature minima in the oscillatory temperature-time record as the environmental temperature corrected for incident solar radiation. This problem has been discussed in a comment by Lindzen (1967) and in a reply by Beyers and Miers (1967) concerning the oscillations observed on the temperature-time traces obtained with the Delta-I temperature sonde.

These comments of Thompson and the replies to them are the only ones which have a bearing on the magnitude of the calculated temperature corrections as stated by Ballard.

In addition, Thompson questions the assumptions of Ballard in Eq. (5.2b) that $T_{f2} = T_{f1}$. It is doubtful that Thompson's assumption is realistic since he does obtain a negative heat transfer coefficient. Consideration of the results of the calculations based upon Eq. (3) for the altitudes of 65, 60, and 50 km indicates that the equilibrium thermistor temperature is quite close to the equilibrium film temperature, the difference being no more than 0.5C over the altitude interval. Thus, the more nearly correct assumption would probably have been that the thermistor and film temperatures are equal. Consideration of Ballard's Eqs. (5.2b)–(5.5) shows that this would have the effect of reducing the semi-empirically determined thermistor convective heat transfer coefficient at 70 km by approximately one-half, from 1.1×10^{-4} to 0.5×10^{-4} cal cm^{-2} sec^{-1} (°K)$^{-1}$. Either value is probably reasonable when compared with a theoretical value for $h = 4.0 \times 10^{-4}$ cal cm^{-2} sec^{-1} (°K)$^{-1}$ at 65 km.

Finally, the assumptions made in Ballard's Eq. (5.9) were for the purpose of establishing, in the absence of knowledge of the extent of aerodynamic heating of the thin-film mount, *lower* and *upper limits* for the temperature correction to the thermistor at 65 km as required by aerodynamic heating. This is all that is indicated in Eq. (5.10). Substitution of $\Delta T_{se} = 0$ and $\Delta T_{se} = \Delta T_{te}$ into Eq. (2) of this reply, along with values of the pertinent parameters stated for 65 km, gives the same result as indicated in Ballard's Eq. (5.10). The calculation made through use of Eq. (3) of this reply for 65 km indicates the actual correction to the thermistor temperature for aerodynamic heating of the film mount and thermistor to be 7.9C, which satisfies the inequality $5\text{C} < 7.9\text{C} < 15\text{C}$.

Perhaps a comment is now in order relative to the work of Thompson and Keily since their results served as a basis for Thompson's comments. Calculations based upon their Eq. (6) for the temperature increase of the thermistor when solar radiation is incident upon both the thermistor and thermistor leads, show that the theoretical temperature increase should be approximately 5C. This is based upon their values for all parameters occurring in Eq. (6). Their experimental results show an increase of temperature for the 15-mil thermistor and thermistor leads at 65 km of approximately 9C (Fig. 4, Thompson and Keily). A similar calculation for the 10-mil aluminized rod gives a theoretical thermistor temperature increase of approximately 2C at 65 km, whereas their experimental results indicate a temperature increase of approximately 4C at 65 km. Thus, in both cases the experimentally determined values deviated considerably from the theoretically determined values for the temperature increase.

In addition, these results of Thompson and Keily, as related to the lesser temperature rise of the 10-mil rod when the rod and its leads were irradiated, demonstrate clearly the principle utilized in the design of the STS instrument. It would be expected, when the larger cross-sectional area associated with the 10-mil rod and its larger diameter leads were exposed to sunlight, that the temperature increase of this thermistor would be greater than that experienced by the 10-mil sphere if no other effect were active. As indicated above, such was not the case. Their result is explained by Thompson and Keily with the statement "... but at high altitudes the measured temperature rise is strongly dependent on the mounting conditions used in the experiment .... Thus, the lower measured temperature rise at high altitude does not represent an improvement but merely that the support post temperature has excessive influence on the rod's temperature."

An evaluation of the quantity pL at 65 km (Thompson and Keily's quantity pd) for a 10-mil thermistor with a lead length of 0.3 cm gives a value of $pL \simeq 0.68$. Evaluation of the quantity pL for a 10-mil rod thermistor with a lead length of 1.22 cm (as used in Thompson and Keily's experiment) gives a value for $pL \simeq 0.64$. It follows that the thin-film mount also has considerable influence on the temperature of its thermistor.

In summary, replies have been made to each of Thompson's comments. Calculations based upon Eq. (3), which includes the effects noted by Thompson, indicate that the correction to the thermistor at 65 km, for aerodynamic heating of the thermistor and thin-film mount, is −7.9C. This is considerably smaller than the −15C suggested by Thompson and 1.7C larger than the value determined by Ballard. Correction values based upon Eq. (3) are −7.9C at 65 km, −4.8C at 60 km, and −1.1C at 50 km. The corresponding values determined originally by Ballard were −6.2C at 65, −3.8C at 60 km, and −1.1C at 50 km, thus showing these values are at least a reliable indication of the temperature corrections, especially when viewed in the light of an uncertainty of at least ±2C as associated with any temperature point as recorded in the present temperature data reduction procedures.

Note that throughout this reply, the word correction has been used in place of the word error as used by Thompson. Although either terminology does not affect the numerical values involved, it is believed that numerical correction, as used here, properly implies the addition or subtraction of a numerical value to an observed value, the correction (not error) resulting from some known physical process.

Thompson's comments are appreciated since it has been demonstrated that the effects to which he calls attention must be considered in determining the necessary corrections to a thin-film mounted rocketsonde thermistor.

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